Problems

Problem 2.1. A monochromatic beam of photons is emitted at $\tau = 0$ in the absence of emission ($j_\nu = 0$) but in the presence of extinction ($\chi \neq 0$).

(a) Integrate the radiation transfer equation to show that the specific intensity in the beam decreases as $I_\nu(\tau) = e^{-\tau} I_\nu(0)$, where τ is the optical depth along the line of sight.

(b) Show that the number of photons in the beam decreases by a factor of $1/e$ for each optical depth traveled.

(c) Show that the optical depth traveled by a photon in the beam has an exponential distribution with a mean of 1. Thus, the mean free path of a photon is 1 in units of τ or $l = \chi^{-1}$ in units of distance.

Problem 2.2. Prove the Eddington-Barbier relation, that if S_ν is a linear function of τ, then $I_\nu(0, \mu) = S_\nu(\tau = \mu)$ and $F_\nu(0) = \pi S_\nu(\tau = 2/3)$. These results underpin the Eddington-Barbier approximation.