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NONSINGULAR COSMOLOGICAL MODELS

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RESUMEN

En este artículo presento una breve introducción al estudio de modelos cosmológicos que pasan de una era de contracción acelerada a una fase de expansión, sin ser singulares.

ABSTRACT

A short introduction to the study of cosmological models that go from an era of accelerated collapse to an expanding era without displaying a singularity is presented.

Key Words: cosmology: theory

1. INTRODUCTION

The standard cosmological model (SCM) (see for instance Nakamura et al. 2010 for an updated review) furnishes an accurate and succesful description of the evolution of the universe, which spans approximately 14 billion years. The main hypothesis on which the model is based are the following:

- Gravity is described by General Relativity,
- The Cosmological Principle,
- Above a certain scale, the matter content of the model is described by a continuous distribution of matter/energy, which is described by a perfect fluid.

In spite of its success, the SCM suffers from a series of problems such as the initial singularity, the cosmological horizon, the flatness problem, the baryon asymmetry, and the nature of dark matter and dark energy[2]. Although inflation (which for many is currently a part of the SCM) partially or totally answers some of these, it does not solve the crucial problem of the initial singularity[3].

The existence of an initial singularity is disturbing: a singularity can be naturally considered as a source of lawlessness[4], because the spacetime description breaks down "there", and physical laws presuppose spacetime. Regardless of the fact that several scenarios have been developed to deal with the singularity issue, the breakdown of physical laws continues to be a conundrum after almost a hundred years of the discovery of the FLRW solution[5] (which inevitably displays a past singularity).

The existence of an initial singularity is disturbing for many other reasons[6]. To name just two, the Cauchy problem is not well-formulated in spacetimes with a singularity, and the initial singularity is inconsistent with the entropy bound. There are also hints that quantum gravitational effects may tame the singularity, as a consequence of the discreteness of the spectrum of some operators. As a consequence of all these arguments indicating that the initial singularity may be absent in realistic descriptions of the universe, many cosmological solutions displaying a bounce were examined in the last decades, starting from the first explicit solutions for a bouncing geometry obtained by Novello & Salim (1979) and Melnikov & Orlov (1979). In fact, there is a "window of opportunity" to avoid the initial singularity in FLRW models at a classical level by one or a combination of the following assumptions:

- Violating strong energy condition in the realm of GR;
- Working with a new gravitational theory, as for instance those that add scalar degrees of free-

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[2]Some "open questions" may be added to this list, such as why the Weyl tensor is null, and what the future evolution of the universe is.

[3]In fact, inflation presents some problems of its own, such as the identification of the inflaton with a definite field of some high-energy theory, the functional form of the potential V in terms of the inflaton, and the Transplanckian problem. See for instance Brandenberger (2009).

[4]For a discussion of the singularity theorems and of the concept of singularity see for instance Earman (1995).

[5]This acronym refers to the authors that presented for the first time the solution of EE that describes a universe with zero pressure (Friedmann) and nonzero pressure (Lemâitre), and to those who studied its general mathematical properties and took it to its current form (Robertson & Walker). For historical details, see Merleau-Ponty (1965).

[6]See Novello & Perez Bergliaffa (2008) for a complete list, as well as a detailed revision of nonsingular cosmological models.

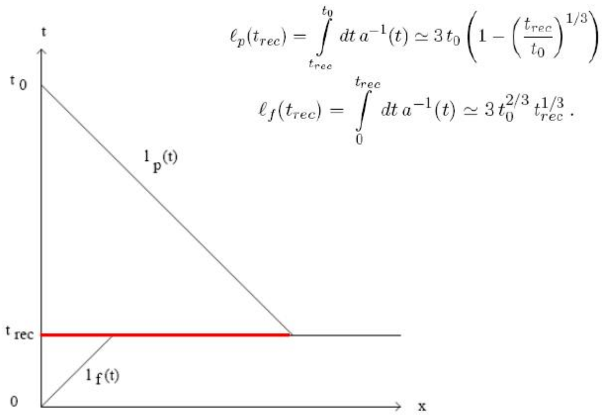

Fig. 1. The plot of the physical distance x versus time t illustrates the homogeneity problem: the past light cone $\ell_p(t)$ at the time $t_{\rm rec}$ (in red) is much larger than the forward light cone $\ell_f(t)$ at $t_{\rm rec}$. Adapted from Brandenberger (2004).

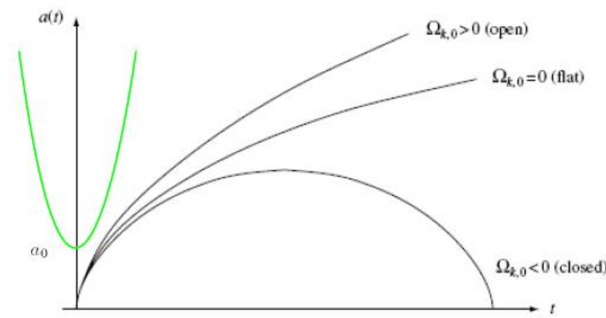

Fig. 2. Typical evolution of the scale factor in a nonsingular cosmological model (in green), as opposed to the singular big bang model.

dom to gravity (Brans-Dicke theory being the paradigmatic example of this type), or by adopting an action built with higher-order invariants.

Other ways to avoid the singularity are:

- Changing the way gravity couples to matter, from minimal to non-minimal coupling;
- Using a non-perfect fluid as a source.

Finally, quantum gravitational effects also give the chance of a bounce[7]. In the next section we shall briefly discuss how a bounce can solve some of the problems that the cosmological model pre-1980 had.

2. THE BOUNCE AND THE PROBLEMS OF THE STANDARD COSMOLOGY

In addition to the initial singularity, SCM had other problems. Among them we can cite the following (Brandenberger 1999):

- The homogeneity problem: the comoving region over which the CMB is observed to be homogeneous to better than one part in 10^{-4} is much larger than the comoving forward light cone at the time of recombination (see Figure 1).
- The flatness problem: the quantity $|\Omega - 1|$ decreases with the evolution of a universe dominated by matter or radiation. Since $\Omega \approx 1$ today (Komatsu et al. 2011), Ω must have been incredibly close to 1 in the past.
- The generation of primordial perturbations: clusters of galaxies have nonrandom correlations on scales larger than 50 Mpc. This scale is comparable to the comoving horizon at $t_{\rm eq}$. If the initial density perturbations were produced much before $t_{\rm eq}$, the correlations cannot be explained by a causal mechanism[8].

Except for the inital singularity, these problems were addressed by inflation (which has problems of its own as we mentioned before)[9]. We shall see next that a model with a bounce may also face these issues succesfully. Let us state that by a nonsingular model with a LFRW geometry we mean a model in which the scale factor attains a minimum value (Figure 2).

Consequently, a model with a bounce solves the problem of the initial singularity by construction. Regarding the homogeneity problem, the future light cone is given by

$$\ell_f(t) = a(t) \int_{t_i}^{t} \frac{dt}{a(t)}.$$

Assuming the equation of state $p = \omega\rho$ it follows that $a(t) \propto (-t)^{\frac{2}{3(1+\omega)}}$. Hence,

$$\ell_f(t) \propto (-t_i)^{\frac{1+3\omega}{3(1+\omega)}} (-t)^{\frac{2}{3(1+\omega)}} + t.$$

If there is a contracting phase led by a perfect fluid with $\omega > -1/3$, then $\ell_f(t)$ diverges for $t_i \to \infty$, thus solving the horizon problem. The flatness problem, is encoded in the equation

$$\frac{d}{dt}|\Omega - 1| = -2\frac{\ddot{a}}{\dot{a}^3}.$$

Since the standard evolution drives Ω to 1, an era during which the evolution of the universe forces $|\Omega - 1|$ away of zero is needed. This can achieved by an

[7]See Novello & Perez Bergliaffa (2008) for details about all these items.

[8]Actually, standard cosmology cannot explain how primordial density perturbations are generated.

[9]For a review of inflation, see for instance Bauman (2009).

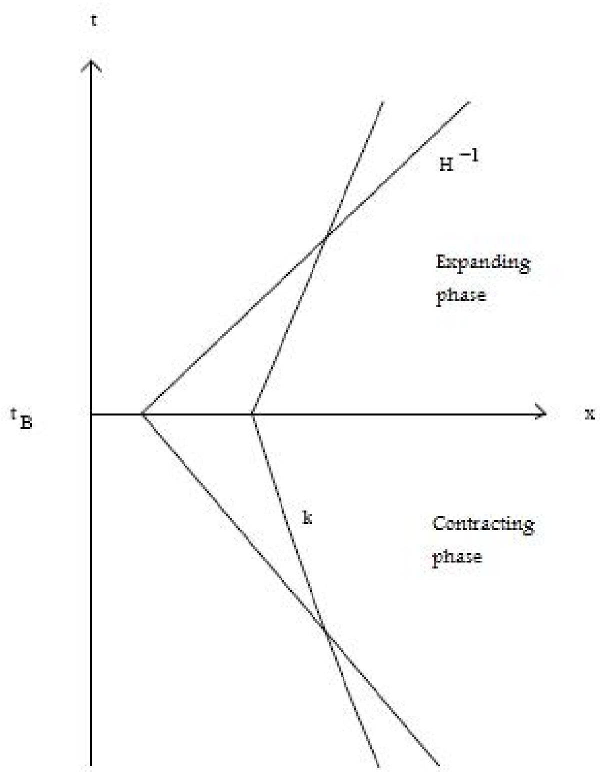

Fig. 3. The plot shows the evolution of the Hubble radius H^{-1} and of a fixed comoving scale, the bounce taking place at $t = t_B$. Adapted from Brandenberger (2004).

expansion such that $\ddot{a} > 0$ and $\dot{a} > 0$ (which is the case of inflation), or through a long decelerated phase of contraction before the bounce, characterized by $\ddot{a} < 0$ and $\dot{a} < 0$.

Regarding the generation of primordial perturbations in nonsingular models, during the contracting phase the Hubble radius H^{-1} contracts faster than the physical length corresponding to a fixed comoving scale k (see Figure 3). Quantum vacuum fluctuations generated causally on sub-Hubble scales in the contracting phase are assumed to be the seeds of the inhomogeneities observed today. The scale of these fluctuations is amplified and evolves according to GR during the (long) time when it is larger than the Hubble radius.

Finally, if the bounce is such that $a_0 \gg \ell_{\rm Pl}$ there is no Transplanckian problem.

3. AN EXAMPLE

Having shown in the previous section that a nonsingular model may in principle furnish a solution to the problems of standard cosmology, let us review in this section a specific model of this type, and what kind of predictions can be obtained from it. The

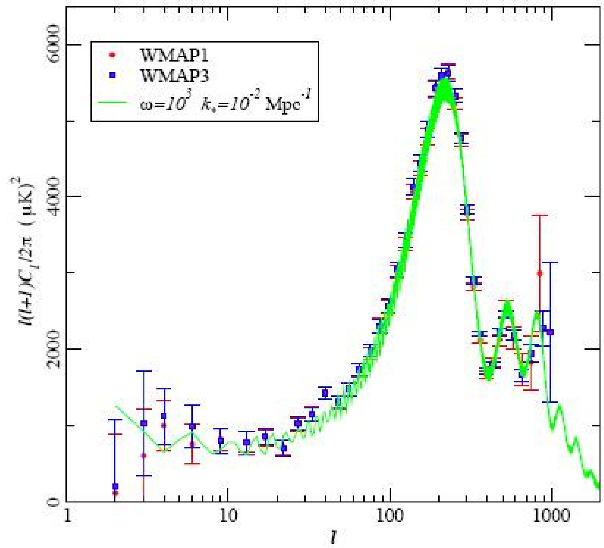

Fig. 4. The plot shows the multipoles C_ℓ for a typical bouncing model, as well as WMAP data (Falciano et al. 2008).

model in question was developed in Peter & Pinto-Neto (2008), in the framework of GR plus a perfect fluid with equation of state $p = \omega\rho$, the spacetime geometry being of the FLRW type. The quantization of both the background and the perturbations of this model following the Bohmian approach (see Peter & Pinto-Neto 2008 for details), furnishes for the evolution of the background

$$a(\tau) = a_0 \left[1 + \left(\frac{\tau}{T_0}\right)^2\right]^{1/[3(1-\omega)]},$$

with $d\eta = [a(\tau)]^{3\omega-1} d\tau$, and η is the conformal time. This solution has no singularities and tends to the classical solution when $\tau \to \pm\infty$. An analysis of the perturbations shows that they behave exactly as shown in Figure 3. The result obtained in Peter & Pinto-Neto (2008) for the power spectra is

$$n_s = 1 + \frac{12\omega}{1+3\omega}, \qquad n_T = \frac{12\omega}{1+3\omega},$$

in such a way that both the scalar and the tensor spectrum tend to a scale-invariant spectrum in the dust limit. Finally, a fit of the amplitude of the perturbations to the CMB data yields $a_0 \approx 1000\ell_{\rm Planck}$, thus avoiding the Transplanckian problem. Notice also that the model predicts a tensor to scalar ratio of $T/S \propto \sqrt{n_s - 1}$, while inflationary models tipically predict a linear relation.

4. BOUNCING MODELS AND OBSERVATION

As we have seen in the previous sections, nonsingular models solve the problems of standard cosmology. The example discussed in § 3 shows that some particular models produce predictions that are not incompatible with observations. Some other predictions generic to bouncing models are:

- The spectrum of primordial perturbations displays a small oscillatory component, see Figure 4 (Falciano et al. 2008).
- Copious production of particles near the bounce. This has been estimated in the case of gravitons in the Pre-Big Bag model (Gasperini & Veneziano 2003), and for photons in the WIST theory (Salim et al. 2005, 2007).

5. CONCLUSION

Nonsingular models solve the problems of standard cosmology, and furnish predictions that may be contrasted with observation in the near future. There are still problems to be solved (such as the influence on the perturbations of the matter creation at the bounce, the amount of matter created, and the possible growth of initial perturbations), but the bottom line is that models with a bounce are certainly worth studying, on their own sake and/or as a complement to inflation[10].

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[10]See for instance Cai et al. (2009).