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# CONTRAST DENSITY AND MASS FUNCTION FOR SPHERICAL COLLAPSE OF LEMAITRE-TOLMAN-BONDI METRIC FROM FRACTAL POINT OF VIEW

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Recent works about large structure in the universe put in doubt the homogeneity transition almost universally accepted, (Joyce et al. 2005), (Gaite 2007), (Chacón-Cardona & Casas-Miranda 2012). In the present work we develop theoretically the density contrast for the spherical collapse of an overdensity of dark matter which evolve in a inhomogeneous universe inside a fractal cosmology presented by (Ribeiro 1993). For a spherically symmetric dust (no pressure) only de first term, the proper density in $T_{\mu\nu}$ is not zero. The element of this metric can be written in the form (Bondi 1947):

$$ds^2 = dt^2 - \frac{R^2(r,t)_r}{1+2E(r)}dr^2 - R^2(r,t)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

where $x^\mu = \{t,r,\theta,\varphi\}$, $R(r,t) > 0$ , $R(r,t)_r = \partial R(r,t)/\partial r$, $R(r,t)_t = \partial R(r,t)/\partial t$ and $E(r)$ is a radial function. From non-zero components of Einstein's tensor we have an equation of motion:

$$\frac{\left(R(r,t)_t\right)^2}{2} - \frac{M(r)}{R(r,t)} = E(r)$$

like an energy equation and the proper density relation:

$$8\pi\rho(r,t) = \frac{2M(r)_r}{R^2(r,t)\ R(r,t)_r}$$

We are interested in the $E(r) > 0$ solution (Hyperbolic). Using the parameterization $R(r,t) = \mathrm{A}^2\sinh^2(\psi/2)$ we can find:

$$t - t_\beta(r) = \frac{M(r)}{\left(2E(r)\right)^{3/2}}(\sinh\psi - \psi)$$

$$R(r,t) = \frac{M(r)}{2E(r)}(\cosh\psi - 1)$$

We introduce fractality in the mass function $M(r) = \alpha r^D/4$ with $\alpha$ a pre-factor related with the average distance between neighbours and $D$ the fractal dimension of the dust. In the present work we have $t_\beta(r) = 0$ by simplicity.

Using a Taylor's series of hyperbolic functions and taking the first terms we found:

$$R(r,t) \approx \frac{1}{2}(3t)^{2/3}\left(\alpha r^D\right)^{1/3}\left[1 + \frac{2E(r)}{5}\left(\frac{3t}{\alpha r^D}\right)^{2/3}\right]$$

The derivative of this expression is:

$$R(r,t)_r \approx \left[\frac{D}{3r} + \frac{\frac{2}{5}(3t)^{2/3}\left(\frac{E(r)}{(\alpha r^D)^{2/3}}\right)_r}{\left(1 + \frac{2E(r)}{5}\left(\frac{3t}{\alpha r^D}\right)^{2/3}\right)}\right]R(r,t)$$

In order to calculate the density contrast:

$$\delta = \frac{\rho(r,t) - \langle\rho(r,t)\rangle}{\langle\rho(r,t)\rangle} = \frac{\rho(r,t)}{\langle\rho(r,t)\rangle} - 1$$

is necessary to determine the proper density $\rho(r,t)$ and the average density $\langle\rho(r,t)\rangle = \frac{3\alpha r^D}{16\pi R(r,t)^3}$, in this form:

$$\delta = -\frac{\frac{2}{5}(3t)^{2/3}\left(\frac{E(r)}{(\alpha r^D)^{2/3}}\right)_r \frac{3r}{D}}{\left[1 + \frac{2}{5}(3t)^{2/3}\left(\frac{E(r)}{(\alpha r^D)^{2/3}}\right)_r \frac{3r}{D}\right]}$$

The function $E(r)$ would be found to form structure. A possibility for the derivative is a periodic function:

$$\left(\frac{E(r)}{\left(\alpha r^D\right)^{2/3}}\right)_r = A\sin\left(\frac{2\pi r}{L}\right)$$

where $A$ is the amplitude of perturbation. Integrating the last equation the energy take the form.

$$E(r) = \left(\alpha r^D\right)^{2/3}\frac{L}{2\pi}A\left(-\cos\left(\frac{2\pi r}{L}\right) + 1\right)$$

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