

THE RELIABILITY OF THE TITIUS-BODE RELATION AND ITS IMPLICATIONS FOR THE SEARCH FOR EXOPLANETS

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The major semiaxes of the planets in our Solar System obey a simple geometric progression known as the Titius-Bode Relation (TBR), whose physical origin remains disputed. Poveda & Lara (2008) found that the 55 Cancri system follows a TB-type exponential relation ($a_n = a_0 e^{bn}$), with a correlation coefficient greater than 0.992, if a vacancy is considered between a pair of planets, that is, a planet not yet discovered due to observational limit. The objective of this work is to quantify the capacity of the TBR to predict exoplanets in planetary systems from observational data (see also Bovaird & Lineweaver 2013).

The periods of the planets of the solar system will help us to discern if all the planets of planetary systems are detected. The selected periods were those of Mercury, Venus, Earth, Mars, Jupiter and Saturn, with a vacancy between Mars and Jupiter. When we make the TB fit to this orbital configuration, we obtain a correlation coefficient $R^2 = 0.9920$, a coefficient that was used as a reference in our method. In order to evaluate the predictive capacity of the TBR, we only used observational data from exoplanetary systems. The sample includes 27 exoplanetary systems with 5 planets or more.

The methodology was as follows:(1) The first adjustment is made taking the values of the first 4 periods observed. (2) If the fit does not have a correlation coefficient of $R^2 \geq 0.992$; vacancies are inserted between the observed planets until the fit meets the desired criteria. (3) Once the desired fit was found, we analyze whether the vacancies or extrapolations of that fit reproduce the periods of the next discovered planets. (4) If the system has more than 5 observed planets, the previous procedure is repeated adding to the first four data, the value of the period of the fifth planet discovered to see if the predictions hold. Applying the described method, it is considered as a successful case when vacancies (predictions) are maintained in every fit; and when the relative er-

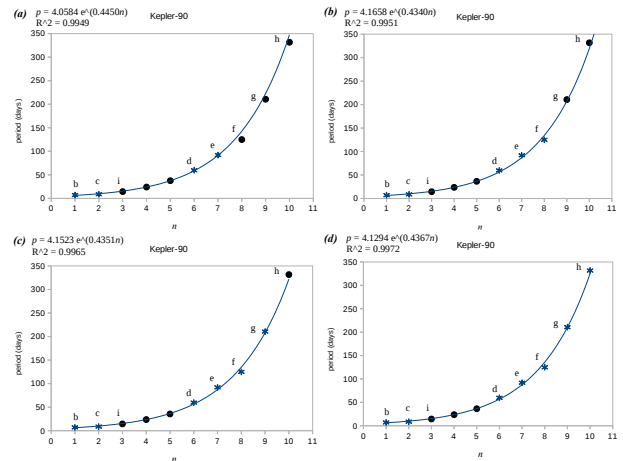


Fig. 1. Method applied to the planetary system Kepler-90. The asterisks denote the planets used in the fit and the dots are the vacancies.

ror between the value of the period predicted by the TBR and the observed value is not greater than 33%. Figure 1 shows an example of a successful case. Using this method, we obtain 21 successful cases and 6 unsuccessful cases.

To assess the statistical significance of our results, we applied our method on randomly generated systems. We generated 900 random systems using 3 types of distributions: normal, exponential and decreasing exponential. We found that there were fewer successful cases than failed ones, which is contrary to what was found for real systems (see Lara et al. 2020).

Using the sample and proposed method, we conclude that TBR fits to planetary systems with at least four planets allow exoplanets to be predicted with a 78% probability. Using a statistical test, we found that the periods of the planets in real exoplanetary systems are not consistent with a random distribution. Vacancies may represent undiscovered planets, asteroids, or rings that could be below the observational limit.

REFERENCES

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