

THE ENTROPY OF BLACK SHELLS

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The formalism of Darmois-Israel is used to calculate the entropy of a thin spherical shell that contracts from infinity down to near its gravitational radius. It was found that the entropy contained in the thin shell is proportional to the horizon area depending on the number of species N , the distance own above the horizon α and two constants C_1 and C_2 which can be measured observationally. Certain thermodynamic parameters were calculated, when thin shell it is near the horizon of events for a static space-time, this parameters are agree with reported.

If we consider a riemannian manifold and under Darmois-Israel formalism. We used the first condition and second condition of juncture

$$[h_{ab}] = h_{ab}^+|_{\Sigma} - h_{ab}^-|_{\Sigma} = 0. \quad (1)$$

$$[K_{ab}] = K_{ab}^+|_{\Sigma} - K_{ab}^-|_{\Sigma} = 0. \quad (2)$$

Then, for an asymptotically distant observer to appreciate a thin shell of mass m contracting to near its gravitational radius r_s , such that

$$r(t) = r_s + \Delta r e^{-t/\tau} \quad (3)$$

$$\Delta r = r_0 - r_s, \quad r_s = \frac{2Gm}{c^2}, \quad \tau = \frac{4Gm}{3c^3}. \quad (4)$$

In the Brick Wall model reviewed by Mukohyama-Israel, the entropy arises from hot fields close to the horizon once the black hole is formed. Under the condition that such entropy is the same for the black shell model for an asymptotically distant observer

$$S_{shell} = \left[\frac{N}{90\pi\alpha^2} \left(\frac{T_{\infty}}{\kappa/2\pi} \right)^3 \frac{A}{4} \right] \frac{C_1 C_2^2}{4} \quad (5)$$

$$\alpha = l_P \sqrt{\frac{C_1 C_2^2 N}{360\pi}}, \quad (6)$$

where

$$C_1 = \frac{1}{\tau} \int_0^{t_u} e^{t/\tau} dt, \quad C_2 = \frac{1}{\tau} \int_0^{t_u} e^{-t/\tau} dt. \quad (7)$$

Near to horizon, we have that the constants take values of

$$C_1(r_s + \epsilon) = 1, \quad C_2(r_s + \epsilon) = 2. \quad (8)$$

It is possible define a new constant like

$$n|_{r_s+\epsilon} = \left(\frac{C_2}{C_1} \right)_{r_s+\epsilon} = 2. \quad (9)$$

So, we have that the minimum value of ϵ near to horizon is

$$\epsilon|_{r_s+\epsilon} = n\Delta r = 2\Delta r. \quad (10)$$

In this analysis we must take into account that such constants have their origin in The kinematics of the Black Shell. Once specify the entropy of the shell in contraction, it is possible to calculate some thermodynamic parameters of the shell with the standard recipe:

$$S_{shell} = \frac{N\pi^2}{180\alpha^2} \left(\frac{T_{\infty}^3}{\kappa^3} \right) AC_1 C_2^2, \quad (11)$$

The internal energy

$$E = \frac{N\pi^2}{240\alpha^2} \left(\frac{T_{\infty}^4}{\kappa^3} \right) AC_1 C_2^2, \quad (12)$$

The heat capacity is

$$C_V = C_P = \frac{N\pi^2}{60\alpha^2} \left(\frac{T_{\infty}^3}{\kappa^3} \right) AC_1 C_2^2 \quad (13)$$

and the internal pressure

$$P = \frac{N\pi^2}{720\alpha^3} \left(\frac{T_{\infty}^3}{\kappa^3} \right) C_1 C_2^2. \quad (14)$$

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