Problems

Problem 4.1. Prove the Eddington-Barbier relation, that if S_{ν} is a linear function of τ , then $I_{\nu}(0, \mu) = S_{\nu}(\tau = \mu)$ and $F_{\nu}(0) = \pi S_{\nu}(\tau = 2/3)$. These results underpin the Eddington-Barbier approximation.

Problem 4.2. The variation of emergent specific intensity of the Sun is given approximately by

$$I_{\lambda}(0,\mu) \approx I_{\lambda}(0,0)[a_0 + a_1\mu + 2a_2\mu^2],$$
 (4.109)

with the parameters $I_{\lambda}(0,0)$, a_0 , a_1 , and a_2 having the values given in the table.

λ	a_0	a_1	a_2	$I_{\lambda}(0,0)$
μ m				$\mathrm{erg}\mathrm{s}^{-1}\mathrm{cm}^{-3}$
0.3727	0.1435	0.9481	-0.0920	4.2×10^{14}
0.4260	0.1754	0.9740	-0.1520	4.5×10^{14}
0.5010	0.2593	0.8724	-0.1336	4.0×10^{14}
0.6990	0.4128	0.7525	-0.1761	$2.5 imes 10^{14}$
0.8660	0.5141	0.6497	-0.1657	1.6×10^{14}
1.2250	0.5969	0.5667	-0.1646	7.7×10^{13}
1.6550	0.6894	0.4563	-0.1472	3.6×10^{13}
2.0970	0.7249	0.4100	-0.1360	1.6×10^{13}

- (a) Assuming that the solar atmosphere is in LTE and ignoring scattering, describe how to calculate the temperature T(τ) as a function of optical depth τ and the optical depth τ(T) as a function of temperature T and wavelength λ. (There is no need to solve these equations explicitly, just describe clearly how to obtain them.)
- (b) Plot the relative values of the extinction coefficient χ at 5800 K as a function of wavelength λ .

Note that if the source function in a plane-parallel atmosphere is given by

$$S_{\nu}(\tau) = \sum a_{\nu,n} \tau^n, \qquad (4.110)$$

then one can show that the emergent intensity is given by

$$I_{\nu}(0,\mu) = \sum a_{\nu,n} n! \mu^n.$$
 (4.111)

This, is a generalization of the Eddington-Barbier relation.

Problem 4.3. Show that in the diffusion approximation the Eddington factor f is 1/3 to first order in $dB_{\nu}/d\tau$. This result is used in the development of the solution to the grey atmosphere.

Problem 4.4. Consider a gas with two components A and B with extinction coefficients χ^A and χ^B given by

$$\chi^{A} = \begin{cases} p & \text{for } v < v_{0} \\ q & \text{for } v > v_{0} \end{cases}$$
(4.112)

and

$$\chi^B = \begin{cases} q & \text{for } v < v_0 \\ p & \text{for } v > v_0, \end{cases}$$
(4.113)

where *p* and *q* are constants. Show that in general $\chi_R \neq \chi_R^A + \chi_R^B$, in which χ_R^A and χ_R^B are the Rosseland mean opacities of the individual components.

A consequence of this is that we cannot publish, say, Rosseland mean extinction coefficients for electrons, hydrogen, helium, etc., and then combine these to give the correct Rosseland mean opacity for a specific mixture. Instead, we must first calculate the appropriate total coefficient χ and *then* calculate the total Rosseland mean extinction coefficient.

Problem 4.5. Consider a grey atmosphere radiating into free space.

- (a) Derive an expression for f(0), the Eddington factor at the surface, when $S = a + b\tau$.
- (b) Using this expression, show that the Eddington approximate solution $S = 3H(\tau + 2/3)$ gives f(0) = 17/42.
- (c) Show that the Eddington approximate solution is not selfconsistent in its predictions of f(0).

Problem 4.6. Consider an atmosphere in LTE in the absence of scattering, and whose temperature structure is given by the Eddington approximation to the grey atmosphere,

$$T^4 = \frac{3}{4} T_{\rm eff}^4 (\tau + \frac{2}{3}). \tag{4.114}$$

- (a) Consider the case in which the opacity is grey. Use the Eddington-Barbier approximation to show that the emergent flux F_{ν} is approximately equal to that of a black body at the effective temperature of the atmosphere radiating into free space.
- (b) Consider the case in which the opacity is slightly non-grey, with

$$\tau(\nu) = \begin{cases} (2/3)\tau & \nu < \nu_0\\ (3/2)\tau & \nu > \nu_0 \end{cases},$$
(4.115)

but with the same temperature structure in terms of τ . Use the Eddington-Barbier approximation to obtain an approximate expression for the emergent flux.

(c) Graph the approximate expressions for the emergent fluxes for both cases, assuming $hv_0 = 3kT_{\text{eff}}$. You may graph the result either for F_v and v for a representative temperature of 10,000 K or in the normalized quantities α and \hat{H}_{α} .

Note that the atmospheres in both cases are not in radiative equlibrium, in (a) because we use an approximate solution and in (b) because we make no attempt to correct the temperature structure

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for the non-greyness of the atmosphere. Nevertheless, the result obtained in (b) is quantitatively correct in many atmospheres; regions of lower opacity tend to have higher flux and regions of higher opacity tend to have lower flux. This is a simple model for the change in flux in the vicinity of an ionization edge, in which there is a step-like increase in opacity with increasing frequency.