A POSSIBLE HYDRODYNAMICAL EQUIVALENCE FOR ASTROPHYSICAL JETS

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RESUMEN

La idea de un modelo unificado para cuasares y μ-cuasares ha sido considerada por bastante tiempo, a pesar de los distintos ambientes y condiciones físicas sobre los cuales ambas clases de objetos residen. En este trabajo mostramos la existencia de una ley de escalamiento simple, que relaciona el tamaño máximo de un chorro con las propiedades físicas del gas del entorno sobre el cual se expande. Este resultado es aparentemente válido para todo tipo de chorros hidrodinámicos y puede considerársele como un modelo general de unificación. La velocidad de expansión del chorro y las propiedades físicas del gas alrededor de éste se combinan de tal manera que ponen un límite máximo al tamaño que los chorros de distintas clases pueden tener.

ABSTRACT

The idea of a unified model for quasars and μ-quasars has been considered for a long time, despite the different environments and physical conditions where both classes of objects reside. Here we show the existence of a simple scaling law, relating the maximum size of a jet to the properties of the gas medium into which it expands. This appears to be valid for all types of hydrodynamical jets, and can be thought of as a broad unified model. The expansion velocity of the jet and the physical properties of the surrounding gas combine in such a way that a limit to the maximum extent of jets at different scales can be obtained.

Key Words: GALAXIES: JETS — GAMMA RAYS: BURSTS — HYDRODYNAMICS — ISM: JETS AND OUTFLOWS — QUASARS: GENERAL

1. INTRODUCTION

When astrophysical jets are formed, they begin to expand into their surrounding medium. A jet is capable of this expansion because the outflow, produced by the central engine, generates a ram pressure capable of elongating it. At the head of a particular jet, the kinetic energy of the particles is thermalised through a shock wave or hot spot. After a particular fluid particle crosses the hot spot, it is recycled back to the neighbourhood of the jet itself. This generates a cavity or cocoon that, together with the jet, expands through the environment. The cocoon keeps the jet collimated so that its transverse length does not grow too much as it expands. This standard picture for the expansion of an extragalactic (hydrodynamical) radio jet was first proposed by Scheuer (1974).

Today, we know that jets are not only associated with quasars, but that they occur at a range of different astrophysical scales. Some jets are observed in recently formed star systems. These are associated with Herbig-Haro objects and are commonly named Herbig-Haro jets (Reipurth & Bally 2001). Jets can also form in galactic stellar binary systems (Mirabel & Rodríguez 1994), and are so similar to their quasar extragalactic counterparts that they were termed μ-quasar jets. Finally, observations of gamma ray burst afterglows (of the long variety) in distant galaxies have led recently to the idea that jets are associated with these sources (see Mészáros (2002) for a review).

Aside from a difference of $6 \sim 7$ orders of magnitude covered by the maximum size of jets, their kinetic power output varies depending on the nature
of the jet itself. Observations show that some extragalactic, and all Herbig-Haro, jets expand at non-relativistic velocities. On the other hand, the fluid particles inside quasar, μ-quasar and γ-ray burst jets expand at relativistic velocities. Scheuer’s (1974) simple hydrodynamical model has been used as the basic model for all kind of jets that we observe in the universe.

Using dimensional analysis arguments, we propose in this article that there is a simple way to relate the maximum size of an astrophysical jet with the properties of its surrounding environment. The result appears to be valid for both low-velocity jets in the Newtonian regime and for highly relativistic outflows such as those found in γ-ray bursts.

Dimensional analysis techniques have been applied to the study of powerful jets in radio galaxies interacting with their external environment by Falle (1991) and Kaiser & Alexander (1997). However, the nature of their models show a self-similar indefinite growth of cocoon and jet, possibly due to the omission of gravity from the forces acting on the growing cocoon, since they concentrated their analysis on the initial growth phase.

2. NON-RELATIVISTIC HYDRODYNAMICAL JETS

Let us first consider a non-relativistic jet, for example an AGN or a Herbig-Haro object jet, that has a characteristic length $r$. The physical processes and the dimensionless combination of important parameters that determine the dynamics of the expansion of the jet are many and complicated (Begelman, Blandford, & Rees 1984; Blandford 1990). For example, internal shock waves that develop inside jets, radiative cooling, kinetic power, magnetic fields, mass of the central engine and accretion power are some of the important physical quantities that enter into the description of an expanding jet. For the sake of simplicity, suppose that we do not include magnetic fields in the description of the jet’s dynamics and consider a purely hydrodynamical jet. Of all the possible dimensionless combinations of physical parameters that describe the problem, one was constructed by Mendoza & Longair (2001). This parameter measures the strength of the kinetic energy in the jet as compared to the gravitational energy of the gas cloud where it is embedded. In order to build such a parameter, we proceed as follows: the gravity of the surrounding cloud is taken into account by introducing Newton’s gravitational constant $G$, and the physical properties of the cloud are determined by its average density $\rho$. In general terms, this density does not only represent the gas density of the surrounding material. It represents the total average density evaluated at a certain distance from the progenitor jet source. This means that the average density contains the gas density plus the stellar mass density (for galactic jets) and dark matter mass density (for extragalactic jets). Finally, since the expanding jet is roughly in pressure equilibrium with the surrounding cloud, the jet is roughly characterised by its bulk velocity $v$.

If dissipative processes inside the jet (such as radiative cooling and drag acting on the flow) are negligible then the four fundamental quantities ($r$, $G$, $\rho$ and $v$) are a good estimate of the dynamical behaviour of the expanding jet in the cloud. This assumes that the expansion of the jet is close to ballistic and is, of course, a first order approximation, dealing only with the purely hydrodynamical aspect of the problem through dimensional analysis. One of the many simplifications being made is to consider a constant density medium, which is in all likelihood not true. However, under the reasonable assumption of a jet expanding from the centre of a spherically symmetric region, or from an overdensity within a cloud, the average density is well defined at each radius.

For any mechanical problem there are three independent dimensions, namely the dimensions of time $t$, length $l$, and mass $m$. Having restricted the problem to the above physics, Buckingham’s theorem of dimensional analysis (Sedov 1993) demands the existence of a unique non-dimensional parameter $\Lambda$ that will fully determine the solution of the problem. In the case of jets, this parameter is given by

$$\Lambda \equiv G \frac{\rho r^2}{M^2 a^2},$$

where $M \equiv v/a$. Formally, this number is not the Mach number of the jet flow, since it is defined as the ratio of the jet velocity to the sound speed $a$ in the cloud (Mendoza & Longair 2001). These authors showed that, for the particular problem they were discussing, the dimensionless parameter $\Lambda$ measured in units of $M^{-2}$ had the same value for jets formed in giant molecular clouds and for the gaseous haloes of galaxies (the length $r$ in their analysis was a characteristic length for which deflection of jets by pressure gradients might occur). This equivalence hints at an underlying mechanism that makes jets look the same at such different scales.

As noted by Mendoza & Longair (2001) the parameter $\Lambda$ can also be written as
\[ \Lambda = \frac{3}{4\pi} \frac{GM(r)}{v^2} \]
\[ \approx 1.2 \times 10^{-17} \left( \frac{M(r)}{M_\odot} \right) \left( \frac{c}{v} \right)^2 \left( \frac{r}{\text{kpc}} \right)^{-1}, \quad (2) \]

where \( M(r) \) is the total mass within a sphere of radius \( r \) and \( c \) is the speed of light. The right hand side of Eq. (2) is roughly the ratio of the gravitational potential energy to the kinetic energy of a fluid element of the jet evaluated at position \( r \). Formally, the total gravitational potential energy acting on a fluid element on the jet needs to include not only the gas, but also the central mass and the dark matter (for the case of extragalactic jets) within radius \( r \). Let us consider from now on that \( r \) represents the maximum length that a specific class of jets can have. This maximum length is constrained by observations.

If a particular jet is such that \( \Lambda \gg 1 \), then the total gravity acting on the jet is so strong that the jet cannot expand away from the central engine that is producing it. On the other hand, when \( \Lambda \ll 1 \) the jet can freely expand away from the progenitor source, drilling a hole through the parent cloud. Since the numerical factor appearing in Eq. (2) is very small, it follows that for any real jet, the parameter \( \Lambda \ll 1 \). For example, if we consider a Herbig-Haro jet, for which the total mass within a radius \( r \) is a few \( M_\odot \) and the expansion velocity \( v \approx 10^{-3} c \) (where \( c \) is the speed of light) and in which typical lengths \( r \approx 1 \text{pc} \), we then obtain a value of \( \Lambda \approx 10^{-7} \). For the case of powerful extragalactic jets, as will be discussed later, \( M(r) \approx 10^{15} \), \( r \approx 1 \text{Mpc} \) and \( v \approx c \), which implies that \( \Lambda \approx 10^{-5} \). As will be shown below, in the case of a relativistic jet we would replace \( \beta \) by \( \gamma \beta \), where \( \beta = v/c \) and \( \gamma \) is the Lorentz factor of the flow. This replacement reduces even more the value of \( \Lambda \).

Since \( \Lambda \ll 1 \) for all types of jets (including \( \mu \)-quasar and \( \gamma \)-ray burst jets), it would appear that from merely considering gravitational potential and kinetic energies, jets should still be capable of growing several orders of magnitude in size beyond what is observed. Typical sizes of jets are much smaller than what the energy balance of \( \Lambda \sim 1 \) suggests. This indicates that the mechanism determining the maximum extent of a jet must lie elsewhere.

In view of this fact, let us return to Eq. (1) in its original form, keeping the ratio \( v/a \) explicit, and performing a re-scaling. The maximum length that a Herbig-Haro jet can have is \( r \approx 1 \text{pc} \) (Reipurth \& Bally 2001) and the surrounding medium where those jets expand is composed of cold molecular clouds for which the particle number density \( n_H \sim 10^2 \text{cm}^{-3} \) and the temperature \( T \sim 10 \text{K} \) (Spitzer 1998; Hartmann 1998). We can thus write Eq. (2) as
\[ \Lambda \approx 10^{-1} \left( \frac{M}{10 M_\odot} \right) \left( \frac{r}{1 \text{pc}} \right)^{-1} \left( \frac{T}{10 \text{K}} \right)^{-1}. \quad (3) \]

The total (stellar plus gas) mass \( M(r) \) within a radius of \( \sim 1 \text{pc} \) is such that \( 2 \lesssim M(r) \lesssim 10 M_\odot \), so that the upper limit of Eq. (3) is \( \Lambda \approx 10^{-1}/M^2 \).

The largest extragalactic jets have typical lengths \( r \approx 2 \text{Mpc} \) (Ferrari 1998) and their surrounding intergalactic medium is such that \( n_H \approx 10^{-4} \text{cm}^{-3} \), and \( T \approx 10^9 \text{K} \) (Longair 1992, 1998). The total gas mass within this radius is \( M_{\text{gas}} \approx 10^{13} M_\odot \). Clusters of galaxies are such that the fraction of gas mass to dark matter is \( \sim 0.1 \) (Allen, Schmidt, \& Fabian 2002; Allen et al. 2004, and references therein). This means that the total mass within a radius of \( \sim 2 \text{Mpc} \) is \( M(r) \approx 10^{14} M_\odot \). With these values Eq. (3) implies that \( \Lambda \approx 10^{-1}/M^2 \), the same as the one obtained for Herbig-Haro jets.

The fact that the numerical value of \( \Lambda \) in units of \( M^{-2} \) is the same for Herbig-Haro jets and for non-relativistic extragalactic jets appears to indicate that a certain underlying physical mechanism is involved, fixing a maximum jet size. In order to investigate this, let us now rewrite Eq. (1) as
\[ \Lambda \approx \frac{1}{M^2 \lambda_j^2}, \quad (4) \]
where \( \lambda_j \approx a/\sqrt{G \rho} \) is the cloud’s Jeans length. Note that \( \lambda_j \) is the scale-length for gravitational instability in a fluid having sound speed \( a \) and subject to the gravity produced by a total density \( \rho \), which includes all matter present. The above comes from setting \( \rho^{-1} \nabla \rho = \nabla \phi \), where \( \phi \) is the gravitational potential.

Comparing Eq. (4) with Eq. (3) we find that if the relation \( \Lambda \approx 10^{-1}/M^2 \) can be treated as universal, then
\[ \lambda_j \approx \sqrt{10} r. \quad (5) \]

One way of interpreting this general result is the following. It appears that jets can grow inside gaseous clouds, up to the point where they can no longer do so because a gravitational instability of the surrounding material sets in. In fact, the numerical factor of \( \sim \sqrt{10} \) that appears in Eq. (5) can be thought of as a form factor for the problem. The Jeans stability criterion can be applied to the gas inside the cocoon plus the swept up gas contained within the outer shell surrounding it in the presence of an extra
central gravitational potential including dark matter or central star. The relevant pressure $p_e$ inside the cocoon decreases as $p_e \propto t^{-\alpha}$ with the expansion of the source in time $t$. Here $\alpha = 4/5 \ldots 2$, depending on the density profile of the external gas (Kaiser & Alexander 1997). Since the gravitational force produced by the shocked gas inside the shell at the boundaries of the cocoon remains the same and since the pressure inside the cocoon tends to zero with the expansion of the source, there might be a time when a gravitational instability on the “ellipsoidal” cocoon will develop, leading to a collapse of the shell. In fact, the regions closest to the head of the jet will not collapse because $\Lambda \ll 1$. However, regions of the shell closest to the central engine will eventually collapse for sufficiently large cocoon sizes, leading to the inflow of the external gas, which perhaps destroys the jet.

There is however, another physical interpretation that can be given to Eq. (5). Alexander (2002) has proposed that the important instability which affects the late evolution in powerful FR-II radio galaxies is the Rayleigh-Taylor instability (Frieman 1954; Chandrasekhar 1961). Due to the fact that Rayleigh-Taylor instabilities grow proportional to $\exp (4\pi G \rho t)$ in time $t$, they scale in exactly the same manner as the Jeans length when the wavelength $\lambda$ of the oscillations is such that $\lambda \gg \lambda_{\text{J}}$. Once the gravitational force of the swept up gas is of the same order as the cocoon pressure, the interface between the swept up gas and the cocoon becomes Rayleigh-Taylor unstable leading to the break in of the surrounding medium, most likely inhibiting the growth of the jet.

The mean size of powerful radio sources is $\sim 300$ kpc with maximum sizes reaching up to $\sim 2$ Mpc. In the scenario presented here, these mean sized objects are understood as transient, being in the process of expansion. They will continue to grow until they reach the limit given by Eq. (5) provided the central engine remains active long enough. The fact that no jets larger than this limit have been found, and the otherwise extraordinary coincidence of Eq. (5) at stellar and extragalactic scales suggest that the physics behind these objects is the same. Of course, other dimensionless parameters can be constructed, related to the many other physical aspects of the problem, such as magnetic fields or nature of the central engine. However, the numerical results of the previous section suggest that the particular feature of the problem which we are examining, i.e., maximum jet length, relates directly, or at least primarily, to $\Lambda$ through Eq. (5).

3. RELATIVISTIC JETS-GENERALISING FURTHER

When a fully relativistic jet is introduced in the discussion, the dimensional analysis is slightly more complex. This is because the speed of light $c$ has to be introduced as a fundamental parameter. In this case, the quantity $M$ is given by the ratio of the proper velocity of the jet $\gamma v$ to the proper velocity of sound $\gamma_a a$ in the medium (Chiu 1973; Mendoza & Longair 2002),

$$M = \frac{\gamma v}{\gamma_a a},$$

where

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2}$$

and

$$\gamma_a = \left(1 - a^2/c^2\right)^{-1/2}$$

represent the Lorentz factors of the jet material and the sound speed respectively. Since all quantities (except the quantity $M$) that appear in Eq. (1) are measured in the proper frame of the gas, and because the sound speed of the gas in the cloud is non-relativistic, it is clear that the dimensionless parameter $\Lambda$ given by Eq. (1) also has the same form in the relativistic case. The only difference is that the number $M$ is now defined by Eq. (6). Following the same procedure as before, it follows that Eq. (4) should be valid in this situation as well.

For the case of relativistic extragalactic jets, i.e., quasar jets, that expand through an intergalactic medium similar to their non-relativistic counterparts, it follows that $\Lambda \sim 10^{-1}/M^2$, so for this particular case Eq. (5) is also valid.

We can now repeat the calculations for the case of $\mu$-quasars to test the generality of the result. Most $\mu$-quasars show jets with typical lengths $r \approx 1$pc (see Corbel et al. 2002, and references therein). The total (stellar plus gas) mass within 1 pc is such that $M(r) \approx 10 M_\odot$. Since the temperature of the interstellar medium is $T \approx 10$ K it follows that $\mu$-quasar jets satisfy the condition given by Eq. (5).

As a final application in an extreme case, consider cosmological $\gamma$-ray bursts, GRBs (Fishman & Meegan 1995; Meszaros 2002). The relevant parameters are less well known than for the previous examples, but a comparison is nevertheless possible. These events typically release $10^{51}$ erg for a few seconds in a highly relativistic outflow, with Lorentz factors $\gamma \approx 100 - 1000$. The long variety, for which X-ray, optical and radio counterparts have been observed (van Paradijs, Kouveliotou, & Wijers 2000), is probably related to the collapse of massive stars (Woosley 1993; Stanek et al. 2003; Hjorth et al. 2003). The outflow itself is apparently highly col-
Hydrodynamical jets are stable because there is a basic equilibrium between the maximum length of the jet \( r \) and the Jeans length \( \lambda_J \) of the surrounding medium given by \( \lambda_J \sim \sqrt{\rho r} \). The plot shows this linear relationship for all known jets: \( \gamma \)-ray burst (GRB) jets, Herbig-Haro (H-H) jets, \( \mu \)-quasar (micro-qsr) jets and active galactic nuclei (AGN) jets. In our analysis the region below the curve appears as a forbidden region, with jets in the process of expansion or contraction lying above it.

If the progenitors of GRBs are massive stars, it is reasonable to assume that they will be found in star-forming regions within the ISM. The density of the circumburst environment has also been determined from broadband observations in several cases, and ranges from 0.1 cm\(^{-3} \) to 100 cm\(^{-3} \), with a canonical value of 10 cm\(^{-3} \) (Panaitescu & Kumar 2002). If the jet is directly interacting with the external ISM, the temperature could be as low as 10 K. This gives a value of \( \Lambda \) that does not violate our constraints, and is lower than in the previous cases. If however, the jet is within the wind-fed bubble produced by the massive progenitor star prior to the GRB, the temperature will be much higher, perhaps 10\(^6 \) K (see e.g., Chevalier, Li, & Fransson 2004). Inspection of Eq. (3) shows that this will decrease \( \Lambda \) even further, thus also satisfying our proposed constraint that the right-hand side of Eq. (3) is an upper stability limit for \( \Lambda \). One additional fact makes GRB jets different from the other types considered here: the central engine typically lasts for only a few tens of seconds for long-duration bursts. Most likely, this is what determines the extent of the jet, but it nevertheless does so within the limits imposed by Eq. (3) under the above interpretation.

4. DISCUSSION

The results presented in this article suggest that astrophysical jets at all scales seem to obey the same physical law given by Eq. (5), regardless of their surrounding environment or the nature of the jet itself. It is remarkable that a class of objects where the physics is super-relativistic and where the overall phenomena have a duration of milli-seconds, over 15 orders of magnitude below that of large radio galaxies, appears to satisfy (or at any rate not to deviate significantly from) the predictions of Eq. (5).
The idea of unification of jets at all physical scales that expand in very different environments has of course been considered before. Much work has been made regarding the mechanism by which jets are produced, from \(\mu\)-quasar to AGN, with the magnetic field usually playing an important role (Blandford & Payne 1982; Meier, Koide, & Uchida 2001; Ghisellini & Celotti 2002; Price, Pringle, & King 2003). Using the observed time variability, the scaling is in terms of the mass of the central object, be it a black hole or a young star (e.g., Mirabel & Rodríguez 1994). In this article we have proposed a unification scheme based on the physical properties between the expanding jet and its surroundings, described pictorially in Figure 1. The straight line is given by Eq. (1), and we have argued here that real jets must lie above it.

Other physical causes of a jet’s maximum size are possible, most obviously the cut of an injection of particles and energy generated by the central engine. Also, for a purely hydrodynamical jet with a fixed opening angle \(\theta\) that expands through an external gaseous environment at constant pressure, lateral expansion of the jet becomes important (Begelman et al. 1984). As a constant momentum is distributed over a growing area, ram pressure decreases, and a natural maximum extent might be reached through pressure equilibrium. Jet opening angles have been studied and observed on many astrophysical sources, mainly on extragalactic radio sources. These observations show globally well-collimated jets over their large scale structure. Indeed, magnetic fields and internal shock waves (Begelman, Blandford, & Rees 1984; Blandford 1990) are probably important physical mechanisms that recollimate the jet flow at large scales. We hence believe that this explanation does not account properly for the maximum length of jets.

Also, well collimated self-similar jets and cocoons have been modelled very successfully (Falle 1991; Kaiser & Alexander 1997), albeit neglecting self-gravity of the medium into which the jet expands, other than in fixing the initial density profile of the surrounding medium. This very probably explains why no characteristic or limiting size for jets appears in their analysis (their jets actually continue to expand indefinitely). The analysis of the previous sections would indicate a limiting size which is perhaps due to gravitational instabilities in the growing cocoon, which collapses onto the jet. Most probably this instability is a Rayleigh-Taylor instability arising from the gravitational field produced by all matter within the cocoon interacting with the swept up gas as the source expands (Alexander 2002).

To conclude, we propose in this paper that purely hydrodynamical jets of characteristic sizes are stable because there is a fundamental combination of definite physical quantities that limits their growth. Once the surrounding cocoon of jets reaches a certain limit, it probably becomes Rayleigh-Taylor unstable and might frustrate the growth of the jet. Evidently, additional physical phenomena (e.g., magnetic fields) play a role in the formation and evolution of astrophysical jets, and should be considered in a more detailed description.

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