Problems

Problem 3.1. Show that in plane-parallel symmetry and when j_ν and χ are isotropic, the local and global statements of the condition of radiative equilibrium are equivalent. That is, show that

$$\frac{dF}{dz} = 0 \quad \Leftrightarrow \quad \int_0^\infty dv\, \chi J_\nu = \int_0^\infty dv\, j_\nu. \tag{3.18}$$

(Hint: integrate the equation of radiation transfer in a plane-parallel symmetry over frequency and solid angle.)

Problem 3.2. The variation of emergent specific intensity of the Sun is given approximately by

$$I_\lambda(0,\mu) \approx I_\lambda(0,0)[a_0 + a_1\mu + 2a_2\mu^2], \tag{3.19}$$

with the parameters $I_\lambda(0,0)$, a_0, a_1, and a_2 having the values given in the table.

λ μm	a_0	a_1	a_2	$I_\lambda(0,0)$ erg s^{-1} cm^{-3}
0.3727	0.1435	0.9481	−0.0920	4.2×10^{14}
0.4260	0.1754	0.9740	−0.1520	4.5×10^{14}
0.5010	0.2593	0.8724	−0.1336	4.0×10^{14}
0.6990	0.4128	0.7525	−0.1761	2.5×10^{14}
0.8660	0.5141	0.6497	−0.1657	1.6×10^{14}
1.2250	0.5969	0.5667	−0.1646	7.7×10^{13}
1.6550	0.6894	0.4563	−0.1472	3.6×10^{13}
2.0970	0.7249	0.4100	−0.1360	1.6×10^{13}

(a) Assuming that the solar atmosphere is in LTE and ignoring scattering, describe how to calculate the temperature $T(\tau)$ as a function of optical depth τ and the optical depth $\tau(T)$ as a function of temperature T and wavelength λ.

(b) Plot the relative values of the extinction coefficient χ at 5800 K as a function of wavelength λ.

Note that if the source function in a plane-parallel atmosphere is given by

$$S_\nu(\tau) = \sum a_{\nu,n}\tau^n, \tag{3.20}$$

then one can show that the emergent intensity is given by

$$I_\nu(0,\mu) = \sum a_{\nu,n} n! \mu^n. \tag{3.21}$$

This, is a generalization of the Eddington-Barbier relation.