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DISTRA: A CODE TO FIND INVISIBLE EXOPLANETS

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RESUMEN

Dados los instantes de tránsito de un exoplaneta, que diferirán de una serie kepleriana de tránsitos de un problema de dos cuerpos si un segundo planeta que no transita está perturbando a aquél, resolvemos el problema inverso de encontrar los seis elementos orbitales y la masa de este segundo planeta. Esto es equivalente a un problema de optimización en siete dimensiones, en el cual la función a minimizar es alguna medida de la diferencia entre los tránsitos observados y los obtenidos al integrar el problema de los tres cuerpos con el planeta que transita y el invisible; las siete variables dependientes son los elementos y la masa de este último. Resolvemos este formidable problema numérico en dos etapas, aplicando como primer paso un algoritmo genético, y luego puliendo este resultado con un algoritmo simplex en 7 dimensiones. Aplicamos el algoritmo al sistema Kepler-9, en el cual hay dos planetas que transitan y por lo tanto el segundo planeta tiene elementos orbitales y masa conocidos.

ABSTRACT

Given the transit times of an exoplanet, which will differ from a Keplerian two-body series of transits if a second, non-transiting exoplanet is perturbing it, we solve the inverse problem of finding the six orbital elements and the mass of that second planet. This is equivalent to an optimization problem in seven dimensions, in which the function to minimize is some measure of the differences between the observed transits and the transits obtained with a three-body integration of the transiting planet and the invisible one; the seven dependent variables are the elements and the mass of the latter. We solve this formidable numerical problem in two stages, applying a genetic algorithm as a first step, and then polishing this result with a 7D simplex algorithm. We applied the algorithm to the Kepler-9 system, in which two planets transit and therefore the second planet has known orbital elements and mass.

Key Words: celestial mechanics — planet-star interactions — planets and satellites: dynamical evolution and stability

1. AIM OF THE WORK

The observation of the transits of an exoplanet allows, among other things, to determine whether there are other planets orbiting the same star. This is possible because the presence of planets perturbs the Keplerian, constant period of the transiting planet, and these deviations are measurable.

The dynamical description of an exoplanet (or any other celestial body) requires the knowledge of the potential in which it moves (the mass of the central star), its own mass, and at least six initial conditions, which can be expressed as a set of orbital elements. So, the question arises: is it possible, based solely on the perturbation of the transit times of an exoplanet, to recover the mass and the six orbital elements of the invisible, non-transiting exoplanet responsible for those perturbations? At first sight, it seems impossible to recover all this information. In this work we will show that, within some limitations, it is indeed possible.

2. METHOD

We then seek to solve the following inverse problem: to find the orbital elements and the mass (hereafter called "the elements") of a perturber exoplanet, given the transit time variations (TTVs), i.e., the differences between the observed transit times and those computed as if the orbit were purely Keplerian, that that planet induces on the transiting one. The implementation of this idea is achieved by means of a two-step procedure, which we called DisTra, for DIsturbed TRAnsits.

The first step consists in using a genetic algorithm (e.g., Davis 1991; Charbonneau 1995) in order to obtain a set of elements near the true ones. This is the main stage, since it allows to explore the seven-dimensional space of solutions in a reasonable period of time. To implement this we have to define a *fitness function*, the value of which is the greater the

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smaller is the difference between the observed transits and those induced by the solution. Thus, each planet, randomly chosen from a population of possible perturbing planets, and called an *i*ndividual, will have its own fitness, or ability to reproduce the observed transits. To compute the fitness function, an integration of the equations of motion of the three bodies (star and both planets) is performed, and the transits of the visible planet are recorded; then, we compute the reciprocal of the sum of the differences between the observed and computed transits as the fitness of the individual. Then the algorithm selects a set of planets consisting of the best individuals thus generated, and, using a mathematical implementation of the characteristic properties of the genetic evolution (parent selection, crossing of genes, mutation, etc.) generates a new population of individuals. After computing their own fitnesses, the procedure is repeated until some predefined number of generations have passed. The outcome of this is a set of invisible planets that can generate well the observed transits. An outstanding characteristic of genetic algorithms is that, although a solution space has countless maxima and the maximised function displays a high non-linearity, as is our case, the solutions found are in general near the absolute maximum, or at least near a local maximum which has a value similar to the absolute one.

The second step consists in taking the best individuals of the last generation and improve them as solutions by means of a seven-dimensional simplex optimization algorithm (e.g., Press et al. 1992), using as a funtion to maximise the same function used with the genetic algorithm. The simplex will take the planet to its local maximum of the solution space. The final solution will be the planet with the final best fitness.

3. NUMERICAL DETAILS

The algorithm has lots of numerical parameters. Some of the most important are reported here as a reference. The number of generations was 500; the population was set to 10,000 individuals. The selection of the parents was achieved by a roulette wheel algorithm; the selection pressure was set proportional to the fitness. The space of search was limited to typical inner Solar System values for the semimajor axis and the eccentricity; the mass was searched for in the interval from tens of terrestrial masses to Jovian ones; the inclination with respect to the plane of the sky was limited between 80 and 90 degrees, and the rest of the angles were searched for along their full possible range.

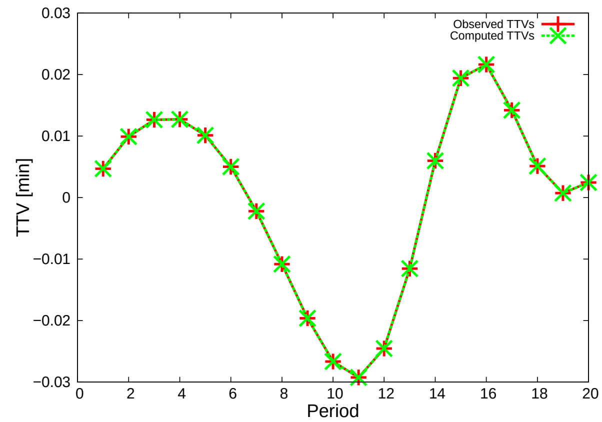

Fig. 1. Synthetic TTVs (red, pluses) and computed TTVs (green, crosses). The reconstruction is notably good. The color figure can be viewed online.

The integration of the equations of motion was performed with a Bulirsch-Stoer integrator; the energy conservation was better than one part in 10^{-11} in all cases. All collisional and/or otherwise inestable orbits were discarded by assigning them zero fitness.

4. SYNTHETIC RESULTS

We tried the DisTra algorithm just described with a synthetic system as a test bed: a transiting planet and an invisible one with known chosen elements. We integrated the equations of motion and recorded the resulting transits. These transits were then used as observed ones, feeding our algorithm with them. From this information, the two-step procedure should then obtain the elements of the invisible planet.

Figure 1 shows the result of one such experiment. As can be seen, although this is a difficult case to reproduce because it generates TTVs of only seconds, the algorithm is able to find a planet which generates those TTVs almost without error. Most important, the elements of this planet coincide with the original ones to the fourth decimal level. This indicates that, tough the problem may appear as degenerate (i.e., with several possible solutions), DisTra can find the correct one.

5. RESULTS WITH OBSERVED VALUES

Whereas the foregoing result shows that the DisTra algorithm is viable, the reconstruction was made based on synthetic data, i.e., data without error. We then look for an observed test bed, that is, an observed case in which there are two transiting planets: the elements of any of them considered as the invisible one, are known and therefore

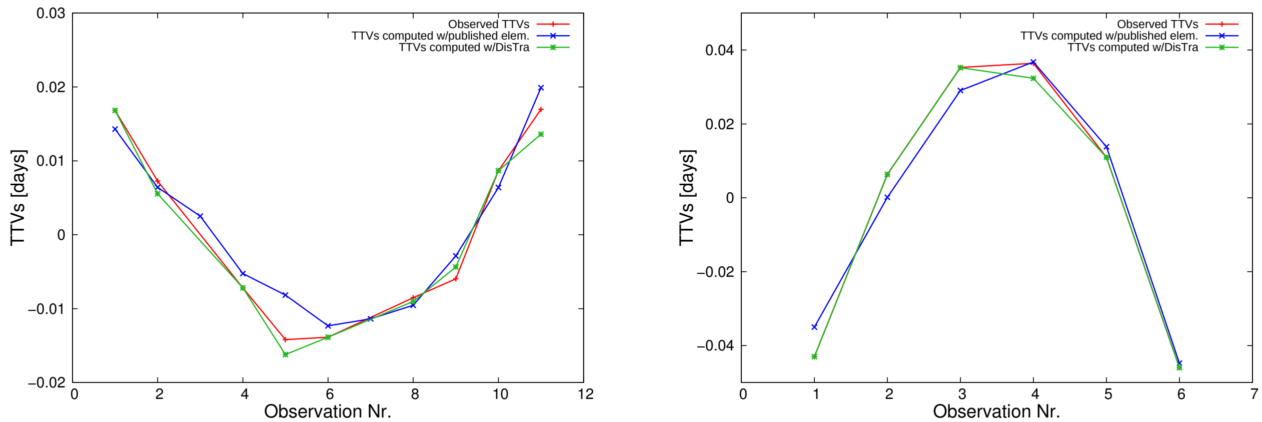

Fig. 2. TTVs of the Kepler-9 system. Red, pluses: observed; blue, crosses: computed with the published elements; green, asterisks: computed with DisTra. The color figure can be viewed online.

we can gauge the ability of the algorithm in reproducing the system. The Kepler-9 system is one such system (Holman et al. 2010). Although this system may possess other minor planets, the masses of these last ones are probably small enough so that their perturbing actions will be below the observational errors.

Figure 2 shows the observed TTVs, the TTVs found with DisTra, and, as a reference, the TTVs obtained integrating the equations of motion with the elements reported in Holman et al. (2010). As can be seen, the TTVs obtained with our algorithm are closer to the observed ones than those computed with the published original elements. Unfortunately, the set of elements thus obtained does not coincide with those published, in spite of its better fit. Table 1 compares the two sets of elements.

TABLE 1

OBSERVED AND COMPUTED ELEMENTS OF PLANET KEPLER-B[a]

Element	Computed	Observed
a [AU]	0.1868	0.22503
e	0.1827	0.1316
i [°]	82.48	88.70
Ω [°]	282.60	0.90
ω [°]	156.18	101.53
λ [°]	224.88	8.74
m [10^{-4} $M_\odot$]	3.04	1.68

[a]Letters on the first column stand for: semimajor axis, eccentricity, inclination with respect to the plane of the sky, longitude of the ascending node, argument of the periastron, mean anomaly and mass.

6. CONCLUSIONS

We have developed a code which allows to obtain the orbital elements and the mass of an invisible exoplanet which modifies the transit times of another transiting one, thus allowing to obtain the neccesary information. Synthetic experiments show the viability of the algorithm. Experiments with real observed values of two transiting planets show that a solution can be obtained which reproduces the times of transit, although the elements thus obtained do not coincide with those published. Our next goal will be to improve our code in order to obtain coherent results, i.e., to obtain the elements of each of the planets when the other is considered the transiting one.

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