# A simple accretion model of a rotating gas sphere onto a Schwarzschild black hole

Sergio Mendoza[1] & Eliu A Huerta [2]
[1]Instituto de Astronomía, UNAM, México
[2]DAMTP, Cambridge, UK

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http://www.astroscu.unam.mx/~sergio/talks

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# 1 Summary

- ☆ Spherical accretion. Bondi (1952).
- ☆ Accretion of a rotating gas cloud towards a star. Ulrich (1976).
- ☆ Relativistic spherical accretion. Michel (1972).
- ☆ Accretion of a rotating gas cloud towards a Schwarzschild black hole. Huerta & Mendoza (2007): gr-qc/0603039.

# 2 Spherical accretion

- ☆ Model built "simply for mathematical curiosity" (Bondi 2005) by Bondi (1952).
- ☆ It considers a central gravitational potential produced by a central mass $M$ embedded into an infinite gas cloud.
- ☆ Cloud is at rest at $\infty$ with a pressure $p_\infty$ and a density $\rho_\infty$.
- ☆ Radial symmetry and stationary situation.
- ☆ Polytropic relation: $p \propto \rho^\kappa$.
- ☆ Continuity equation applied to a streamline implies that:

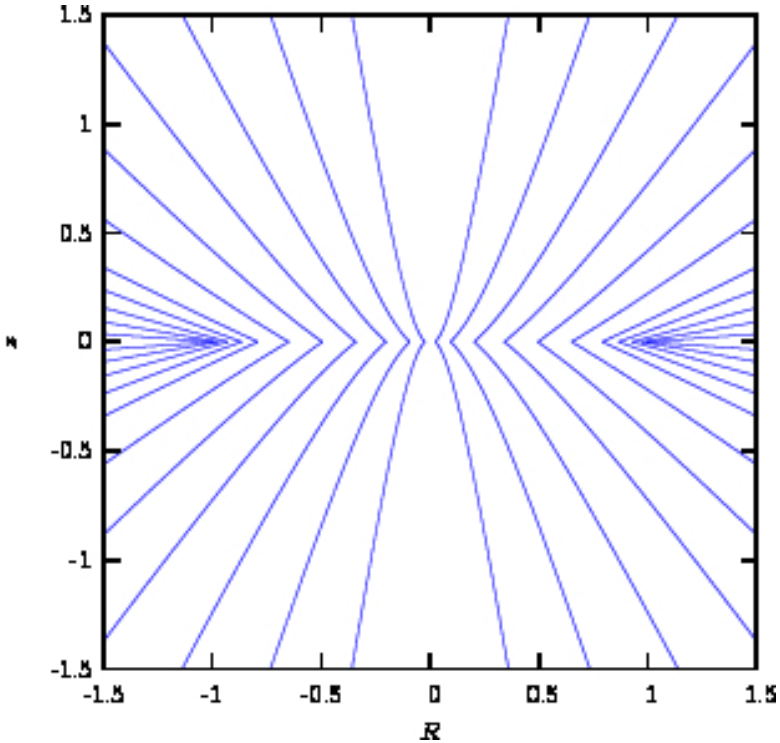
$$\dot{m} = 4\pi r^2 \rho v = \text{const.} \tag{1}$$

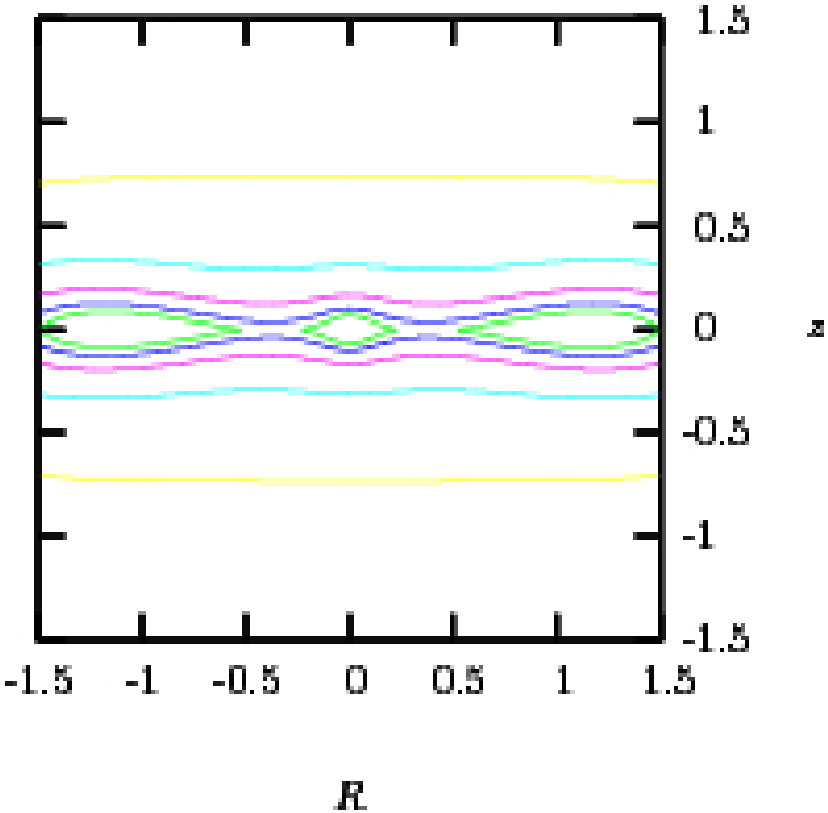
- ☆ Bernoulli's equation can be integrated to obtain velocity field.
- ☆ This model "provides a good estimate to the order of magnitude of the accretion rate in all cases of physical interest" (Bondi, 1952).

# 3 Spherical accretion with rotation (Ulrich 1976)

- ☆ Small perturbation to Bondi's model: the gas cloud rotates as a rigid body in such a way that its specific angular momentum $h$ has a value $h_\infty$ far away from the rotation axis which satisfies $h_\infty \ll 1$.
- ☆ Since the accretion process is sufficiently slow $\Rightarrow$ mass $M$ of the central object is constant.
- ☆ Self–gravity of the gas is negligible compared to the gravity of central object.
- ☆ Collisions between particles arriving to the equatorial plane $\Rightarrow$ formation of an accretion disc of radius $r_d = h_\infty^2/GM$.
- ☆ Assuming the mass of the disc $M_d \ll M$, and no viscosity $\Rightarrow$ trajectory of a fluid element is described by a Newtonian central potential since its specific energy $E \approx 0$ along all the fluid's particle trajectory.

☆ Streamlines and density isocontours (Mendoza, Cantó & Raga 2005).



# 4 Spherical accretion on a Schwarzschild space–time (Michel 1972)

- ☆ Gas cloud without rotation. Particles fall from $\infty$ to the black hole in a stationary way.
- ☆ Accretion rate given by:

$$\dot{n} = 4\pi r^2 n u = \text{const.} \tag{2}$$

- ☆ Relativistic Bernoulli's equation can be integrated to find the solution (Huerta & Mendoza 2006).
- ☆ Michel (1972) obtained his solutions using the time component of $\nabla_\mu T^{\mu\nu} = 0$ + continuity equation.

# 5 Relativistic accretion with rotation

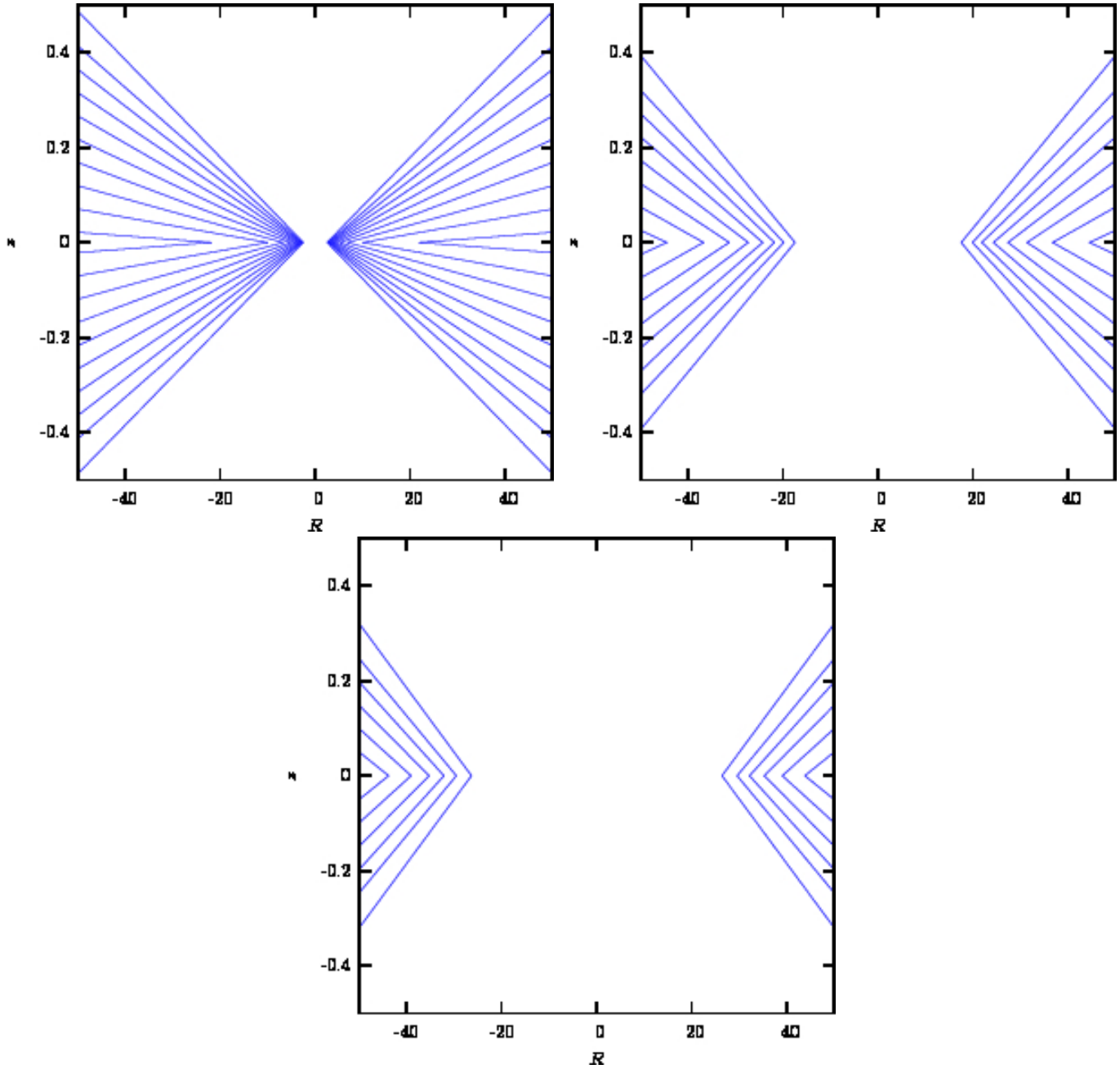
- ☆ Generalisation of Ulrich's accretion model in a Schwarzschild space–time.
- ☆ Convergence with Ulrich and Michel's models.
- ☆ Problem: integrate equation of motion for material particles taking into account general relativity, without approximations, i.e. no perturbations and no pseudo–Newtonian potentials.
- ☆ Solution: exact (analytic) integral of motion in terms of Jacobi elliptic functions.
- ☆ Under these assumptions one can obtain analytical solutions for the streamlines, velocity field and accretion flow density (Huerta & Mendoza, 2007)

# 6 Results

- ☆ The model describes "parabolic" orbits and so, it turns out that it depends on a parameter $\alpha$, s.t., when $\alpha \to 0$, then $c \to \infty$. In fact, $0 \leq \alpha \leq 1/8$ and when $\alpha \to 1/8$ the specific angular momentum $h = 2r_{\mathrm{g}}$ reaches its minimum value.
- ☆ Solutions converge to Ulrich's model when $\alpha \to 0$, and to Michel's one with null pressure in the absence of angular momentum, i.e. $h = 0$.

Why is it a coherent model?

- ☆ Resulting orbits satisfy: $E_{tot} = 0$.
- ☆ Orbits are the relativistic counterpart of the parabolic Newtonian model obtained when $\alpha = 0$.
- ☆ Exact analytic model that gives information about velocity field and density.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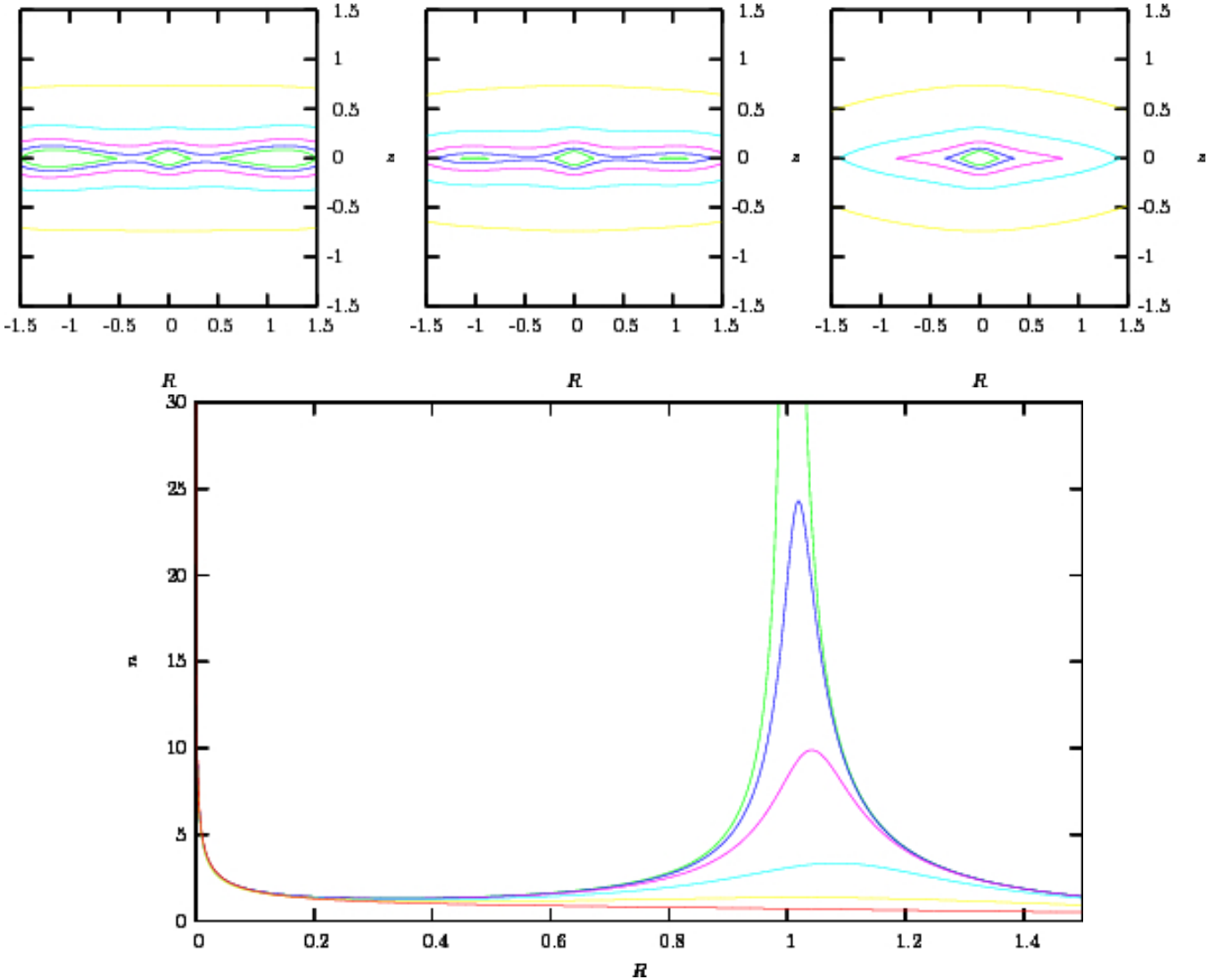
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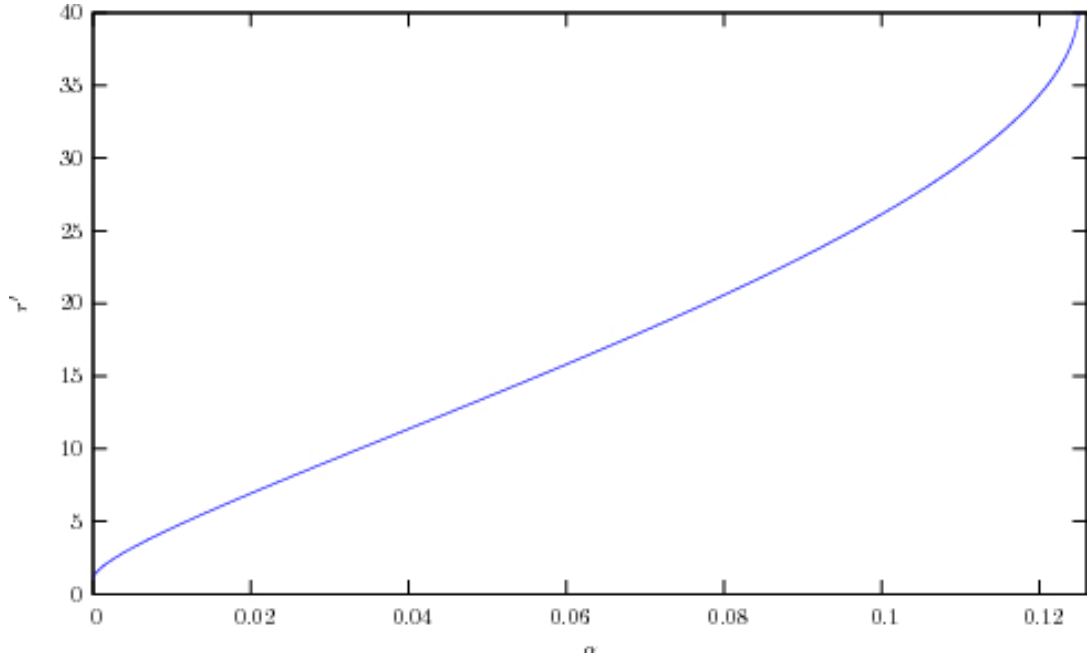
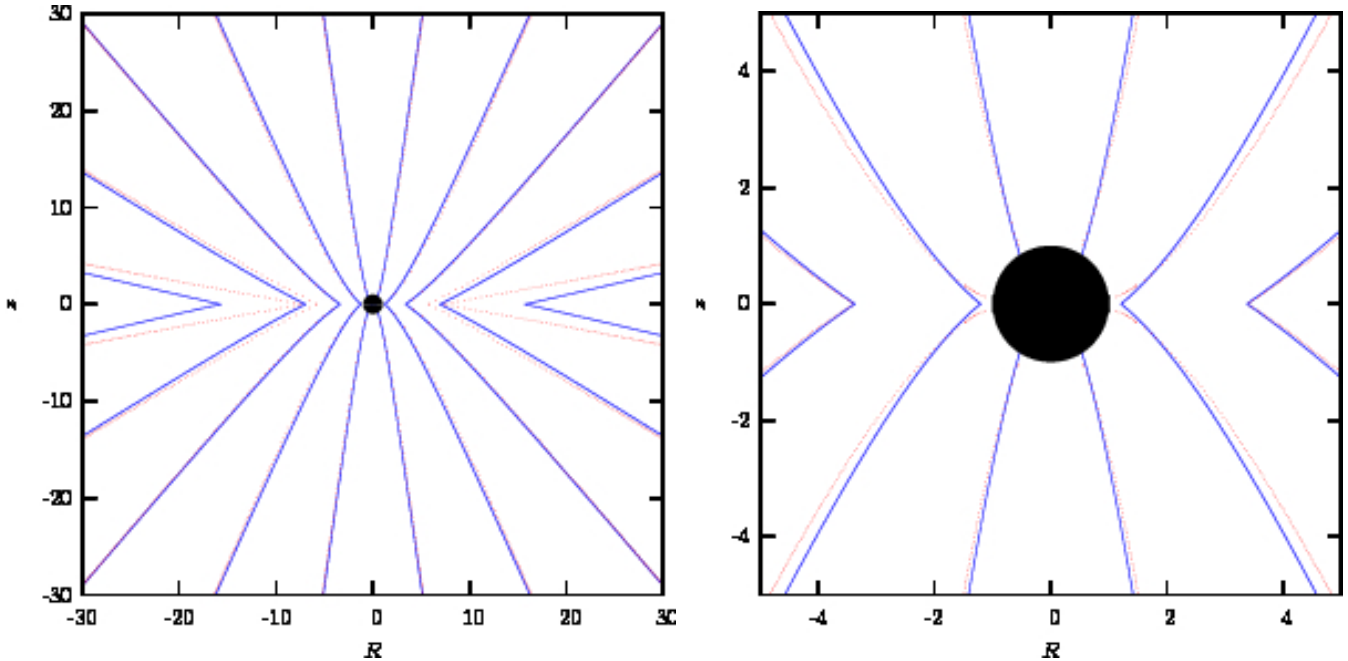



Unexpected results:

- ☆ Softening of the density divergence at the border of the disc.
- ☆ Streamlines are not packed at the border of the disc.
- ☆ Accretion disc extends to all the equatorial plane since its radius $r_{\mathrm{d}} \to \infty$ as $\alpha \to 1/8$. ¡This can be prooved analytically!

- ☆ Lee & Ramirez–Ruiz (2006) built a similar Pseudo-Newtonian accretion model with the same conditions as our model.




- ☆ Notable differences at small and large scales in the streamlines.
- ☆ In the full relativistic model, particles approach faster the equatorial plane.
- ☆ The hole does not swallow directly all particles on its surroundings, they are previously injected to the disc.

# 7 Conclusions

- ☆ We had built a relativistic generalisation of Ulrich's accretion model.
- ☆ Essential features are preserved.
- ☆ Accretion disc radius increases without limit when $h = 2r_{\mathrm{g}}$.
- ☆ Divergence in the particle number density at the border of the disc occurs only for the Newtonian case.

Future: together with E. Tejeda y E. Ley Koo we are building an accretion model of a rotating gas cloud towards a Kerr black hole.