SimpleX Radiative Transfer as a Markov Process

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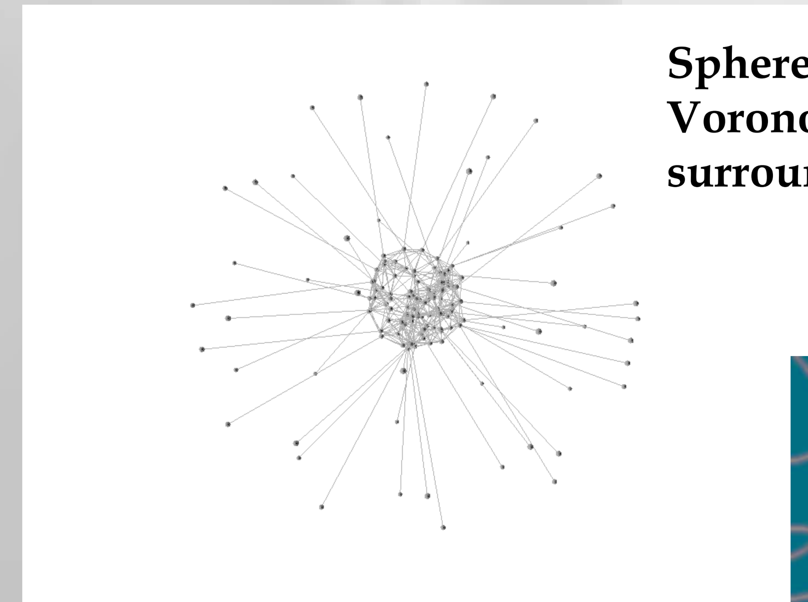
Summary

Photons are our main research tool. Therefore, mastery of radiative transfer is essential. Another truism is, that the necessary equations look innocuous but are very hard to solve quickly in practical cases. Especially when radiation has hydrodynamical consequences, this is a serious problem. Our SimpleX algorithm (Ritzerveld & Icke 2006) transports parcels of radiation along lines connecting points (nodes) that represent the scattering medium. We make suitable choices for (1) the node distribution, (2) their connectivity, (3) their scattering and absorption properties, (4) sources and boundaries (sinks). The transport problem is then cast into the form of a classical Markov process. Inversion of the Markov matrix yields the stationary solution of the corresponding radiative transfer problem. By making judicious use of the sparseness of the matrix, this method was made to run extremely fast. Furthermore, the eigenvalue spectrum of the transport matrix is a sharp tool for judging the robustness of the solution.

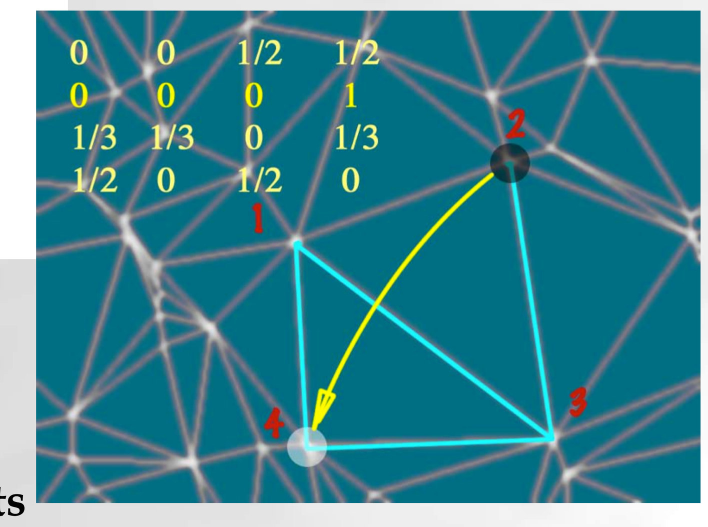
Ritzerveld, J. & Icke, V., 2006 Phys.Rev.E, 74, 026704

Conclusion

The Markov-Process variant of the SimpleX tessellation-based algorithm for radiative transfer is robust, fast, and realistic.



Sphere with 100 nodes, with Voronoi-Delaunay connections, surrounded by 50 boundary points



Example of a Markov matrix with only four nodes, one of which is a source (white) and one is a sink (black). The yellow arrow represents the radiation that is fed back to the source to create a stationary solution. This is merely a computational trick and has no consequences for the relative values of the eigenvector

The Basic Algorithm

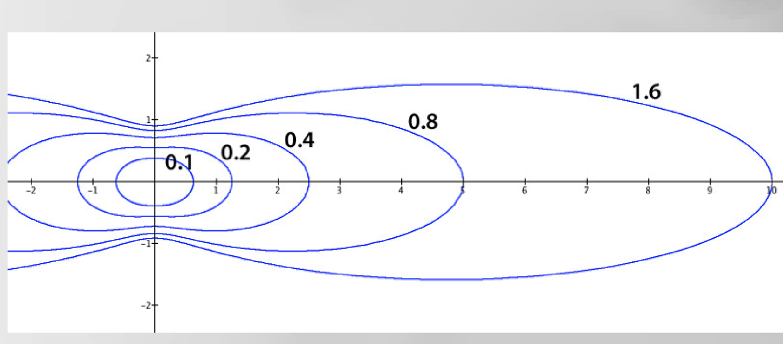
I have formulated the SimpleX algorithm for radiative transfer as a stationary Markov process. In this algorithm, parcels of radiation are shuttled along lines connecting points (nodes) that represent the scattering medium. By choosing suitable connections between sources and boundary points, this problem can be cast into the form of a classical Markov process. Inversion of the Markov matrix then yields the stationary solution of the corresponding radiative transfer problem. By making judicious use of the sparseness of the matrix, this method was made to run extremely fast.
The grid is fully connected. Unless the trasnsition probabilitiess are very contrived, the Markov matrix has distinct eigenvalues and therefore is diagonalizable.
The sum of the row entries is 1 to guarantee photon conservation. So there will be one eigenvector with eigenvalue 1, the steady state.
If the matrix is fully reciprocal, the steady state value is trivial: it is proportional to the number of lines at each node. When sources and sinks are added, the solution is non-trivial.

SimpleX represents the optical density by a point process. These points (nodes) are connected by a suitable nearest-neighbour scheme. What is "nearest" depends on the choice of distance recipe, e.g. plain linear distance (Voronoi-Delaunay graph), mean free path, percolation length, isotropy-weighted linear distance, or anything else that is useful.
The radiation is then transported along the lines of the resulting conectivity graph.

The Voronoi-Delaunay graph has "density gradient bias": most lines will point in the direction of the highest density. This makes the grid artificially anisotropic.
To circumvent this bias, I use a distance recipe that includes a weight factor for the isotropy.

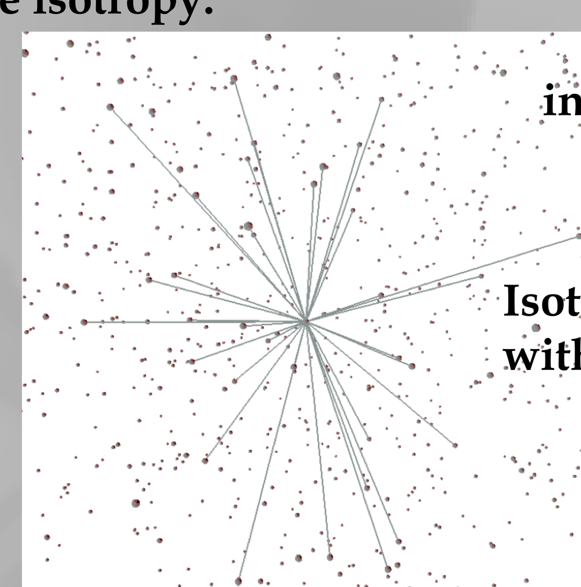
Computational Considerations

The Markov matrix associated with the SimpleX algorithm is extremely sparse. Typically, each network node has of the order of 25 connecting lines, so that a realistic representation of an astrophysical situation corresponds to an NxN matrix with only 25N nonzero entries. Such a very sparse array resembles a vector more than a full matrix. The chosen diagonalisation algorithm (Van der Vorst, 1991) is so fast that finding the eigenvectors and eigenvalues uses almost no time compared to the other computational steps, in particular building the radiative transfer network.



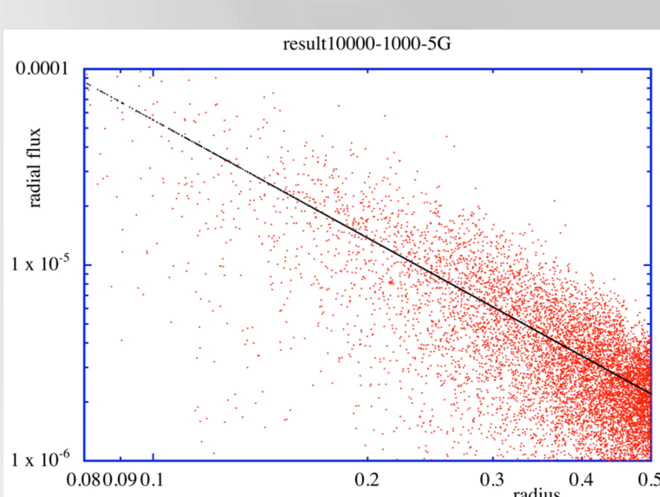
$r(w+\sin 2\,\theta)=1$ with constant w (indicated on the contours)

Polar diagram of the function that finds the isotropized nearest neighbours. The function is a combination of the linear distance r and the angle θ with respect to a fixed coordinate direction.

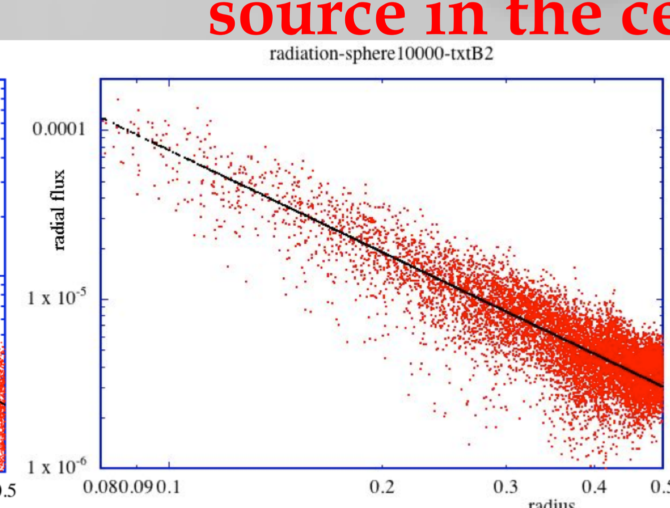


Isotropized connections with 25 nearest-neighbours

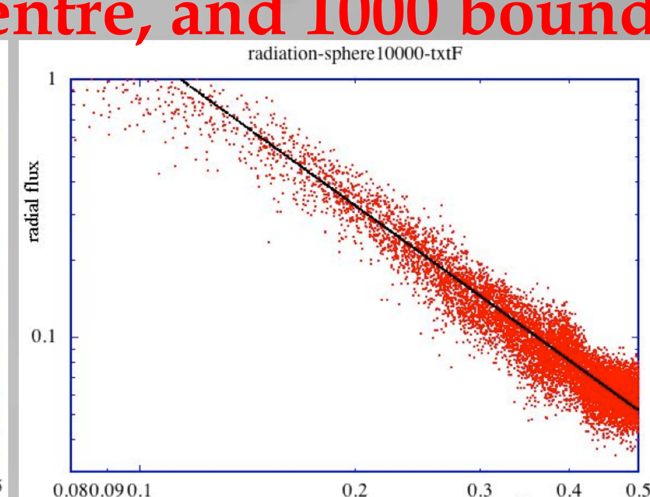
Red dots: radial flux, black line: analytic stationary solution for a sphere with 10,000 nodes, a single source in the centre, and 1000 boundary points



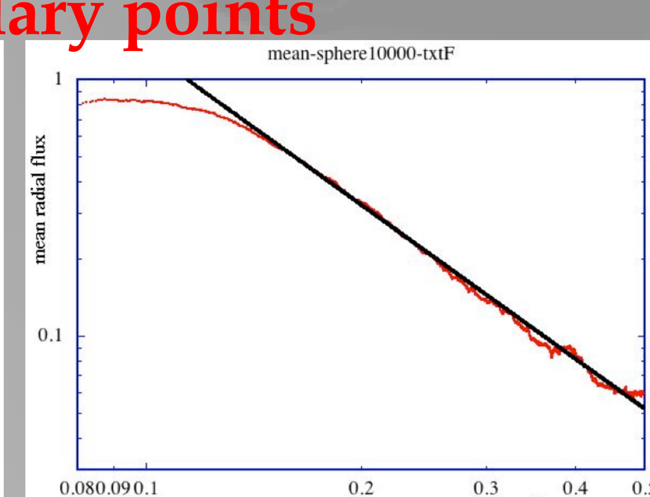
5 isotropic neighbours



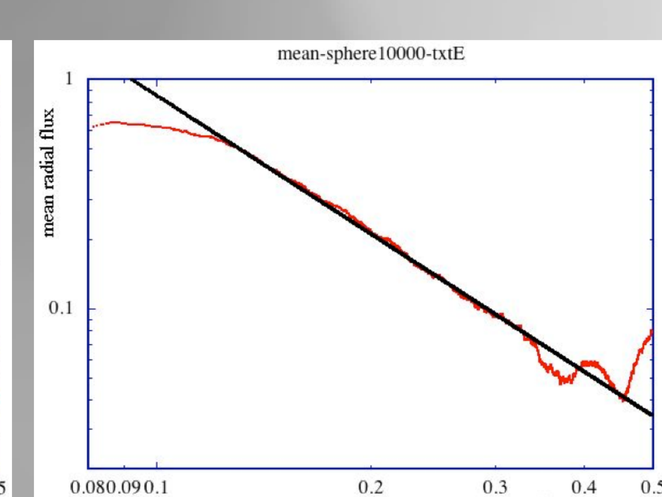
15 isotropic neighbours



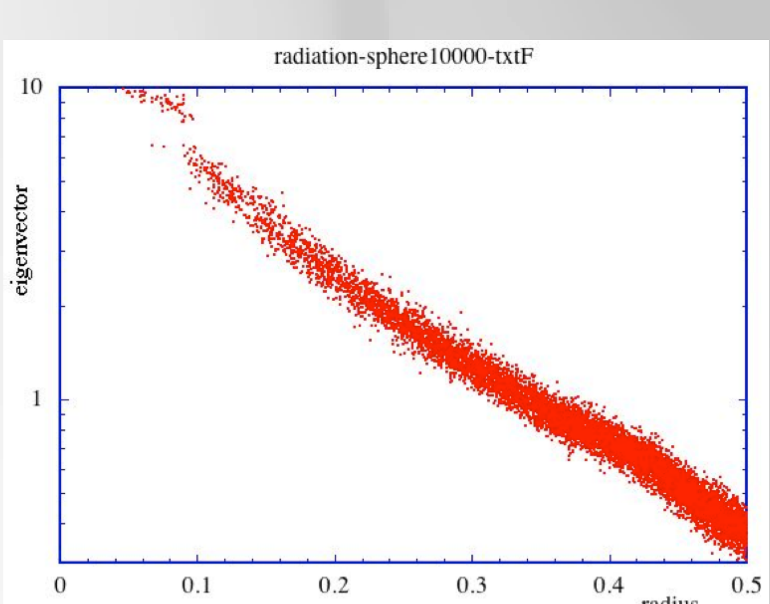
50 isotropic neighbours



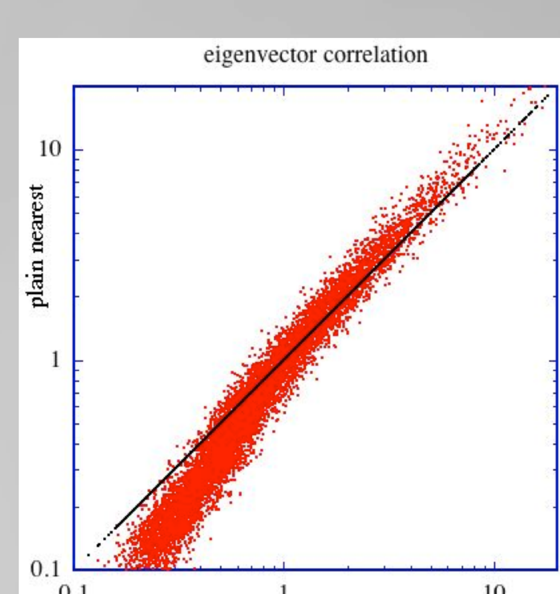
50 isotropic neighbours



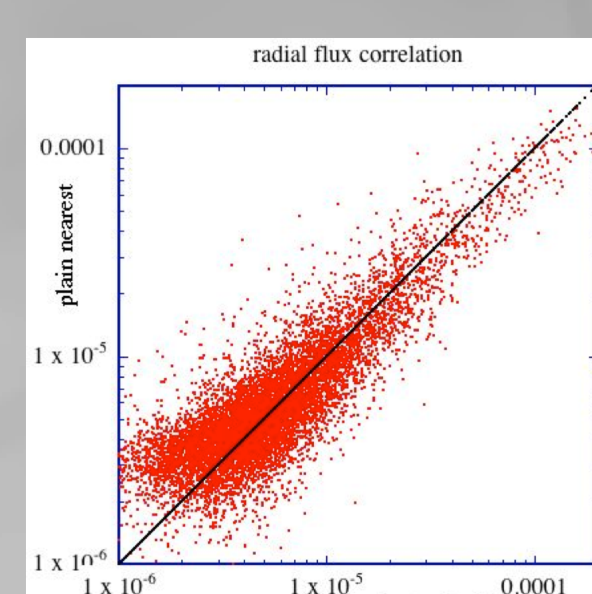
50 nearest neighbours



Red dots: eigenvector (node occupation values) of the stationary solution for a sphere with 10,000 nodes, a single source in the centre, 1000 boundary points, and 50 isotropic neighbours



Systematic difference between the eigenvector values in the Voronoi-Delaunay nearest-neighbour node distribution and the isotropic case

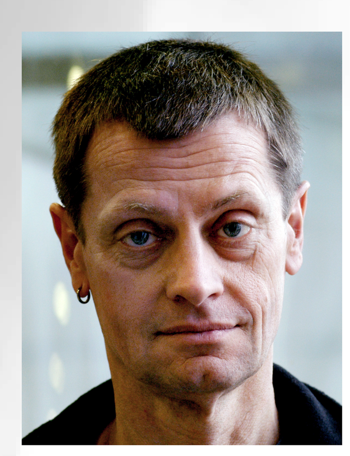


Systematic difference between the radial flux values in the Voronoi-Delaunay nearest-neighbour node distribution and the isotropized case

Ask Me

This is a poster, i.e. an advertisement. For details and demos of the real thing, please contact me, either at this meeting or by mailing

icke@strw.leidenuniv.nl



Future

After completion of this phase of the project, we will refine our hydrodynamical models by dynamically coupling the hydro and radiation into a true radiation-hydrodynamics code.