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THE BAYESIAN CRAMÉR-RAO LOWER BOUND IN ASTROMETRY

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RESUMEN

La determinación de la máxima precisión que es posible obtener en la medición de la ubicación de un objeto puntual ha sido un tópico de interés permanente dentro de la comunidad astrométrica. El así denominado límite (no-paramétrico o no-Bayesiano) de Cramér-Rao (CR a partir de ahora) nos proporciona un límite inferior para la varianza con la cual es posible estimar la posición de un objeto puntual. Esto ha sido recientemente estudiado por Mendez y colaboradores (2014, 2015). En este trabajo presentamos una aproximación distinta al mismo problema (Echeverria et al. 2016), haciendo uso de un esquema Bayesiano del límite de CR, el cual presenta varias ventajas con respecto al escenario paramétrico.

ABSTRACT

A determination of the highest precision that can be achieved in the measurement of the location of a stellar-like object has been a topic of permanent interest by the astrometric community. The so-called (parametric, or non-Bayesian) Cramér-Rao (CR hereafter) bound provides a lower bound for the variance with which one could estimate the position of a point source. This has been studied recently by Mendez and collaborators (2014, 2015). In this work we present a different approach to the same problem (Echeverria *et al.* 2016), using a Bayesian CR setting which has a number of advantages over the parametric scenario.

Key Words: astrometry — methods: analytical — methods: data analysis — methods: statistical

Astrometry is the foundation of classical astronomy and modern astrophysics, and it remains a cornerstone of the field for the 21$^{\text{st}}$ century. Nowadays, astronomers take for granted resources such as the ESA Gaia astrometric satellite which will deliver a catalogue of over 10^9 stars, with accuracies smaller than 10-20 micro-arcseconds for objects brighter than V = 15.

In astrometry the CR bound offers meaningful expressions that can be used to analyze the complexity of the inference task in terms of key observational and design parameters such as: source properties (flux, shape), position of the object in the measuring array, background structure, pixel resolution of the instrument, and other design specifications of the detector. Mendez *et al.* (2014, 2015) have developed closed-form expressions for this bound and have studied its structure and dependency with respect to important observational parameters. Complementing these results, Lobos *et al.* (2015) have studied the conditions under which the CR bound can be achieved by a practical estimator.

In this contribution, we present a summary of a novel analysis (Echeverria *et al.* 2016) of the best precision that can be achieved to determine the location of a point source on a CCD-like detector array in a Bayesian CR setting (BCR hereafter). This changes in a fundamental way the nature of the inference problem: from a parametric context - in which we are estimating a constant parameter from a set of random observations - to a setting in which we estimate a random object from observations that are statistically dependent with the position.

We derive new closed-form expressions for the BCR as well as expressions to estimate the gain in astrometric precision. An insightful corollary of this analysis is that the Bayes setting *always* offers a better performance (tighter bound) than the parametric setting, even in the worse-case prior (i.e., that of a uniform distribution or of a very loose prior). This is a very important since, as we demonstrate, the bound can be reached through the use of the conditional expectation estimator (see our full paper for details).

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REFERENCES

Echeverria, A., Silva, J. F., Mendez, R. A., and Orchard, M., 2016, A&A, 594, A111

Lobos, R. A., Silva, J. F., Mendez, R. A., & Orchard, M. 2015, PASP, 127, 1166

Mendez, R. A., Silva, J. F., Orostica, R., & Lobos, R. 2014, PASP, 126, 798

Mendez, R. A., & Silva, J. F. 2015, Revista Mexicana de Astronomia y Astrofisica Conference Series, 46, 77