QUANTIFICATION OF HABITABILITY IN ROCKY PLANETARY BODIES

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RESUMEN

Primero, se presenta un modelo conceptual general para la abiogénesis-biogénesis basado en las leyes de la Física y la Química, potencialmente aplicable en todo el Universo observado. De acuerdo con este modelo se hace una breve revisión del desarrollo de las métricas de habitabilidad, con énfasis en los ambientes de los cuerpos planetarios rocosos. Se incluyen varios estudios de casos relevantes para la Astrobiología.

ABSTRACT

First, it is presented a general conceptual model for abiogenesis-biogenesis based in laws of Physics and Chemistry, potentially applicable through the entire observed Universe. According to this model a brief review on the development of habitability metrics is done, with emphasis in environments in rocky planetary bodies. Several case studies relevant for Astrobiology are included.

Key Words: astrobiology — habitability — rocky planetary bodies

1. GENERAL REMARKS ON THE QUANTIFICATION OF HABITABILITY

The Quantification of Habitability is an interdisciplinary and emerging area of Natural and Exact Sciences. There are three (complementary) approaches to address it (Shock & Holland 2007). The astrobiological one focuses on investigating the most basic conditions for the existence of primary producers anywhere in the Universe, the biogeochemical focuses on the organism-environment interaction, while the ecological pays special attention to the interactions between organisms within the context of the ecosystem. Despite their complementarity, much remains to be done to better understand the complex phenomenon of life in the Universe. Ecologists began to develop habitability metrics during the 1970's: the so called habitat suitability indexes. On another hand, the discovery of exoplanets in recent decades has motivated the development of habitability metrics for Astrobiology. Therefore, in this talk we review the emergent topic of Quantitative Habitability for Astrobiology as related with Environmental Sciences. It is included the modeling of habit-ability for planetary subsurfaces, something relatively overlooked in many previous studies. A natural question arises: can be devised a general conceptual model for Abiogenesis (origin of Life)Biogenesis (evolution of Life), in principle valid for the entire Universe?

Accepting that the most basic laws of Nature, formulated by Physics and Chemistry, are valid in all the observed Universe (something favored by an abundant set of astrophysical observations) leads to a general conceptual model for Abiogenesis-Biogenesis, in principle applicable to any part of our observed Universe. Actually, this model can be inferred from the following four premises for life to arise and evolve (Hoehler 2007); (Cockell et al. 2016):

(1) The presence of biogenic chemical elements in adequate concentrations (on Earth all known species contain at least CHON, P and S).

(2) A solvent in which the above mentioned elements can react to form the complex biological molecules (water plays this role in life on Earth).

(3) An external energy source to overcome the activation barriers of biochemical reactions, to maintain the high degree of organization of living things (low entropy), and to do work. In life known on Earth these sources are luminous energy for photosynthetic species, and energy of redox chemical re-actions for chemosynthetic ones (there are also mixotrophs, capable of using both kinds of energy sources, according to their availability).

(4) The existence of a favorable physical-chemical environment allowing the viability of living entities (background radiation, temperature, pH, salinity, etc.).

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2. QUANTITATIVE HABITABILITY THEORY

Within the astrobiological school of Quantitative Habitability, of special interest is the emerging and interdisciplinary Quantitative Habitability The-

ory (QHT), which traces a bridge between Ecology and Astrobiology, and whose main objective is to explain the distribution, abundance and productivity of life. It is scalable in time and space, so that it can be applied both at planetary and ecosystem scales, and to any life at any stage of evolution of the Universe (Méndez 2010), (Cardenas et al. 2014). Its main postulate suggests that, in principle, a habitability index HI may be written as a product of functions of environmental variables $fi\{xj\}$ influencing life:

$$HI = \prod_{i=1}^{n} = fi(\{xj\}) \qquad (1)$$

A crucial aspect of habitability indexes is that with them net primary productivity (NPP) can be estimated (Mendez 2010):

$$NPP = HI * NPP_{\max} \qquad (2)$$

where $NPPmax$ is the maximum possible NPP. Combining the conceptual model for abiogenesis-biogenesis with QHT (eq. 1), a generic habitability index can be formulated as:

$$HI = f_{M} f_{K} f_{E} f_{PC} \qquad (3)$$

where f_{M}, f_{K}, f_{E} and f_{PC} are functions representing, respectively, the chemical (mineral), kinetic, energetic and physicochemical (environmental) premises for abiogenesis and biogenesis.

3. QUANTIFICATION OF THE ENERGETIC ASPECT OF LIFE

As said in section 1, on Earth we know photosynthetic organisms, which use light as source of energy, and chemosynthetic ones, which use the energy released in redox chemical reactions. The so called primary producers obtain carbon from inorganic sources (like CO_2), so they are called autotrophs, not depending on other living beings to get carbon, (unlike heterotrophs, who obtain carbon from organic matter synthesized by other beings). So photoautotrophy and chemoautotrophy are the basis of the biosphere, while photoheterotrophy and chemoheterotrophy are at a higher trophic level. Because CO_2 is closely related to the evolution of the inorganic world (carbonate rocks, karst), autotrophs are also often called photolithotrophs or chemolithotrophs, depending on whether they use

light or chemical energy as primary energy source. They form the basis of the trophic or food assembly, of which depend all other organisms, that's why they are called primary producers. Therefore, to estimate the habitability of natural environments, these two groups of organisms are very important and emphasis on them is put in this talk. While photoautotrophs can be unicellular (e.g., phytoplankton) or multicellular (e.g., higher plants), all chemoautotrophs discovered so far are unicellular organisms (prokaryotes). In most cases in the literature, like in this talk, for simplicity autotrophs are called photosynthetic or chemosynthetic organisms. It is noteworthy that until the 1970's, photosynthesis was by far considered the dominant mechanism of primary production in our planet. However, after the detection in 1977 of ecosystems based on chemosynthesis in the Galapagos Fault in the eastern Pacific, frequent discoveries in oceanic and continental depths have shown diverse ecosystems very dependent on chemosynthesis as primary production mechanism (Sarbu et al. 1996);(Pohlman 2011),(Por 2008). The ubiquity of chemosynthesis at planetary depths has motivated to propose a new model for the biosphere on Earth (Por 2008). It consists of a surface biosphere (eubiosphere), which basically depends on photosynthesis and has an oxidant redox state; a bacteriosphere in the deep crust where only live prokaryotes that perform chemosynthesis from compounds originating from the mantle (especially sulfur compounds), and has a reducing redox state; and an intermediate deuterobio-sphere in the oxidizing-reducing interface dependent on both mechanisms of primary production, although probably more on chemosynthesis. The main biomes (major subclasses of ecosystems) that make up the deuterobiosphere are caves, anchialine caves (connected underground with the sea), cold seeps into the deep ocean, and hydrothermal vents at the junction of tectonic plates on the ocean floor or near underwater volcanoes (Por 2008). The deuterobiosphere is largely an avegetal aquatic world, mainly populated by chemosynthetic prokaryotes and invertebrate animals. Some authors suggest that in our planet subsurface life is comparable in mass and volume to the surface one, and even that life on Earth may have emerged deep and not on the sea surface (Gold 1992). This makes evident the relevance of chemosynthesis when quantitatively estimating the habitability of a rocky planetary body. For an excellent review on energy sources and their availability in the Solar System we recommend (Cockell et al. 2016).

3.1. *Habitability Metrics for Photosynthesis-Based Environments*

Model E for Photosynthesis

Photosynthesis needs light. The so-called band of photo-synthetically active radiation (PAR), although slightly dependent of the species, approximately coincides with the band of visible light for humans (400-700 nm). In a mathematical model for photosynthesis, it is also needed to include part of the ultraviolet band (UV), as it can inhibit photosynthesis. The part of the needed UV band depends on the parent star and the planetary atmosphere. For instance, in current Earth the atmosphere blocks UV with wavelengths below 280 nm. Thus, for photosynthesis calculations are used UV-B band (280-320 nm, mostly absorbed by ozone, water vapor, oxygen and carbon dioxide) and the UV-A band (320-400 nm), much less absorbed by the atmosphere, but less energetic and less damaging for biosphere). Then calculations of the photosynthetic potential of ecosystems can involve solving the equation of radiative transfer for multilayer atmosphere and ocean models (Thomas & Stamnes 2002):

$$\frac{1}{c}\frac{\partial I_\lambda}{\partial t} + \Omega.\nabla I_\lambda = K(\lambda)(S_\lambda - I_\lambda) \qquad (4)$$

where c is the speed of light in vacuum, I_λ is the spectral radiance, t is time, Ω is a unit vector perpendicular to the plane for which the spectral radiance is calculated, K_λ is the attenuation coefficient of light (including absorption and scattering) and $S\lambda$ is a function of the light sources. In several contexts, researchers prefer to work with spectral irradiances E_λ, (instead of spectral radiances), which implies to use equation (4) written for E_λ. Spectra at planetary surface can be calculated with several computer codes of radiative transfer. A very good one (source free for download) is TUV (Tropospheric Ultraviolet and Visible), developed at the National Center for Atmospheric Research of the United States of America. A very interesting approach is the one developed by the group of Solar Radiation and Clouds of NASA, which assumes the ocean as the lowest layer of the atmosphere: Coupled Ocean-Atmospheric Radiative Transfer code (COART) and can be used online at https://clouds.larc.nasa.gov/jin/coart.html. In (Castillo Alvarez 2017) its coupling to a preliminary proposal for a Cuban coastal thermohydrodynamic model is proposed, whereas in (López et al. 2019) an initial assessment of their applicability to inland water systems (freshwater) is made. Sim-

pler radiative transfer calculations of spectral irradiances E_λ,z) at depth z in aquatic ecosystems can be done considering a field of stationary light (neglecting emission) in equation (4) and using the Lambert-Beer's law of Optics:

$$E(\lambda, z) = E(\lambda, 0^-)e^{-K(\lambda).z} \qquad (5)$$

where $E(\lambda, 0^-)$ are spectral irradiances just beneath the air-water interface. They are obtained by subtracting the reflected light at this interface:

$$E(\lambda, 0^-) = (1 - R)E(\lambda, 0^+) \qquad (6)$$

where $E(\lambda, 0^+)$ are the incident spectral irradiances just above the air-water interface and R is the reflection coefficient, calculated by the Fresnel formulae of Optics. The set of attenuation coefficients $K(\lambda)$ can be obtained by linear interpolation from the reference tables of optical classification of oceanic and coastal waters in (Jerlov 1976). This method gave attenuation coefficients $K(\lambda)$ in the range 220-700 nm for each wavelength (Peñate et al. 2010).

Once obtained the underwater light field, we can proceed to calculate the photosynthetic potential, using a particular model of photosynthesis. For the case of microalgae (unicellular organisms: phytoplankton) it has been widely used the so-called E model (Fritz et al. 2008):

$$\frac{P(z)}{P_S} = \frac{1 - \exp(-\frac{E_{PAR}(z)}{E_S})}{1 + E_{UV}^*(z)} \qquad (7)$$

where $P(z)$ and P_S are photosynthetic potentials at depth z and the maximum possible, respectively. $E_{PAR}(z)$ is the irradiance of photosynthetically active radiation, while $E_{UV}^*(z)$ is the irradiance of ultraviolet radiation, convolved with a biological action spectrum $\epsilon(\lambda)$, which weights the ultraviolet wavelengths according to their potential to inhibit photosynthesis. The expressions for calculating the irradiances are:

$$E_{PAR}(z) = \sum_\lambda E(\lambda, z)\Delta\lambda \qquad (8)$$

$$E_{UV}^*(z) = \sum_\lambda \epsilon(\lambda)E(\lambda, z)\Delta\lambda \qquad (9)$$

where the spectral irradiances $E(\lambda,z)$ are calculated by the aforementioned law of Lambert-Beer.

The parameter E_S is the irradiance giving 63 percent of the maximum possible photosynthesis potential if ultraviolet radiation is negligible. It is a measure of the efficiency of the species in the use of photosynthetically active radiation: the lower the value, the greater the efficiency.

Inclusion of Particulate Ionizing Radiation in the E Model of Photosynthesis

The E model of photosynthesis considers only the inhibitory effect of UV radiation. However, the radiational environment plays an important role in abiogenesis-biogenesis. This motivated some of us to make a first modification of this model to include the effect of particulate ionizing radiation in more severe radiational contexts (Rodriguez et al. 2013):

$$\frac{P(z)}{P_S} = \frac{1 - \exp(-\frac{E_{PAR}(z)}{E_S})}{f_{ri}(z) + E_{UV}^*(z)} \qquad (10)$$

where $fri(z)$ is a function of particulate ionizing radiation. This function can in principle have several forms, depending on the radiational context. For instance, in above mentioned reference it is proposed to calculate this function by:

$$fri(z) = \frac{D_{inc}}{D_n} \qquad (11)$$

where D_{inc} is the dose deposited by ionizing radiation in the severe radiational regime, while D_n is the dose in the "ordinary" radiational regime (note that when returning to a normal mode, these doses are equal and model E of photosynthesis particular case is recovered). Given that the E model was developed assuming a radiational back-ground "ordinary" or "normal" obtaining the specific form of $fri(z)$ involves an elaborate radiation dosimetry dependent on the type of ionizing particle.

Case Study: The radiational Extragalactic shock

Several astrophysical phenomena within our galaxy can deposit significant doses of ionizing radiation at the top of planetary atmospheres, such as stellar explosions. This has raised the possibility of significant influence of radiations in biological evolution. Actually, radiations play a dual role: sterilizing for many species, and for other mutagenic (stimulating speciation). On the other hand, there are indications that even radiation from outside our galaxy have had influence on life on our planet. In (Rodriguez et al. 2013) this possibility is examined in the scenario of extragalactic radiational shock, according to which the Earth receives an increased amount of high-energy cosmic rays when it is in the north of the galaxy. This is because the solar system orbit around the center of the Milky Way is not in a plane, it has periodical ascents (towards the galactic north) and descents (towards the galactic south). When it is relatively close to the galactic north, Earth receives a higher dose of high-energy extragalactic radiation due to the shock wave of the accelerated motion of the Milky Way towards the galaxy cluster Virgo. It is a very interesting coincidence that the period of this cycle roughly coincides with a period of declines in biodiversity detected in the fossil record: in both cases a cycle of approximately 62 million years is followed. This constituted the motivation for the work published in above mentioned reference. As a first consideration it was taken into account that when cosmic rays of high energy reach the top of the atmosphere, interacting with atomic nuclei (primarily nitrogen and oxygen if it is a "modern" atmosphere as Proterozoic and Phanerozoic) it generates various cascades of lower energy particles, which are called secondary cosmic rays. At sea level this are mainly protons, neutrons and muons. Of these particles, the greater penetrating power in the ocean are muons, recognizing that can travel hundreds of meters into the ocean water column. Therefore, in this first modeling muons were the only particles considered. Studies on the biological damage of muons in non-human samples are scarce. However, some suggest that the doses are proportional to the total flux F of muons, and the yield/dose factor has little variation with energy. So, in the case of irradiation with high-energy muons it was assumed:

$$\frac{Finc}{Fn} = \frac{Dinc}{Dn} \qquad (12)$$

The subscript n refers to the respective magnitudes during ordinary or "normal" regime. In a first approximation, it was considered that $fri(z)$ is constant in the water column, partly because this function is a ratio (not an absolute magnitude), and partly because due to the availability of PAR photosynthesis takes place primarily in the first 200 m of the water column. Two extreme cases were considered when Earth is in the galactic north (Atri & Melott 2011).

✓1) Minimum increment of ionizing radiation: $fri(z) = 1.26$.

✓2) Maximum increase of ionizing radiation: $fri(z) = 4.36$.

Table I quantifies the rate at which photosynthesis is kept under the radiational background muons. It is noted that, in the case of minimal increase in

radiation, for all latitudes the reduction of photosynthetic potential is about 20 percent. When the increase is maximum, it is drastically reduced: this falls 4 or 5 times and sometimes even more. As phytoplankton is one of the most basic links in the oceanic trophic assembly, this could lead to a significant decline in biodiversity, reinforcing the hypothesis of extragalactic ionizing radiation as a potential cause of the 62 million years periodic decrease of biodiversity observed in the fossil record (Atri & Melott 2011).

Standard Primary Habitability

The Standard Primary Habitability (SPR) (Méndez 2010) was developed in the Planetary Habitability Laboratory of the University of Puerto Rico at Arecibo (http://phl.upr.edu). It is intended to assess habitability in environments dominated by terrestrial plants. It is a multiplication of a function of relative humidity of air $f(RH)$ by a function of temperature $f(T)$:

$$SPH = f(RH) f(T) \qquad (13)$$

Both functions are defined through similar expressions:

$$f(T) = \left[\frac{(T - T_{min})(T - T_{max})}{(T - T_{min})(T - T_{max}) - (T - T_{opt})^2} \right]^{W_T} \qquad (14)$$

$$f(RH) = \left[\frac{(RH - RH_{min})(RH - RH_{max})}{(RH - RH_{min})(RH - RH_{max}) - (RH - RH_{opt})^2} \right]^{W_{RH}} \qquad (15)$$

In above equations subscripts min, max and opt mean, respectively, minimum, maximum and optimum values for terrestrial plants to live. The parameters W_T and W_{RH} weight the contribution of each function to the habitability metrics SPH. As with many other habitability indexes, SPH take values in the range 0-1, being 0 the value for dead environments and 1 the value for the optimum of life. According to this, the application of SPH has given very interesting outcomes concerning habitability of Earth and other rocky planetary bodies (Méndez 2010); http://phl.upr.edu).

Aquatic Primary Habitability The Aquatic Primary Habitability APH was devised at the Planetary Science Laboratory of Universidad Central Marta Abreu de Las Villas (PSL-UCLV), Cuba. It is intended to assess the habitability of aquatic ecosystems. It is usually acknowledged that the main environmental variables controlling life in these ecosystems are light, nutrients, temperature and salinity. So far, the most general version of APH is for aquatic environments without salt stress (Cardenas et al. 2014):

$$APH = f(L)f(N)f(T) \qquad (16)$$

where $f(L)$, $f(N)$ and $f(T)$ are functions of light, limiting nutrient and temperature, respectively. Particular versions of this index are:

$$APH_I = f(L)f(T) \qquad (17)$$

and

$$APH_{II} = f(L)f(N) \qquad (18)$$

The index represented in equation (17), was applied to environments where light, rather than nutrients, is a limiting variable (Cardenas et al. 2014). Common examples are rocky planets and satellites orbiting red dwarfs, stars with a light emission much smaller than emission of solar-type stars. The index represented by equation (18) was applied to Ana Maria Gulf, a Cuban gulf of importance for tourism and fisheries. The function of light was inspired in the E model for photosynthesis. Photosynthesis rates are depth-dependent, so the actual $f(L)$ used in the aquatic habitability indexes presented above is an average in all the photic zone normalized respect to the optimum average:

$$f(L) = \langle \frac{P(z)}{P_S} \rangle / \langle \frac{P(z)}{P_S} \rangle opt \qquad (19)$$

The calculation of the optimum average was presented in (Cardenas et al. 2014). The function on nutrients used in equation (18) was inspired in an eutrophication index.

3.2. *Habitability Metrics for Chemosynthesis-Based Environments*

The chemosynthetic habitability index QHI was also devised at PSL-UCLV. The light function $f(L)$ in APH was substituted by a function of the chemical energy $f(Q)$, which chemoautotrophic organisms are able to use:

$$QHI = f(Q) f(N) f(T) \qquad (20)$$

The form of the chemical function $f(Q)$ was inspired by suggested analogies between photosynthesis and chemosynthesis concerning the use of energy (Shock & Holland 2007). Ignoring the stage of chemoinhibition, and by analogy with the non-inhibitory part of the E model for photosynthesis, the following chemical function was proposed (Cardenas et al. 2019):

$$f(Q) = 1 - e^{-\frac{I}{q}} \quad (21)$$

where I is the chemical energy per unit area and per unit time that the chemoautotrophic organism receives, and q a parameter of chemosynthetic efficiency. The function of nutrients $f(N)$ is inspired in the well-known Michaelis-Menten kinetics (normalizing respect to its maximum value):

$$f(N) = \frac{v_{max}[LN](K_M + [LN])}{f_{max}(N)} \quad (22)$$

where $[LN]$ stands for the concentration of limiting nutrient, v_{max} is the maximum assimilation rate of the limiting nutrient, and K_M is the semisaturation constant known as Michaelis constant. As function of temperature $f(T)$ it was taken an inverted parabola symmetric respect to the optimum temperature for life $Topt$:

$$f(T) = 1 - \left(\frac{T_{opt} - T}{T_{opt} - 273}\right)^2 \quad (23)$$

A Case Study: Black Smoker TY in the East Pacific Ridge

Hydrothermal vents, such as the black smoker TY located at 2.3 km depth in the East Pacific Ridge, can host very dynamic ecosystems. They are mostly supported by chemosynthesis, although low levels of photosynthesis from geothermal photons have been reported. Typically the reaction which most contributes to chemosynthesis is the oxidation of hydrogen sulfide:

$$H_2S + 2O_2 \rightarrow SO_4^{2-} + 2H^+ \quad (24)$$
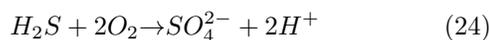
in which the limiting reactant is dioxygen. For the calculation of $f(Q)$ it was assumed a null concentration of dioxygen in the vent and a constant increase up to 5.0 mg/mol at 2.5 m from it. It was also considered the dependence on temperature of the diffusion coefficient D. For the calculation of

the temperature function $f(T)$ it was assumed that the optimum temperature for living organisms is 298 K (25°C), while for the calculation of the nutrients function $f(N)$ we considered the wide range of variation of nitrogen concentration [N] in hydrothermal vents: 10, 100 and 1000 mol/L. Then the chemosynthetic habitability index QHI was calculated. It turned out to be highly sensitive to temperature and to the parameter of chemosynthetic efficiency q, and weakly sensitive to the limiting nutrient nitrogen. However, it should be noticed that the value taken for the Michaelis constant KM for the evaluation of the function of nutrients is one reported for a generic phytoplankton organism (Amemiya et al. 2007), given the scarcity of data for chemoautotrophs. Scarcity of data for these organisms still limits the refinement and applications of this index, especially in the deep biosphere. We hope that ongoing and planned expeditions involving deep sea and continental crust drilling will improve this situation.

4. MERGING THE ASTROBIOLOGICAL AND ECOLOGICAL SCHOOLS

In (Rodríguez-López et al. 2019) some of us showed a way to link the astrobiological and ecological schools of Quantitative Habitability, modifying the phytoplankton-zooplankton dynamics presented in (Ferrero et al. 2006). First we introduced the average net primary production $< NPP >$ in the photic zone, estimating it using an averaged version of equation (2):

$$\langle NPP \rangle = \langle HI \rangle . NPP_{max} \quad (25)$$

Then the dynamics phytoplanktonzooplankton was described by:

$$\frac{dA}{dt} = A[\frac{\langle NPP \rangle}{A_s}] - qH] \quad (26)$$

$$\frac{dH}{dt} = H[e_T qA - \mu] \quad (27)$$

where A and H are (volumetric) biomass densities of phytoplankton and zooplankton, respectively; μ is the mortality rate of zooplankton, q is predation efficiency, while e_T is the transformation efficiency, i.e., conversion efficiency of predated (phytoplankton) matter to zooplankton biomass. For the sake of dimensional homogeneity, it was introduced the (surface) density of phytoplankton carbon biomass A_S.

A Case Study: Movile Cave in Rumania

The paradigmatic Movile Cave in Rumania was the first discovered continental ecosystem totally dependent on chemoautotrophy (Sarbu et al. 1996). In caves usually energy and nutrients are the limiting factors affecting biological productivity, so in this preliminary modeling we are assuming it is energy release through the oxidation of sulfur compounds (although other chemoautotrophs, such as methanotrophic bacteria, are present). In PSL-UCLV we are considering a prey-predator model in which sulfur oxidizing bacteria are the prey and sulfur reducing bacteria are predators. In Movile Cave sulfur oxidation to sulfate in general passes through intermediate-oxidation chemical species, such as thiosulfate, tetrationate and elemental sulfur. However, in this first model we assume the oxidation is straight until sulfate (equation 4). The inverse process is done by sulfur reducing bacteria, which are predators of the sulfur oxidizing bacteria. The equations of the model are then:

$$\frac{d\,[Sob]}{dt} = \frac{r_1\,[H_2S]\,[Sob]}{k_1 + [H_2S]} - \frac{f_1\,[Sob]\,[Srb]}{k_2 + [Sob]} \\ -d_1\,[Sob] + r_e\,[Srb] \quad (28)$$

$$\frac{d\,[Srb]}{dt} = \frac{\eta f_1\,[Sob]\,[Srb]}{k_2 + [Sob]} - d_2\,[Srb] \quad (29)$$

where $[Sob]$ and $[Srb]$ are biomass concentrations of sulfur oxidizing bacteria and sulfur reducing bacteria, respectively, t is time, $[H_2S]$ is the concentration of hydrogen sulfide, k_1 and k_2 are semi-saturation constants, f_1 is the feeding rate of predators and their assimilation rate, while d_1 and d_2 are mortality rates of prey and predator, respectively. In equation (28) the last term is the respiratory model for sulfur reducing bacteria, appearing in the prey equation because H_2S is released in this process (sulfurous respiration), so being indirectly beneficial for the prey (sulfur oxidation bacteria). The critical or singular points ($[Sob]$, $[Srb]$) of the system (28)-(29) are the trivial one (0,0) and:

$$[Sob] = \frac{d_2 k_2}{\eta f_1 - d_2} \quad (30)$$

$$[Srb] = \frac{\eta d_2 k_2 \left(d_1 \left(k_1 + [H_2S]\right) - r_1\,[H_2S]\right)}{\left(k_1 + [H_2S]\right)\left(d_2 - \eta f_1\right)\left(d_2 - \eta r_e\right)} \quad (31)$$

All variables and parameters of this model should be finite and equal or greater than zero, thus from equation (30) it is implied:

$$\eta f_1 > d_2 \quad (32)$$

which leads to two possibilities in equation (21):
✓ (1) Numerator and denominator are both negative.
✓ (2) Numerator and denominator are both positive.

The first option implies for the denominator $d_2 > \eta r_e$, which in turn implies for the numerator:

$$[H_2S] > \frac{d_1 k_1}{r_1 - d_1} \quad (33)$$

We have not found numerical values for chemoautotrophic species for the parameters in equation (33), thus we used those for the predator-prey part of the model appearing in (Amemiya et al. 2007). This gives:

$$[H_2S] > \frac{d_1 k_1}{r_1 - d_1} = 0.0025 \frac{mg}{L} \quad (34)$$

This threshold concentration for H_2S is far smaller than its typical concentration in hydrothermal fluids (the main source of this substance in Movile Cave), which is around 1800 mg/L. Therefore, even using data for other metabolisms (Amemiya et al. 2007), we hypothesize that item number 1 holds for the system (30)-(31), and not number 2, which would imply the inverse situation:

$$[H_2S] < (d_1 k_1) / (r_1 - d_1) = 0.0025 \frac{mg}{L} \quad (35)$$

This is just an example of how mathematical modeling can help in obtaining in-formation on ecosystems in the deep biosphere. Further directions include analyzing the stability of the singular points (30)-(31), which could shed light on management of the peculiar chemoautotrophy-based ecosystem in Movile Cave, and perhaps on other sulfurous caves.

5. HABITABILITY METRICS FOR PLANETARY SCALES

5.1. *Earth Similarity Index*

The index of similarity with Earth (ESI) is applicable on a global scale, so in principle includes aquatic and terrestrial environments (Schulze-Makuch et al. 2011). It is defined through:

$$ESI = \prod_{i=1}^{n} \left(1 - \left|\frac{x_i - x_{i0}}{x_i + x_{i0}}\right|\right)^{\frac{w_i}{n}} \quad (36)$$

TABLE 1

AVERAGE PHOTOSYNTHESIS POTENTIALS COMPARED WITH ORDINARY ONES

Radiational Regime	Latitude (degrees)	Optical type of ocean water	Average potentials for photosynthesis <P/Ps>; %	
			ES = 2 W/m^2	ES = 100 W/m^2
Minimum increment of ionizing radiation	0	I	80.2	81.8
		III	80.7	84.5
	30	I	80.2	81.6
		III	80.7	84.7
	60	I	81.1	84.2
		III	81.0	87.0
Maximum increase of ionizing radiation	0	I	24.1	26.2
		III	25.0	32.2
	30	I	24.0	26.1
		III	22.7	31.5
	60	I	13.4	2.5
		III	25.9	39.1

where x_i is a planetary property, x_{i0} is its value in the current Earth, w_i is a weight exponent and n is the number of planetary properties considered for the index. This set of variables was selected to consider two environments on the planet: inner and outer.

The surface temperature and the escape velocity relate, respectively, with the rate of biochemical reactions at the planetary body's surface and with the retention of the atmosphere (outer ESI). This part of the index mostly reflects life based on photosynthesis, which concentrates at the planetary surface. On another hand, the average radius and density are related to the planetary geodynamics (inner ESI). This part of the index mostly reflects life based chemosynthesis, which concentrates at the planetary subsurface. From equation (36) it is obvious that the present Earth gives $ESI = 1$ (as this index measures how similar is a rocky planetary body to current Earth. Planets with ESI in the range 0.8-1 are considered suitable to host life more or less similar to that of the present Earth. Those with an index in the range 0.6-0.8 (like current Mars) are very cold or very hot, and could host extremophiles. A planetary body with $ESI < 0.6$ is considered inhabitable.

5.2. Planetary Habitability Index (PHI)

This habitability metric is defined as the geometric mean of separate values of the variables: stable substrate (S), available energy (E), appropriate chemistry (C) and a liquid solvent (L) in a given rocky planetary body (Schulze-Makuch et al. 2011):

$$PHI = (S.E.C.L)^{\frac{1}{4}} \qquad (37)$$

Details on this metric are available in (Schulze-Makuch et al. 2011).

6. CONCLUSIONS

Rocky planetary bodies seem to be the places most amenable for Life in our observed Universe. Astrobiology, in interplay with Environmental Sciences, should give answers to many issues on Habitability currently under research. Mathematical Modeling, including Quantification of Habitability, shall be of much help in this great human endeavor in understanding the origin, evolution and distribution of Life in this vast Universe we live in.

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