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ANALYSIS OF THE K-FACTOR IN THE COBB- DOUGLAS HABITABILITY FUNCTION FOR EXOPLANETS FROM THE BUCKINGHAM THEOREM

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The proper determination of the habitability factor of extrasolar planets can contribute to their differentiation. At present, more than 3500 extrasolar planets have been detected (Christiansen et al. 2018) by methods that allow the identification of physical parameters of these exoplanets and the inference of their velocity. In this regard, strategies inspired by econometric studies have been used to evaluate the habitability of extrasolar planets (Bora et al. 2016). The habitability of these extrasolar planets is based on the Cobb-Douglas function, which is reformulated as a habitability production function, thus presenting itself as a new model for calculating the habitability factor of an extrasolar planet (Bora et al. 2016). This procedure has been validated with the TRAPPIST-1 system (Saha et al. 2018).

The function $\mathcal{Y}$ establishes a relation between some physical parameters of the exoplanets as the radius (R), density (D), escape velocity (v_e) and surface temperature (T_s) and is given by:

$$\mathcal{Y} = KR^{\alpha}D^{\beta}T_s^{\gamma}v_e^{\delta}. \qquad (1)$$

To calculate the habitability from equation (1), K is given as a constant and arbitrary. In fact, some authors assume $K = 1$ (Bora et al. 2016; Saha et al. 2018). However, the optimization of the exponent's elasticity into the function is determined using the statistical information from the extrasolar planet catalogs (Christiansen et al. 2018). The associated physical variables (Schulze-Makuch et al. 2011) allow us to estimate the Earth Similarity Index (ESI) from some pairs of variables, proposing the ESI as a measure of Earth similarity. The K-factor is not introduced in this formulation since the similarity indices are dimensionless at the input because they relate only one variable.

By understanding the habitability factor $\mathcal{Y}$ as a dimensionless number, we relate the physical parameters involved to the Cobb-Douglas function of equation (1) using the Buckingham's theorem, which allows to determine an analogous functional relation that is dimensionally consistent with certain conditions on the elasticity in the exponents, in addition to an explicit form for the factor-K and to the fact that is establishes evidence between this relation with the Boltzmann constant, k_B in equation 1

$$\mathcal{Y} = k_B R^{\alpha-3}D^{\beta-1}T_s^{\gamma+1}v_e^{\delta-2}. \qquad (2)$$

Thus, we find an analogous relation and certain conditions on the elasticity of the exponents, initially to preserve dimensional homogeneity. By applying the Buckingham's theorem it is identified that the density contains information about the mass and volume of the planet, its radius and, by estimating the Earth Similarity Index (ESI) with this pair of variables, information would be repeated.

Initially $E = k_B T$ is introduced as a scaling factor for the energy values, but this variable can be changed by the luminosity of the primary star and, from the Steffan-Boltzman law, the temperature of the primary to the fourth power and the semi-major axis would be related to the radius, density and escape velocity of the exoplanet. It is possible, in a first instance, to extend the function of the habitability of equation (1) with the inclusion of a set of variables associated with the primary star, such as its temperature, the average distance between the star and the exoplanet itself, making it possible to perform a computation process associated with a method of maximization linked to the elasticity in the exponents such that this function is concave or convex, similar to the work of Bora et al. (2016).

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