RevMexAA (Serie de Conferencias), **49**, 19–23 (2017)

TIDAL EVOLUTION OF PLANETARY SYSTEMS

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RESUMEN

En este trabajo revisamos la evolución orbital y rotacional de sistemas de uno y dos planetas afectados por disipación de mareas. En el contexto de perturbación gravitacional mútua e interacción de marea entre la estrella central y el planeta interior, presentaremos los principales resultados para la variación de excentricidades en ambos casos. Estos resultados fueron obtenidos através de la simulación numérica de las ecuaciones exactas del movimiento planetário. También haremos un análisis de la rotación planetaria, la cual puede ser temporariamente capturada en configuraciones especiales como resonancias spin-órbita. Serán mostrados resultados usando una ley de deformación viscoelastica para el planeta interno. Esta reologia es caracterizada por un tiempo de relajación viscoso, τ, que puede ser visto como el tiempo medio caracteristico que el planeta precisa para alcanzar una nueva figura de equilibrio luego de ser perturbado por un forzamiento externo (la marea de la estrella)

ABSTRACT

We review the orbital and rotational evolution of single and two-planet systems under tidal dissipation. In the framework of mutual gravitational perturbation and tidal interaction between the star and the innermost planet, we shall present the main results for the variations of eccentricities in both cases. These results are obtained through the numerical solution of the exact equations of motions. Moreover, we will also give an analysis of the planetary rotation, which can be temporarily trapped in special configurations such as spin-orbit resonances. Results will be shown using a Maxwell viscoelastic deformation law for the inner planet. This rheology is characterized by a viscous relaxation time, τ, that can be seen as the characteristic average time that the planet requires to achieve a new equilibrium shape after being disturbed by an external forcing (tides of the star).

Key Words: planets and satellites

XV Latin American Regional IAU Meeting (Cartagena de Indias, Colombia, 3-7 October 2016)
Editors: Mario Armando Higuera Garzón & Santiago Vargas Domínguez

1. INTRODUCTION

The application of classical theories (e.g., Darwin 1880; Kaula 1964; Ferraz-Mello et al. 2008) predicts that the tidal interaction between a star and a short-period planet leads to orbital decay and eccentricity damping in timescales that depends on the physical parameters of the system. Recently, other approaches (e.g., Ferraz-Mello 2013; Correia et al. 2014) incorporate a law of deformation which is controlled by a parameter depending on the internal viscosity of the deformed body (creep or viscoelastic deformation). In this work we describe the main results for the orbital and rotational evolution considering a viscoelastic deformation of the tidal interacting planet. We consider single and two-planet systems and apply for real extrasolar planetary systems.

2. TIDAL EVOLUTION OF SINGLE-PLANET SYSTEMS

2.1. *Equations of motion*

We consider a system consisting of a central star of mass m_0 and a companion planet of mass m_1. We assume that the planet is deformed by the tidal action of the central star in such a way that the body responds to deformation following a Maxwell viscoelastic rheology (Darwin 1880). The equations of motions governing the evolutions of the planet's orbit and rotation are:

$$\ddot{\vec{r}} = -\frac{\mu_1}{r^2}\hat{r} - \frac{3\mu_1 R^2}{2r^4}J_2\hat{r} - \frac{9\mu_1 R^2}{r^4}\Big[C_{22}\cos 2\gamma - S_{22}\sin 2\gamma\Big]\hat{r} + \frac{6\mu_1 R^2}{r^4}\Big[C_{22}\sin 2\gamma + S_{22}\cos 2\gamma\Big]\vec{K}\times\hat{r} \tag{1}$$

and

$$\ddot{\theta} = -\frac{6Gm_0m_1R^2}{Cr^3}\Big[C_{22}\sin 2\gamma + S_{22}\cos 2\gamma\Big] \tag{2}$$

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where R is the radius of the planet, C its principal inertia moment, $\mu_1 = G(m_0 + m_1)$, r is the distance between the planet and the star and $\gamma = \theta - f$, with θ and f are the angle of rotation (referred to a fixed reference direction) and the true longitude, respectively (the reader is referred to Correia et al. (2014) for further details). The quantities C_{22}, S_{22} and J_2 are related to the instantaneous shape of the planet figure, accounting for equatorial and polar deformations. Because the planet is not rigid and can be deformed under the action of a perturbing potential, the gravity field coefficients J_2, C_{22} and S_{22} are not constant but vary according to the viscoelastic deformation law (see equation (17)–(19) of Correia et al. (2014)).

A fundamental parameter appearing in the viscoelastic rheology is the relaxation time, τ. This quantity can be interpreted as the time for which the planet acquires its equilibrium figure after being deformed by the tidal interaction with the star. Large and small τ values corresponds to "hard" and "soft" planets, respectively, characterizing its degree of fluidity. Because τ is poorly constrained (or unknown) even for Solar System's bodies, we take values of τ covering a range of several orders of magnitude.

2.2. *Stationary solutions*

Fig.1 illustrates the dissipation regimes resulting from the consideration of the viscoelastic deformation law. For low frequencies ($\tau\omega_k << 1$, where $\omega_k = 2\Omega - kn$, with k an integer, $\Omega = \dot{\theta}$ and n the mean orbital motion), the regime corresponds to the usually known as "viscous" or constant time-lag approximation, widely described in the literature (e.g., Mignard 1979; Hut 1981). For high frequencies ($\tau n > 1$), the dissipation regime is inversely proportional to a given power of the tidal frequency, as proposed by Efroimsky & Williams (2009). One one hand, for low frequencies, the stationary solutions of the planetary rotation corresponds to pseudo-synchronous motion, for which the ratio Ω/n is larger than unity by a quantity depending only on the orbital eccentricity, thus, the synchronous motion is only attained for circular orbits. On the other hand, for high frequencies, the stationary solutions for the rotations are such that $\Omega/n = p/q$, with p and q integers. These configurations are known as spin-orbits resonances (see Fig. 4 Correia et al. (2014)).

2.3. *Numerical simulations*

The spin-orbit evolution is obtained through the numerical integration of Eqs. (1) and (2), together with the deformation law for the gravity coefficients.

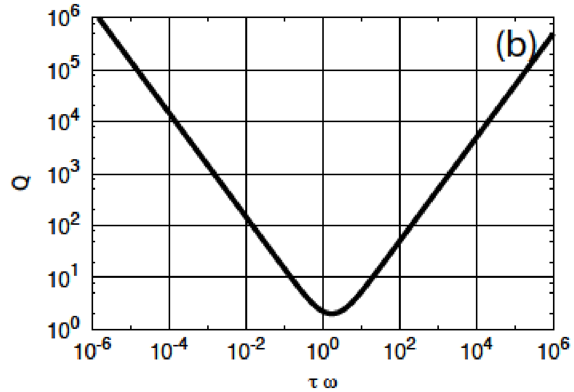

Fig. 1. The quality factor Q, which $1/Q$ is a measure of tidal dissipation, showing the two regimes for low and high frequencies.

Fig. 2 shows the time variation of semi-major axis (a), eccentricity (e), planet rotation (Ω/n) and gravity coefficients (J_2 and $\epsilon = \sqrt{C_{22}^2 + S_{22}^2}$), corresponding to the planet HD 80606b, which is a hot Jupiter of 111.4 d of orbital period around a Sun-like star, having 4.1 Jupiter masses. For all values of τ, we note the orbital decay and circularization (top panels). For small τ, the rotation evolves following the stationary pseudo-synchronization, whereas for larger values of τ, many spin-orbit resonances naturally arises. These temporary trappings are destabilized as the orbit becomes more circular, in agreement with classical results (e.g., Goldreich & Peale 1966). For all τ, the rotation is finally captured in the synchronous motion at the end of the simulations.

Fig. 3 indicates that the instantaneous shape of the planet oscillates around its mean equilibrium value (see Eqs. (85)–(86) of Correia et al. (2014)) for small τ. The equatorial deformation, measured by ϵ, follows the mean equilibrium value for a specific spin-orbit resonance (Eq. (87) in Correia et al. (2014)) for large τ.

Fig. 4 show the evolution of the rotation for two super-Earth planets, namely, Kepler-78 b and 55 Cnc e, with masses of 1.7 $M_\oplus$ and 8.6 $M_\oplus$, respectively, and very short orbital periods (see details in Correia et al. (2014)). For large τ, the relaxation time is permanently larger than the orbital period. So, for the considered values of τ, super-Earth are in the high frequency tidal regime. Hence, the rotation is always temporary trapped in spin-orbit resonance and, as the eccentricity decays, the resonance becomes unstable and the rotations is trapped in the following low-order spin-orbit resonance and is finally captured in the synchronous motion.

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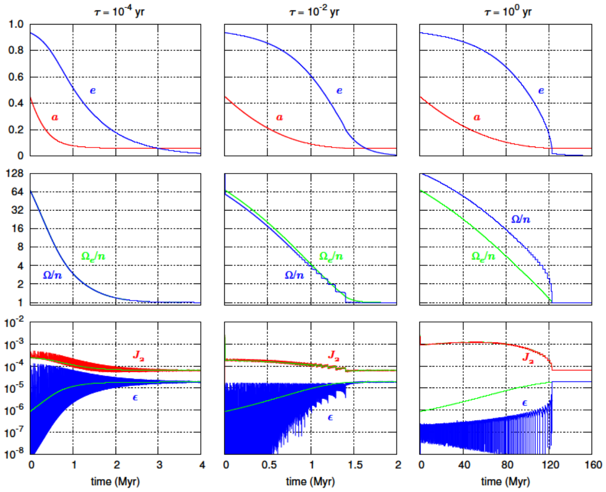

Fig. 2. Time variation, for different values of the relaxation time, of orbital elements (top panels), rotation (middle planes) and instantaneous shape (bottom panels) of the planet HD 80606b.

3. TIDAL EVOLUTION OF TWO-PLANET SYSTEMS

In this section we assume that there exist a second companion in exterior orbit with mass m_2. In addition, we assume that only the inner planet is deformed due to the tides raised by the star. Adding the corresponding mutual gravitational perturbation between the planets, the equation of motions now are

$$\ddot{\vec{r}}_1 = -\frac{\mu_1}{r_1^3}\vec{r}_1 + Gm_2\left(\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|^3} - \frac{\vec{r}_2}{r_2^3}\right) + \vec{f} + \vec{g}_1 + \frac{Gm_2}{\mu_2}\vec{g}_2 \,, \tag{3}$$

$$\ddot{\vec{r}}_2 = -\frac{\mu_2}{r_2^3}\vec{r}_2 + Gm_1\left(\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|^3} - \frac{\vec{r}_1}{r_1^3}\right) + \vec{g}_2 + \frac{Gm_1}{\mu_1}\left(\vec{f} + \vec{g}_1\right) \,, \tag{4}$$

where $\mu_i = G(m_0 + m_i)$ for $i = 1, 2$, $\vec{r}_i$ are the astrocentric positions of the planets, $\vec{f}$ is given by Eq. (1) and $\vec{g}_i$ are the contributions due to the general relativity(see Kidder 1995).

Fig. 5 shows the time variation of eccentricities and rotation of the inner planet for several values of τ, resulting from the numerical solution of Eqs. (3)–(4) and applied to the system CoRoT-7. This system is composed by an inner super-Earth planet ($m_1 = 4.73M_\oplus$) and a Neptune-like mass planet ($m_2 = 13.56M_\oplus$), both in short-period orbits (see details in Rodríguez et al. (2016)).

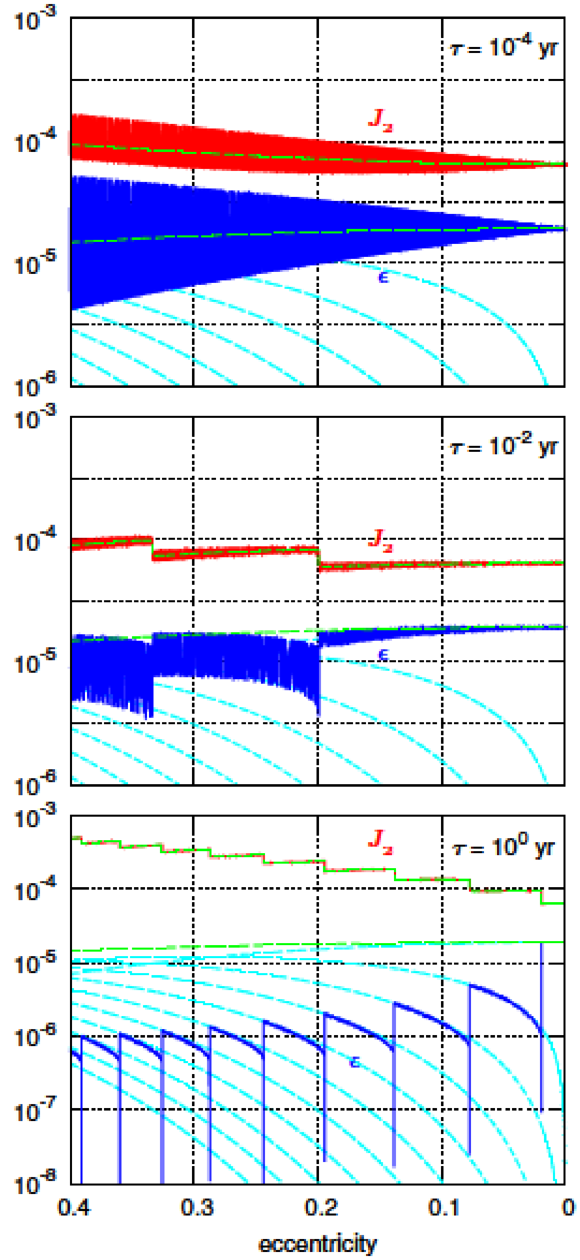

Fig. 3. Evolution of polar (J_2) and equatorial (ϵ) deformations as a function of the eccentricity for different values of τ, corresponding to HD 80606b.

In panel (a), corresponding to $\tau = 10^{-3}$ yr, the planet is in the low-frequency regime since $n_1\tau < 1$. In this regime, the orbital evolution of the system is expected to be similar to the linear tidal model, for which the tidal dissipation is proportional to the corresponding tidal frequency, that is, the eccentricity of both orbits is rapidly damped, in agreement with previous results (e.g., Ferraz-Mello et al. 2011; Rodríguez et al. 2011). According to this model,

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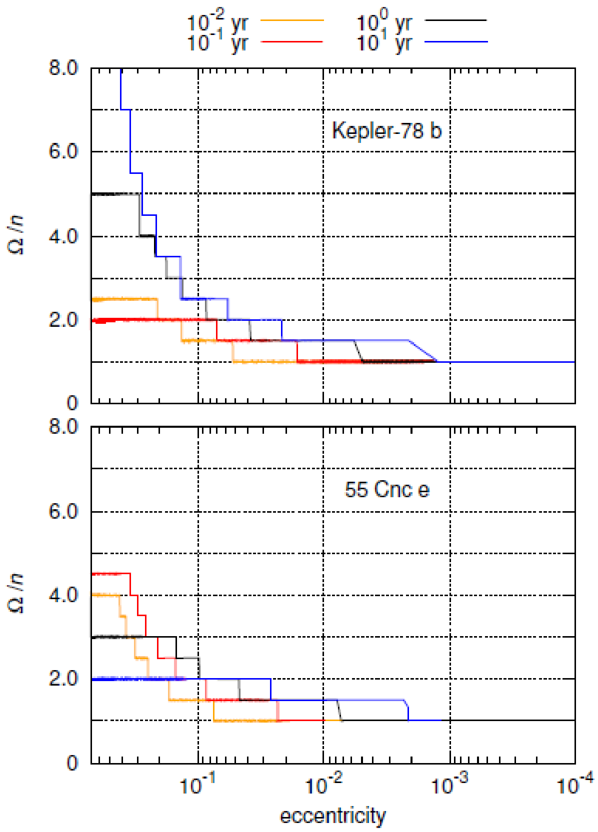

Fig. 4. Evolution of the rotation as a function of eccentricity for Kepler-78 b and 55 Cnc e super-Earth planets.

the rotation of the planet evolves into the pseudo-synchronization and when $e_1 = 0$, the synchronous motions is attained.

In panels (b) and (c), corresponding to $\tau = 10^{-2}$ yr and 10^{-1} yr, respectively, we still observe a rapid synchronization of the rotation with the orbital motion, while both eccentricities are quickly damped to zero. The only difference is that in panel (c) the rotation becomes captured in higher order spin-orbit resonances ($\Omega/n_1 =$ 5:2, 2:1, 3:2) at the beginning of the simulation, that are nevertheless quickly destabilized until the spin reaches the synchronization. Dissipation of the tidal energy only occurs in the inner planet, but both eccentricities are damped since the system is coupled.

In panel (d), corresponding to $\tau = 1$ yr, we observe that the rotation evolves through a succession of temporary trappings in spin-orbit resonances (3:1, 5:2, 2:1, 3:2), ending with synchronous motion (1:1). In this case, the rotation spends more time trapped in higher order resonance than for $\tau = 10^{-1}$ yr. All the resonances are destabilized as the eccentricity decays, in agreement with previous results (Rodríguez et al. 2012), because the capture and escape probability in spin-orbit resonances critically depends on the eccentricity (e.g., Goldreich & Peale 1966).

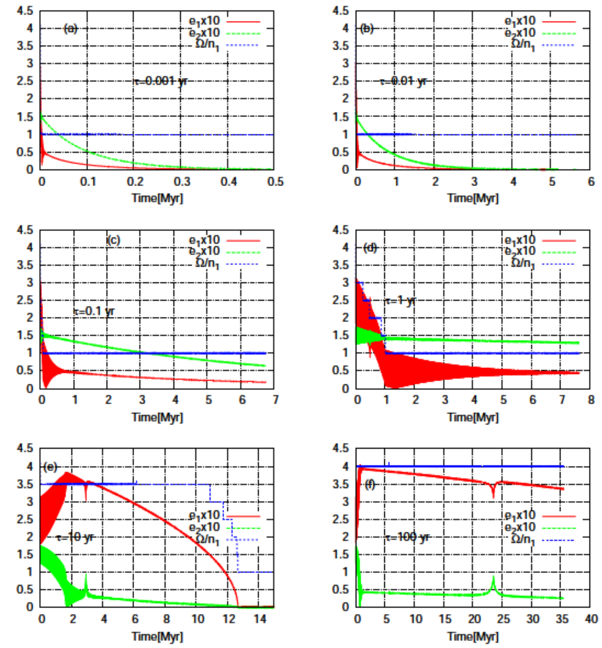

Fig. 5. Time variation of eccentricity and rotation of the inner planet for several values of τ.

In panels (e) and (f), corresponding to the largest values of τ, we observe that the rotation is captured in high order spin-orbit resonance. For $\tau = 10$ yr, the rotation is initially trapped in the 7:2 spin-orbit resonance and for $\tau = 10^2$ yr it is initially trapped in the 4:1 spin-orbit resonance. As explained in (Correia et al. 2014), large τ imply that the relaxation time is much longer than the orbital period, allowing the prolateness of the planet to acquire a large deformation value. This helps the rotation to be captured more easily in spin-orbit resonance.

Unlike previous simulations for lower τ values, in panels (e) and (f) we also observe that the eccentricity of the inner orbit is initially excited to a high value, whereas the outer planet eccentricity is simultaneously damped (due to the angular momentum conservation). The initial excitation of e_1, that we call "eccentricity pumping", is somewhat unexpected, since most studies on tidal evolution of the orbits predict that the eccentricities can only be damped.

4. CONCLUSION

In this paper we have studied the coupled orbital and spin evolution of single and two-planet systems using a Maxwell viscoelastic rheology for the inner planet.

In all situations, the spin evolves quickly until it is captured in some spin-orbit resonance. It then follows through a successive temporarily trappings in some spin-orbit resonances, which are progressively destabilized as the eccentricity decays. Several works on tidal evolution usually assume synchronous motion for the rotation of the close-in companions, as this is the natural outcome resulting from tidal interactions. Nevertheless, for large values of the relaxation times, which is likely the case for most terrestrial planets, we note that the rotation can remain trapped into high-order some spin-orbit resonances for tens of Myr.

For two-planet systems, we observed that there are two different regimes for the orbital evolution. For small τ values, the eccentricity of both orbits is rapidly damped, in agreement with previous results. However, for large τ values, the inner planet eccentricity is pumped to higher values, whereas the outer planet eccentricity is simultaneously damped due to the orbital angular momentum conservation.

REFERENCES

Correia, A. C. M., Boué, G., Laskar, J., & Rodríguez, A. 2014, A&A, 571, A50
Darwin, G. H. 1880, Philos. Trans. R. Soc. London, 171, 713
Efroimsky, M. & Williams, J. G. 2009, CeMDA, 104, 257
Ferraz-Mello, S. 2013, CeMDA, 116, 109
Ferraz-Mello, S., Rodríguez, A., & Hussmann, H. 2008, CeMDA, 101, 171
Ferraz-Mello, S., Tadeu Dos Santos, M., Beaugé, C., Michtchenko, T. A., & Rodríguez, A. 2011, A&A, 531, A161
Goldreich, P. & Peale, S. 1966, AJ, 71, 425
Hut, P. 1981, A&A, 99, 126
Kaula, W. M. 1964, RvGSP, 2, 661
Kidder, L. E. 1995, Phys. Rev. D, 52, 821
Mignard, F. 1979, Moon and Planets, 20, 301
Rodríguez, A., Callegari, N., & Correia, A. C. M. 2016, MNRAS, 463, 3249
Rodríguez, A., Callegari, N., Michtchenko, T. A., & Hussmann, H. 2012, MNRAS, 427, 2239
Rodríguez, A., Ferraz-Mello, S., Michtchenko, T. A., Beaugé, C., & Miloni, O. 2011, MNRAS, 415, 2349