Problems

Problem 6.1. Calculate and plot the partition function for neutral hydrogen from 10^3 K to 10^5 K assuming (a) that the ionization potential is lowered by $\Delta\chi = 0.1$ eV and (b) that the ionization potential is lowered by $\Delta\chi = 0.2$ eV.

Problem 6.2. Consider a pure hydrogen gas. Ignore the formation of molecules and consider only H^-, H^0, H^+, and electrons.

(a) Obtain a cubic equation for the electron density n_e in terms of the total density of hydrogen nuclei n_H.

(b) Calculate and plot the ionization fractions $f_- \equiv n_-/n_H$, $f_0 \equiv n_0/n_H$, and $f_+ \equiv n_+/n_H$ in LTE for temperatures between 10^3 K and 10^5 K and hydrogen densities n_H of 10^{14}, 10^{15}, and 10^{16} K. Plot all of the ionization fractions on the same plot using different line styles for the different densities, with the logarithm of temperature as ordinate and the logarithm of ionization fraction as abscissa.

The ionization potentials of H^- and H^0 are $\chi_- = 0.754$ eV and $\chi_0 = 13.60$ eV. H^- has only one bound state with degeneracy 1 and so its partition function U_- is 1. Since the excited states of H^0 lie so far above the ground state, its partition function U_0 can be assumed to be 2, independent of the temperature. Since H^- has no bound electrons, its partition function U_+ is 1.

You will need to solve a cubic equation. This can be done by using the analytic solution for the roots (Press et al. 1992, §5.6), by standard root-finding methods, such as the simple and robust bisection method (Press et al. 1992, §9.1), perhaps in combination with the more efficient Newton-Raphson method (Press et al. 1992, §9.4), or implicitly using software such as Mathematic. Note that cubic equations in general have more than one root; you will need to make sure you obtain the appropriate one.