ATOMIC LINE WIDTHS BY THERMAL ENERGY FLUCTUATIONS IN STELLAR ATMOSPHERES

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RESUMEN

Un método para encontrar los anchos de las líneas atómicas usando fluctuaciones de la energía térmica se presenta en este trabajo. Se hacen comparaciones con valores publicados experimentales y teóricos.

ABSTRACT

A method for finding the width of the atomic lines using the thermal energy fluctuations is presented in this work. Comparisons are made with published experimental and theoretical values.

Key Words: atomic processes — line: profiles — stars: atmospheres

1. INTRODUCTION

The broadening produced by the radiation of light by the atoms is called natural broadening and it occurs even for isolated atoms. The atoms embedded in a plasma will experience broadening of the lines caused by perturbations by other atoms or charged particles in the gas and is called pressure broadening. The impact and statistical quantum theories are local theories that explain the broadening (Griem 1997). The width at half maximum (FWHM) is used as a diagnostic tool in stellar atmospheres.

The method for finding the width of the atomic lines using the thermal energy fluctuations is presented in this work. Thermal energy fluctuations are a global collective phenomena by nature.

2. FLUCTUATIONS

The physical quantities which describe a macroscopic system in equilibrium are very nearly equal to their mean values. Nevertheless, fluctuations from the mean values occur. The probability of the fluctuations of the thermodynamic variables in a system is given by (Einstein 1907)

$$P(\Delta S) = Q \exp(\frac{\Delta S}{k}). \tag{1}$$

The mean square fluctuation of the energy σ^2 , the variance, is given by (Kittel & Kroemer 1980)

$$\sigma^2 = \tau^2 (\frac{\partial U}{\partial \tau})_V, \qquad (2)$$

where U is the mean energy of the thermodynamic system. The FWHM for the Gaussian distribution

of the fluctuations is given by $\gamma = 2\sqrt{2 \ln 2\sigma}$. Equation (2) is the fundamental expression for developing the model.

3. MODEL

The basic idea is that the change in energy of the levels is produced by the fluctuations in energy. The linear densities of the changes in energy and of the fluctuations are obtained. The principal assumption in this model is that these two linear energy densities are equal to each other. The linear density is used because the orbit is changed slightly by the perturbations, therefore, one has to take into account the size of the orbit to distribute those changes during the motion of the electron in the orbit.

3.1. Hydrogenic Atoms

Given the total number of particles per cubic centimeter N and the temperature T for a system the linear density of the fluctuations is obtained from the following expression (Cardona 2009)

$$\delta U = \frac{kT}{3L} = kT \frac{\sqrt[3]{N}}{3},\tag{3}$$

and the linear density change in energy of the level is given by

$$\frac{\delta E_n}{2\pi r_n} = \frac{\alpha R y}{\pi r_n n^3} \delta n,\tag{4}$$

where r_n is the radius of the orbit with principal quantum number n and α is a numerical factor to produces the FWHM of the level that now is a Gaussian distribution by the perturbations produced by the fluctuations. Our principal assumption requires equating equations (3) and (4) to obtain

$$\delta n = \frac{\pi r_n n^3}{\alpha} k T \sqrt[3]{N}.$$
 (5)

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Substituting equation (5) into equation (4) gives

$$\delta E_n = \frac{\pi r_n n^3}{3\alpha R y} kT \sqrt[3]{N},\tag{6}$$

for the width of the level n. Now using in equation (6) $r_n = a_0 n^2/Z$ the radius of the orbit of the electron, the most probable distance, for the quantum state n, for a hydrogenic atom, where a_0 is the Bohr radius and Z is the atomic number of the atom, to finally obtain

$$\delta E_n = \frac{2\pi a_0}{3\alpha Z} \ kT \sqrt[3]{N} n^2, \tag{7}$$

that represents the width of the level of principal quantum number n in ergs. For a line between two levels n_1 and n_2 by convolution of two Gaussians the FWHM of the line is (Mihalas 1978)

$$w = \frac{2\pi a_0}{3\alpha Z} kT \sqrt[3]{N} (n_1^2 + n_2^2).$$
(8)

This is the FWHM of the lines for Hydrogenic atoms in ergs. The convolution of the Gaussian distribution of the fluctuations with the Lorentzian of the natural broadening produces a Voigt profile, α is composed of some numerical factors to obtain the FWHM and is defined by $\alpha = \pi \sqrt{\pi} (2\sqrt{2 \ln 2})^2$.

3.2. Non Hydrogenic Atoms

For non hydrogenic atoms the effective charge and the effective quantum numbers of the levels are used. The effective charge is $Z_{\text{eff}} = J + 1$, J is the ionization state of the chemical element. The effective quantum numbers n_{eff} are obtained from (NIST 2010; TOP 2010). Then instead of equation (8) one has

$$w = \frac{2\pi a_0}{3\alpha Z_{\text{eff}}} kT \sqrt[3]{N} (n_{\text{eff1}}^2 + n_{\text{eff2}}^2), \qquad (9)$$

for the FWHM in ergs.

4. COMPARISON WITH EXPERIMENTAL RESULTS

The experimental line widths are given for temperature and electron densities and the proportion of the constituents is not given. Therefore the calculations are made for pure elements. Figure 1 shows the FWHM of H α for electron number densities for different temperatures and experimental FWHM with argon (Wiese et al. 1972; Vitel 1987), and helium plasmas (Büscher et al. 2002), and theoretical val-



Fig. 1. Shows the width of $H\alpha$ in Å for several temperatures, compared with experimental and theoretical FWHM in argon and helium.

ues (Gigosos & Cardenosos 1996). There is good agreement between the FWHM.

5. CONCLUSIONS

The results agree well with the experimental results, The widths of the lines would be greater than those calculated with other methods and could help eliminate the so-called "microturbulence" in stellar atmospheres.

The author thanks the technical support of María de los Angeles López Castillo.

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