RevMexAA (Serie de Conferencias), **50**, 52–53 (2018)

PARTICLE FILTER-BASED ESTIMATION OF ORBITAL PARAMETERS OF VISUAL BINARY STARS WITH INCOMPLETE OBSERVATIONS

Rubén Clavería[1], Rene A. Mendez[2], and Marcos Orchard[1]

This work addresses the problem of orbital estimation from a Bayesian point of view, using the Particle Filter technique to approximate the posterior distribution of orbital parameters. Additionally, we present a multiple imputation scheme as a means to include partial measurements into the analysis.

In binary stars, mass estimation can be accomplished through the study of their orbital parameters –Kepler's Third Law establishes a strict mathematical relation between orbital period, orbit size (semi-major axis) and the system's total mass. Astronomers frequently deal with the problem of partial measurements (i.e., observations having one component missing, either in (X, Y) or (ρ, θ) representation), which are often discarded. This work presents a particle-filter-based method to perform the estimation and uncertainty characterization of orbital parameters in the context of partial measurements. The proposed method uses a multiple imputation strategy to cope with partial information. The algorithm is tested on synthetic and real data.

Particle Filter (PF) is a family of Monte Carlo techniques to address the *filtering problem*. This problem consists of estimating the real state of a system that: i) can be represented with a space-state model that evolves over time (Eq. 1); and ii) receive information about its current state through noisy measurements at each time step t (Eq. 2). If the system of interest satisfies conditions of linearity and gaussianity, the optimal solution is the well-known Kalman Filter (Kalman 1960). Since most systems do not comply with the aforementioned conditions, sub-optimal approaches such as PF arise as an interesting alternative (Candy 2009).

$$x_t = f(x_{t-1}, w_{t-1}) \quad (\textit{Evolution equation}), \qquad (1)$$

$$z_t = g(x_t, v_t) \quad (\textit{Observation equation}) \qquad (2)$$

In the equations above, state x_t is related to the past state value x_{t-1} and system noise w_{t-1} through function $f(\cdot)$, whereas observation z_t is a function of current state x_t and observation noise v_t. PF is based on the representation of the probability density function (p.d.f) of state x_t by means of a set of samples $x_t^{(i)}$ with their respective weights $w_t^{(i)}$. At each time step, weights must be updated in order to incorporate the information provided by observation z_t:

$$w_t^{(i)} = w_{t-1}^{(i)} \cdot \frac{p(z_t|\tilde{x}_t^{(i)}) \cdot p(\tilde{x}_t^{(i)}|x_{t-1}^{(i)})}{\pi(\tilde{x}_t^{(i)}|\tilde{x}_{0:t-1}^{(i)}, z_{1:t})}, \qquad (3)$$

In this work, we adopt the Artificial Evolution of Parameters approach (Liu 2001). The PF framework is used to approximate the p.d.f. of orbital parameters (not the *state*) of a binary star (P, T, e, a, ω, Ω, i). Particles are forced to *evolve* according to a random walk (Eq. 4) and the statistical characterization of the Mean Square Error is used as a likelihood function (Eq. 5):

$$x_{t+1}^{(i)} = x_t^{(i)} + \epsilon_t^{(i)}, \text{ with } \epsilon_t \sim \mathcal{N}(0, \Sigma), \qquad (4)$$

$$\mathcal{Y}_t^{(i)} = \frac{1}{N} \sum_{k=1}^{N} \frac{1}{\sigma_x^2(k)} [X_k - X_{comp}^{k,t,i}]^2 + \frac{1}{\sigma_y^2(k)} [Y_k - Y_{comp}^{k,t,i}]^2, \qquad (5)$$

where i, k index the particles and the epochs, respectively. In Eq. 5, observations X_k (Y_k) are compared to the positions $X_{comp}^{k,t,i}$ ($Y_{comp}^{k,t,i}$) defined by particle $x_t^{(i)}$ at epoch k. It can be proven that, under certain assumptions, $\mathcal{Y}_t^{(i)}$ follows a Gamma distribution.

Since the vast majority of data-processing techniques require complete data sets as an input, a common technique to cope with missing data is to fill in or *impute* plausible values. In this work, a multiple imputation (MI) scheme is preferred over the single imputation approach. MI replaces each missing datum with *a set* of plausible values, which represents the uncertainty about the right quantity (Rubin 2004). When performing MI, the most critical aspect to be considered is how the imputed values are obtained (i.e., the distribution from which the imputed samples are drawn), and this aspect is problem-dependent. In this work, we use the conditional p.d.f. of orbital parameters x_t as a proposal distribution for imputed values.

The experiments carried out are described next. Parameters of the well-known binary star Sirius were

[1]Departamento de Ingeniería Eléctrica, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Av. Tupper 2007, Santiago, Chile, Postal Code 8370451 (rclaveria@ing.uchile.cl, morchard@ing.uchile.cl).

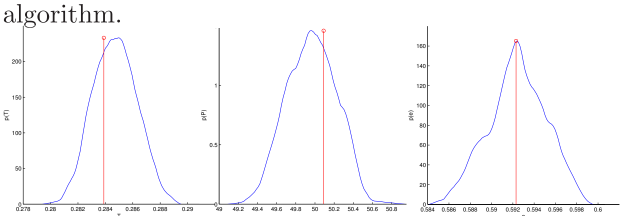
[2]Departamento de Astronomía, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Casilla 36-D, Santiago, Chile, Postal Code 7591245 (rmendez@das.uchile.cl).

TABLE 1

AVERAGE VALUE OF ESTIMATED PARAMETERS

	T	P $[yr]$	e	a $[arcsec]$	ω $[rad]$	Ω $[rad]$	i $[rad]$
Input Parameters	0.2839	50.09	0.5923	7.5000	2.5703	0.7779	2.3829
Complete data	0.2820 (0.0033)	50.4178 (0.5571)	0.5961 (0.0045)	7.5030 (0.0289)	2.5639 (0.0234)	0.7735 (0.0201)	2.3766 (0.0075)
Data Discarding	0.2859 (0.0144)	49.8637 (2.3624)	0.5909 (0.0153)	7.4874 (0.1129)	2.5669 (0.0239)	0.7717 (0.0213)	2.3823 (0.0128)
Multiple Imputations	0.2830 (0.0059)	50.2793 (0.9475)	0.5949 (0.0122)	7.5035 (0.0354)	2.5670 (0.0397)	0.7740 (0.0316)	2.3770 (0.0072)

used to generate synthetic data (values in Table 1), with observation noise having standard deviation $\sigma = 0.075''$ for both X and Y axes. Three scenarios were considered: i) Full data set is available ($N = 11$ measurements; Figure 2a). ii) Data Discarding (Fig. 2b): observations at epochs 10 and 11 (filled circles in the figure) are incomplete –only X is known– and discarded from the analysis. 3) Multiple Imputations (Fig. 2c): same data set as in the previous point, but Multiple Imputation Particle Filter is carried out in order to include partial information. Finally, the algorithm is applied to objects HU177 and I669AB, whose data data is taken from (Tokovinin 2015). Table 1 displays the results obtained from a series of 10 repetitions of the experiments described previously (standard deviation in parentheses). Figure 1 shows an example of the estimated p.d.f. of orbital parameters obtained by using the proposed algorithm.

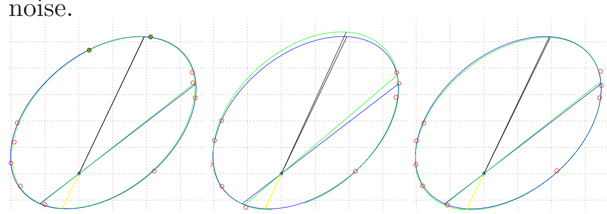


(a) Parameter T **(b)** Parameter P **(c)** Parameter e

Fig. 1. Marginal p.d.f. of parameters (T, P, e). (bar: real value; continuous line: estimated p.d.f.)

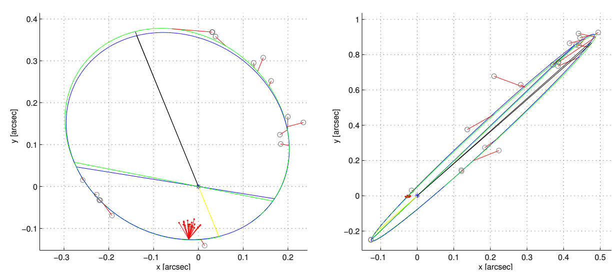
Figure 2 shows a visual comparison of the results obtained in the three scenarios (black line: reference orbit; gray line: orbit estimate). Figure 3 displays a visualization of the results of our method applied to real data (black: orbit obtained with traditional least-squares algorithm; gray: orbit obtained with PF). Real observations are represented by circles, whereas imputed observations are represented by dots. In Figure 2b the orbit estimate shows a certain degree of inconsistency with the reference orbit.

The contribution of this work is twofold: first, it adds robust uncertainty characterization to the problem of orbital parameters estimation; secondly, it provides a strategy to cope with incomplete observations. Results on artificial data suggest that the incorporation of incomplete observations can increase the precision of the estimation without a noticeable decrease in the accuracy. Future work includes the use of uncertainty characterization in the planning of astronomical observation campaigns, investigating alternative convergence criteria for the particle filter and formulating an adaptive evolution noise.



(a) Full Data Set **(b)** Discarding **(c)** Imputations

Fig. 2. Orbit estimation in three scenarios.



(a) HU177 **(b)** I669AB

Fig. 3. Orbit estimates of real objects

Acknowledgements: R.A.M. acknowledges support from the Chilean Centro de Excelencia en Astrofísica y Tecnologas Afines (CATA) BASAL PFB/06, from the Project IC120009 Millennium Institute of Astrophysics (MAS) of the Iniciativa Cientfica Milenio del Ministerio de Economía, Fomento y Turismo de Chile, and from FONDECYT #115 1213.

REFERENCES

Candy, J. 2009, Bayesian Signal Processing Classical, Modern, and Particle Filtering Methods, (Wiley-IEEE Press)

Kalman, R. E. 1960, J. Fluids Eng., 82(1), 35

Liu, Jane & West, Mike. 2001, Combined parameter and state estimation in simulation-based filtering, (Springer, New York, NY)

Rubin, D. 2004, Multiple imputation for nonresponse in surveys, (Wiley-IEEE Press)

Tokovinin, A., Mason, B. D., Hartkopf, W. I., Mendez, R. A., & Horch, E. P., 2015, AJ, 150, 50