Problems

Problem 1.1. The frequency of a photon in a Schwarzschild metric varies such that $h\nu(1-2GM/c^2r)^{1/2}$ is constant (Misner, Thorne, & Wheeler 1973, p. 187 and p. 659, although note that these authors set $G = c = 1$).

(a) Calculate the gravitational redshift $z \equiv \Delta\nu/\nu$ from the bottom to the top of the solar atmosphere. Assume that the thickness of the solar atmosphere is 600 km.

(b) Most computer programs used to model stellar atmospheres use single-precision floating-point numbers that typically are precise to about 10^{-7}. Which is likely to lead to larger errors, using this representation or ignoring the gravitational redshift across the solar atmosphere?

Problem 1.2. Consider a solid sphere whose surface emits with uniform specific intensity I outwards into space. Use the constancy of specific intensity to show that the flux from the sphere drops according to the inverse square law. What is the flux at the surface? (Hint: consider how the solid angle subtended by the sphere drops with distance.)

Problem 1.3. Show that in general (a) the total density of photons is $(4\pi/hc)\int_0^\infty d\nu\; J_\nu/\nu$ and (b) the total energy density is $4\pi J/c$.

Problem 1.4. If I_ν is isotropic over the outward hemisphere and zero over the inward hemisphere, i.e.,

$$I_\nu(\mu) = \begin{cases} I_\nu(1) & \text{for } \mu > 0, \\ 0 & \text{for } \mu < 0, \end{cases} \tag{1.61}$$

where $I_\nu(1)$ is a constant, show that

$$F_\nu = \pi I_\nu(1). \tag{1.62}$$

Problem 1.5. Calculate the effective temperature of the Sun from its luminosity $L_\odot = 3.826 \times 10^{33}\,\mathrm{erg\,s^{-1}}$ and radius $R_\odot = 6.960 \times 10^{10}$ cm.