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THE THREE-DIMENSIONAL EQUATIONS FOR THE LOCAL GALACTIC MASS DENSITY

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RESUMEN

En el 2012 propusimos una nueva formulación tridimensional para estimar la densidad de masa del disco Galáctico. Su aplicación nos dió una densidad de materia oscura en la posición solar sorpresivamente baja. Aquí revisamos la formulación y derivamos ecuaciones mejoradas. Nuestros cálculos preliminares nos indican que estas correcciones no alteran apreciablemente nuestros resultados previos, pero futuros estudios deberían basarse en las nuevas ecuaciones presentadas aquí.

ABSTRACT

In 2012, we proposed a new three-dimensional approach to estimate the mass density of the Galactic disk. Its application returned a surprisingly low dark matter density at the solar position. Here we derive the formulation and present refined equations. Our preliminary calculations indicate that these corrections do not alter our previous results noticeably, but further studies should be based on the new equations presented here.

Key Words: dark matter — Galaxy: kinematics and dynamics — Galaxy: structure

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1. INTRODUCTION

Moni Bidin et al. (2010) and Moni Bidin et al. (2012b) (hereafter M12b) proposed a new formulation to estimate the Galactic mass density by means of a direct three-dimensional equation, whose validity is not limited to small Galactic heights. The classical one-dimensional approach is obtained from this same equation, after some simplifications by means of more assumptions (Bovy & Tremaine 2012). Applying this formulation to the kinematical measurements of Moni Bidin et al. (2012a), M12b found a surprising lack of dark matter at the solar position, as opposed to most previous one-dimensional estimates. Here we show that M12b formulation must be refined, and we present improved equations. We will use the cylindrical Galactic coordinates (R, θ, Z), and their respective velocity components $(\dot{R}, \dot{\theta}, \dot{Z})=(U, V, W)$.

2. REVISION OF THE M12B FORMULATION

The M12b calculation is based on the integrated Poisson equation, which expresses the mass surface density $\Sigma(Z)$ within $\pm Z$ kpc from the Galactic plane:

$$-4\pi G\Sigma(Z) = 2 \cdot F_{\rm z}(Z) + \int_{-Z}^{Z} R^{-1}\partial_R(RF_{\rm R})dz, \quad (1)$$

where $F_{\rm R}$ and $F_{\rm z}$ are the radial and vertical component of the force per unit mass, respectively. Equation (1) makes use of the antisymmetry of $F_{\rm z}(Z)$. $F_{\rm R}$ and $F_{\rm z}$ can be expressed by means of the Jeans equations. If the test disk population is a virialized system in steady state with $\overline{U} = \overline{W} = 0$, and its density distribution follows a double exponential law[3], $\rho(R, Z) \propto \exp\left(-R/h_{\rm R} - |Z|/h_{\rm Z}\right))$, where $\partial_Z h_{\rm R} = 0$ (Cabrera-Lavers et al. 2007) and $\partial_R h_{\rm Z} \approx 0$ (Shaw & Gilmore 1990; Naeslund & Joersaeter 1997)[4], the radial Jeans equation becomes

$$RF_{\rm R} = R\partial_R\sigma_{\rm U}^2 - Rh_{\rm R}^{-1}\sigma_{\rm U}^2 + R\partial_Z\overline{UW} + f_{\rm uw} + \sigma U^2 - \sigma_{\rm V}^2 - \overline{V}^2, \quad (2)$$

where $f_{\rm uw} = R\rho^{-1}\overline{UW}\partial_Z\rho$. M12b assumed $\partial_Z\rho = -h_{\rm Z}^{-1}\rho$, therefore $f_{\rm uw} = -Rh_{\rm Z}^{-1}\overline{UW}$. Hence, the integral $I(f_{\rm uw}) = \int_{-Z}^{Z} R^{-1}\partial_R(f_{\rm uw})dz$ becomes

$$I(f_{\rm uw}) = -(Rh_{\rm Z})^{-1}\int_{-Z}^{Z}\overline{UW}dz - h_{\rm Z}^{-1}\int_{-Z}^{Z}\partial_R\overline{UW}dz. \quad (3)$$

The two integrals in Eq.(3) are null, because both $\overline{UW}(Z)$ and $\partial_R\overline{UW}(Z)$ are antisymmetric (M12b). The term $f_{\rm uw}$ thus gave a null contribution to Eq. (1), and it was removed from the calculation. However, this conclusion is wrong, because

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[3]The vertical density profile should be closer to a sech^2 function (Camm 1950, 1952), but this distinction is relevant only close to the plane (e.g., Jurić et al. 2008).

[4]If $\partial_R h_{\rm Z} \neq 0$, the equations must be refined as indicated in Sect. 4.4 of M12b

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$\partial_Z\rho$ is also antisymmetric with respect to Z, therefore $f_{\rm uw} = -Rh_{\rm Z}^{-1}\overline{UW}\cdot sgn(Z)$. The quantity $\overline{UW}(Z)\cdot sgn(Z)$ is symmetric, and Eq. (3) becomes

$$I(f_{\rm uw}) = -2(Rh_{\rm Z})^{-1}\int_0^Z \overline{UW}dz - 2h_{\rm Z}^{-1}\int_0^Z \partial_R\overline{UW}dz. \quad (4)$$

The contribution of these integrals is not null.

Equation (2) can be derived, and then integrated with respect to Z. The antisymmetry of $\overline{UW}(Z)$ implies $\int_{-Z}^{Z}\partial_Z\overline{UW}dz = 2\overline{UW}(Z)$, and analogously for $\partial_R\overline{UW}(Z)$, while the other quantities are symmetric with respect to Z, hence their integral in $\pm Z$ is twice the integral between $z=0$ and Z. We thus obtain

$$\begin{aligned}-2\pi G\Sigma(Z) &= 2\partial_R\overline{UW} + (2R^{-1}-h_{\rm R}^{-1})\overline{UW} \\ &+\partial_Z\sigma_{\rm W}^2 - h_{\rm Z}^{-1}\sigma_{\rm W}^2 + \int_0^Z(2R^{-1}-h_{\rm R}^{-1})\partial_R\sigma_{\rm U}^2dz \\ &-\int_0^Z(Rh_{\rm R})^{-1}\sigma_{\rm U}^2dz + \int_0^Z\partial_{RR}^2\sigma_{\rm U}^2dz \\ &-\int_0^Z R^{-1}\partial_R\sigma_{\rm V}^2dz - (Rh_{\rm Z})^{-1}\int_0^Z\overline{UW}dz \\ &-h_{\rm Z}^{-1}\int_0^Z\partial_R\overline{UW}dz - R^{-1}\int_0^Z\partial_R\overline{V}^2dz. \quad (5)\end{aligned}$$

Moni Bidin et al. (2015) showed that the last term of this equation should be taken into account, contrary to what was done by M12b, although its contribution is small. Equation (5) substitutes Eq. (13) of M12b, where the last three terms were incorrectly neglected. This new equation is very general because it assumes only basic symmetries, the steady state of the population under study, and the local double-exponential shape of its mass distribution. If the two parameters $(h_{\rm R}, h_{\rm Z})$ and the Z-dependence of the kinematical quantities are obtained from observations, $\Sigma(Z)$ can be directly calculated up to any Galactic height. It is worth noticing that Eq. (5) requires the local radial and vertical trends of the kinematics in the points of calculations, but not their global behavior.

At this point, M12b introduced a further hypothesis, assuming that $\sigma_{\rm U}^2$, $\sigma_{\rm V}^2$, and $\overline{UW}$ decay exponentially in the radial direction[5], with scale length $h_\sigma = h_{\rm R}$. Both theoretical and observational results support the assumption that $\sigma_{\rm X}^2 \propto \exp(-R/h_\sigma)$, with $X = U, V$ (Lewis & Freeman 1989; Cuddeford & Amendt 1992), but we will not force h_σ to match $h_{\rm R}$ in our formulation. Moreover, the assumption $\partial_R\overline{UW} = -h_\sigma^{-1}\overline{UW}$ has not general validity. For example, the equation of Binney & Merrifield (1998) -valid if the dispersion ellipsoid is aligned with the spherical coordinate system- and the approximated expression of Kuijken & Gilmore (1989) -which well reproduce the observations (Moni Bidin et al. 2012a)-, both indicate that $\partial_R\overline{UW} \neq -h_\sigma^{-1}\overline{UW}$. In absence of a unique functional form of $\partial_R\overline{UW}$, we will leave it explicit in the formulation. With the assumptions above, Eq. (5) can be simplified to

$$\begin{aligned}2\pi G\Sigma(Z) &= -2\partial_R\overline{UW} - (2R^{-1}-h_{\rm R}^{-1})\overline{UW} \\ &-\partial_Z\sigma_{\rm W}^2 + h_{\rm Z}^{-1}\sigma_{\rm W}^2 + k_1\int_0^Z\sigma_{\rm U}^2dz \\ &-(R^{-1}h_\sigma^{-1})\int_0^Z\sigma_{\rm V}^2dz + (Rh_{\rm Z})^{-1}\int_0^Z\overline{UW}dz \\ &+h_{\rm Z}^{-1}\int_0^Z\partial_R\overline{UW}dz + R^{-1}\int_0^Z\partial_R\overline{V}^2dz, \quad (6)\end{aligned}$$

where

$$k_1 = R^{-1}h_{\rm R}^{-1} - (h_\sigma h_{\rm R})^{-1} - h_\sigma^{-2} + (2R^{-1}h_\sigma^{-1}). \quad (7)$$

Equation (6) substitutes the expression for $\Sigma(Z)$ used by M12b (their Eq. 14).

3. HOW DO M12B RESULTS CHANGE?

It is very important at this point to check how the corrections presented in the previous section affect the results of M12b. That analysis is undergoing. Preliminary results indicate that the new formulation slightly decreases the inferred mass, hence these corrections do not explain the lack of DM previously found. More investigation is therefore needed to fully understand those puzzling results. Nevertheless, we stress that the new equations presented here rather than those of M12b must be used.

[5]Alternatives are analyzed in Sect. 4.3 of M12b.

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