COMPUTATION OF INITIAL CONDITIONS FOR A THIN DISC AS AN EXTENSION OF DEHNEN ALGORITHM

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Spiral galaxies are characterized by having an angular momentum which gives rise to their shape. The stars in these galaxies can be modeled as particles that follow, in general, elliptical trajectories.

In that case, the energy of a particle located at a radius r with angular momentum L is given by:

$$E = \frac{1}{2}m\dot{r}^{2}(t) + \frac{L^{2}}{2mr^{2}} + \psi(r), \qquad (1)$$

where $\psi(r)$ is the potencial due to the mass distribution of the galaxy. The simplest approximation to model a disk galaxy is by assuming that the orbits of the particles are circles (cold disk); nevertheless, this model does not reproduce accuracy the real trajectories of the stars in these type of galaxies, which are, in general, elliptical (warm disk). For this type of configuration, the best distribution function was proposed by Dehnen (1999):

$$F(E,L) = \frac{\gamma(r_E)\Sigma(r_E)}{2\pi\sigma_r^2(r_E)} \exp\frac{\left[\Omega(r_e)(L - L_C(E))\right]}{\sigma_r^2(r_e)},$$
(2)

where r_E is the radius of the circular orbit at energy E. Σ is the density distribution; σ_r , the velocity dispersion; L_C , the circular angular momentum and Ω , the circular frequency.

The simulated galaxy obeys to a density and a velocity dispersion profiles of the form:

$$\Sigma(r) = \Sigma_0 e^{-r/h}, \qquad (3)$$

$$\sigma(r) = \sigma_0 e^{-r/r_\sigma}, \qquad (4)$$

where h and r_{σ} are length scales. These functions were optimized by means of the Richardson-Lucy technique (Lucy 1974). Using the new density function, we calculate both the mass inside a disk of radius r and the potential. With the mass, we computed numerically the position of each particle; with the potential, we calculated the circular velocity v_c , circular energy E_c and circular angular momentum L_C ; the radial frequency κ , circular frequency ω and γ a constant related to the radial frequency. These



Fig. 1. Numerical Distribution function obtained by using the corrected density and velocity dispersion.

parameters are used to describe the trajectories of the particles in a warm disk. These are defined by the following relations:

$$v_c^2(r) = r\left(\frac{\partial\phi}{\partial r}\right); \qquad L_c^2 = r^2 v_c^2$$
 (5)

$$E_c(r) = \frac{v_c^2}{2} + \phi; \qquad \Omega^2 = r^{-2}v_c^2 \qquad (6)$$

$$\kappa^2 = 2\frac{\partial^2 \phi}{\partial r^2} + \frac{r}{2}\frac{\partial^3 \phi}{\partial r^3}; \qquad \gamma = \frac{2\Omega}{\kappa}.$$
 (7)

With the corrected density and potential functions, we constructed numerically the distribution function shown in Figure 1. Subsequently we incorporated the rejection technique (Press et al. 2007) to calculate the velocities of each particle. The energy E of the system is the circular energy and the total angular momentum L is computed by generating a random number $\zeta \in [0,1]$ and using the fact that $L = L_c + \sigma \ln(\zeta)/\Omega$ (Dehnen 1999).

REFERENCES

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