Problems

Problem 4.1. Show that in the diffusion approximation the Eddington factor f is 1/3 to first order in $dB_\nu/d\tau$. This result is used in the development of the solution to the grey atmosphere.

Problem 4.2. Suppose that a gas consists of a number of distinct components (electrons and diferent ions). The total extinction coefficient χ can in general be considered to be the sum of the extinction coefficients of the distinct components, that is,

$$\chi = \sum \chi_{i,\nu}. \tag{4.87}$$

Show that the total Rosseland mean extinction can *not* in general be considered to be the sum of the Rosseland mean opacities of the distinct components.

A consequence of this is that we cannot publish, say, Rosseland mean extinction coefficients for electrons, hydrogren, helium, etc., and then combine these to give the correct Rosseland mean opacity for a specific mixture. Instead, we must first calculate the appropriate total coefficient χ and *then* calculate the total Rosseland mean extinction coefficient.

Problem 4.3. Consider a grey atmosphere radiating into free space.

(a) Derive an expression for $f(0)$, the Eddington factor at the surface, when $S = a + b\tau$.

(b) Using this expression, show that the Eddington approximate solution $S = 3H(\tau + 2/3)$ gives $f(0) = 17/42$.

(c) Show that the Eddington approximate solution is not self-consistent in its predictions of $f(0)$.

Problem 4.4. Consider an atmosphere in LTE, with coherent and isotropic scattering, and whose temperature structure is given by the Eddington approximation to the grey atmosphere,

$$T^4 = \frac{3}{4}T_{\rm eff}^4(\tau + \frac{2}{3}). \tag{4.88}$$

(a) Consider the case in which the opacity is grey. Use the Eddington-Barbier approximation to show that the emergent flux is approximately equal to that of a black body at the effective temperature of the atmosphere radiating into free space.

(b) Consider the case in which the opacity is slightly nogray, with

$$\tau(\lambda) = \begin{cases} 0.9\tau & \nu < \nu_0 \\ 1.1\tau & \nu > \nu_0 \end{cases}, \tag{4.89}$$

but with the same temperature structure in terms of τ. Use the Eddington-Barbier approximation to obtain an approximate expression for the emergent flux.

(c) Sketch the approximate expressions for the emergent fluxes for both cases, assuming $h\nu_0 \sim kT_{\rm eff}$.

Note that the atmospheres in both cases are not in radiative equlibrium, in (a) because we use an approximate solution and in (b) because we make no attempt to correct the temperature structure for the non-greyness of the atmosphere. Nevertheless, the result obtained in (b) is quantitatively correct in many atmospheres; regions of lower opacity tend to have higher flux and regions of higher opacity tend to have lower flux.

Problem 4.5. Consider an isothermal atmosphere at temperature T with constant mean molecular mas μ.

(a) If the atmosphere is in hydrostatic equilibrium, show that the density varies with z as

$$\rho(z) = \rho_0 \exp(-(z - z_0)/H), \tag{4.90}$$

with $H \equiv kT/\mu m_{\rm H} g$.

(b) If the opacity varies with density as $\chi \propto \chi_0(\rho/\rho_0)^\alpha$, show that the density at the point $\tau = 2/3$ varies with α as

$$\rho(\tau = 2/3) \propto g^{1/(1+\alpha)}. \tag{4.91}$$

(c) Atmospheres typically have $1 \le \alpha \le 2$. What does this tell you about the relative densities of main sequence stars ($\log g \approx 5$), giants ($\log g \approx 3$), and supergiants ($\log g \sim 1$)?

Real atmospheres are not isothermal and do not have uniform mean molecular mass. Nevertheless, the temperature and mean molecular mass typically change by a factor of 2 at most across an atmosphere, whereas the density changes by orders of magnitude. Thus, the results derived here remain qualitatively correct.

Problem 4.6.

(a) Use SIMBAD to find the observed B magnitude of a named star.

(b) Convert the B magnitude into an observed monochromatic flux F_ν or F_λ in physical units.

(c) Assuming that the flux in B is the same as the flux in SDSS g, calculate the observed g magnitude in the SDSS system.

B magnitudes are on a Vega-based system. SDSS g magnitudes are on an AB system. See Bessell & Murphy (2012) for zero points.