RevMexAA (Serie de Conferencias), **40**, 268–268 (2011)

THE IIM STABILIZED WITH A CUBIC SPLINE MODEL FOR THE SOURCE FUNCTION

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The Integral Implicit Method (IIM) (Simonneau & Crivellari 1988) solves the RT problems in stellar atmospheres model computations. A cubic spline model assures the continuity of the source function and that of the first and second derivatives at each discrete optical depth.

The RT equations describe the evolution of the intensities $I^+(\tau,\mu)$ and $I^-(\tau,\mu)$ at optical depth τ and direction μ.The connection between the discrete set of optical depths is formed by the RT equations in their integral form. The single link of the spline of the spline chain is the cubic approximation for $S(\tau)$ in each specific layer (τ_L, τ_{L+1}). Each cubic arc will be defined by the value of the source function $S(\tau_L)$ and $S(\tau_{L+1})$ and of the second derivatives $S''(\tau_L)$ and $S''(\tau_{L+1})$ at the optical depths τ_L and τ_{L+1} boundaries of each specific layer. To connect the neighboring links of the spline chain between two consecutive cubic links of $S(\tau)$ we use the conditions $(\Delta\tau_L = \tau_{L+1} - \tau_L)$

$$\begin{aligned}\frac{1}{\Delta\tau_{L-1}}S(\tau_{L-1}) - (\frac{1}{\Delta\tau_{L-1}} + \frac{1}{\Delta\tau_L})S''(\tau_L) + \\ \frac{1}{\Delta\tau_L}S(\tau_{L+1}) &= \frac{\Delta_{L-1}}{6}S''(\tau_{L-1}) + (\frac{\Delta\tau_{L-1}}{3} + \\ &\frac{\Delta\tau_L}{3})S''(\tau_L) + \frac{\Delta\tau_{L+1}}{6}S''(\tau_{L+1}). \quad (1)\end{aligned}$$

As in the RT chain we need two boundary conditions for the spline chain. These conditions are $S''(\tau_0) = S''(\tau_1)$ and $S''(\tau_{NL}) = S''(\tau_{NL-1})$ (NL is the last depth point). The two extreme links of the chain become parabolic. We come back to the RT equations to obtain

$$\begin{aligned}S(\tau) &= S(\tau_L) + S'(\tau_L)(\tau-\tau_L) + \frac{1}{2}S''(\tau_L) \\ &(\tau-\tau_L)^2 + \frac{1}{6}S'''(\tau_L)(\tau-\tau_L)^3, \quad (2)\end{aligned}$$

$$\begin{aligned}S'(\tau_L) &= \frac{1}{\Delta\tau_L}[S(\tau_{L+1}) - S(\tau_L)] \\ &-\frac{\Delta\tau_L}{3}S''(\tau_L) - \frac{\Delta\tau_L}{6}S''(\tau_{L+1}), \quad (3)\end{aligned}$$

and

$$S'''(\tau_L) = \frac{1}{\Delta\tau_L}[S''(\tau_{L+1}) - S''(\tau_L)]. \quad (4)$$

That allows us to write

$$\begin{aligned}I^+(\tau_L,\mu_j) &= I^+(\tau_{L+1},\mu_j)\exp-(\frac{\Delta\tau_L}{\mu_j}) \\ &+ws_1(j)S(\tau_L) + ws_2(j)S(\tau_{L+1}) \\ &+wd_1(j)S''(\tau_L) + wd_2(j)S''(\tau_{L+1}), \quad (5)\end{aligned}$$

and

$$\begin{aligned}I^-(\tau_{L+1},\mu_j) &= I^-(\tau_L,\mu_j)\exp-(\frac{\Delta\tau_L}{\mu_j}) \\ &+ws_2(j)S(\tau_L) + ws_1(j)S(\tau_{L+1}) \\ &+wd_2(j)S''(\tau_L) + wd_1(j)S''(\tau_{L+1}), \quad (6)\end{aligned}$$

where the interpolation weights $ws_1(j)$, $ws_2(j)$, $wd_1(j)$, $wd_2(j)$ are given by $(\delta = \frac{\Delta\tau_L}{\mu_j})$

$$ws_1(j) = [1 - \frac{1}{\delta}] + \frac{1}{\delta}e^{-\delta}, \quad (7)$$

$$ws_2(j) = \frac{1}{\delta} - [1 + \frac{1}{\delta}]e^{-\delta}, \quad (8)$$

$$wd_1(j) = \mu^2[(1 - \frac{\delta}{3} - \frac{1}{\delta}) - (\frac{\delta}{6} - \frac{1}{\delta})e^{-\delta}], \quad (9)$$

$$wd_2(j) = \mu^2[(-\frac{\delta}{6} + \frac{1}{\delta}) - (1 + \frac{\delta}{3} + \frac{1}{\delta})e^{-\delta}]. \quad (10)$$

At this point we dispose of all the mathematical tools to solve the global RT problem in the same way that in the classic IIM.

REFERENCES

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