Problems

Problem 2.1. Here we will show that the mean free path of a photon is 1 in units of τ or $l = \chi^{-1}$ in units of distance.

(a) Consider a probabilistic process in which an event can occurs over a range of positions x. The probability density function $p(x)$ is defined so that the probability of an event in the interval $(x, x + dx)$ is $p(x)dx$. If the probability that the event occurs after a position x is $P(>x)$, show that the probability density function is given by $p(x) = -dP(>x)/dx$.

(b) A monochromatic beam of photons is emitted at $\tau = 0$ in the absence of emission ($j_\nu = 0$) but in the presence of extinction ($\chi \neq 0$). Show that the optical depth traveled by a photon in the beam has an exponential distribution with a mean of 1.