Problems

Problem 8.1. In FGK stars, hydrogen is predominantly neutral and iron is predominantly singly ionized. Show that at a given temperature (a) $n_{\mathrm{Fe}^0}/n_{\mathrm{H}^-}$ is independent of the electron density; (b) $n_{\mathrm{Fe}^+}/n_{\mathrm{H}^-}$ is inversely proportional to the electron density; and (c) $n_{\mathrm{Fe}^{2+}}/n_{\mathrm{H}^-}$ is inversely proportional to the square of the electron density.

Problem 8.2. Calculate and plot the ratios $R = \chi_-/\chi_+$ of extinction coefficients below and above the Balmer jump in LTE for temperatures between 10^3 K and 10^5 K and hydrogen densities n_H of 10^{14}, 10^{15}, and 10^{16} K.

Plot all of the ratios on the same plot using different line styles for the different densities, with the logarithm of temperature as ordinate and the logarithm of R as abscissa.

To determine the state of matter, ignore the formation of molecules and consider only H^-, H^0, H^+, and electrons. The ionization potentials of H^- and H^0 are 0.754 and 13.60 eV. H^- has only one bound state with degeneracy 1 and so its partition function U_- is 1. Since the excited states of H^0 lie so far above the ground state, its partition function U_0 can be assumed to be 2, independent of the temperature.

To determine the opacities, consider only bound-free absorption from H^0 in the $n = 2$ and $n = 3$ states and H^-, electron scattering, and free-free absorption from H^+. The bound-free absorption cross-sections of H^0 in the $n = 2$ and $n = 3$ states and H^- at the Balmer limit are 1.58×10^{-17}, 2.08×10^{-18}, and 2.05×10^{-17} cm^2. The bound-free cross-sections should be multiplied by $(1 - e^{-h\nu/kT})$ to account for stimulated emission. The scattering cross-section of an electron is 6.65×10^{-25} cm^2. The H^+ free-free absorption coefficient at the Balmer limit is given by

$$\alpha_\mathrm{ff} = 6.61 \times 10^{-37} n_e n_+ T^{-0.5} (1 - e^{-h\nu/kT}) \text{ cm}^{-1}, \qquad (8.19)$$

in which n_e and n_+ are in cm^{-3}, and T is in K.

Explain the dependence of R on density for hot stars. That is, explain why R is almost independent of density for high densities but dependent on density for low densities.