

Magnetic fields and Neutron Star Surface

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# Neutron Stars: Interiors - Surface(s)



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# Neutron Stars: Interiors - Surface(s)

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- Neutron stars exist and have strong magnetic fields.
- Neutron stars do not exist: what are they ?
- How to find "exotic" matter: cooling.
- How to find "exotic" matter: stellar radii.
- Conclusions.

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## Neutron stars do exist !

Let's consider the fastest known pulsar:

PSR J1748-2448ad in Terzan5: rotational period **P = 1.39 msec.**

(Previous fastest pulsar was PSR 1937+21: P = 1.558 msec.)

Velocity at equator < light velocity:

$$v_{\text{equator}} = \Omega R = \frac{2\pi R}{P} \leq c \quad \Rightarrow \quad R \leq 2\pi c P = 65 \text{ km}$$

If the star is bound by gravity:  $a_{\text{gravity}} > a_{\text{centrifugal}}$  at equator:

$$a_{\text{gravity}} = \frac{GM}{R^2} \geq a_{\text{centrifugal}} = \Omega^2 R = \frac{4\pi^2 R}{P^2} \quad \text{or} \quad \frac{M}{R^3} \geq \frac{4\pi^2}{GP^2}$$

$$\Rightarrow \quad \bar{\rho} = \frac{M}{\frac{4}{3}\pi R^3} \geq 8 \cdot 10^{13} \text{ g/cm}^3$$

## Pulsar spin-down

Spin-down power:  $\dot{E} \equiv \frac{d}{dt} \left[ \frac{1}{2} I \Omega^2 \right] = I \Omega \dot{\Omega}$

Magneto-dipolar power:  $\dot{E}_{md} = - \frac{R^6 B_p^2 \Omega^4 \sin^2 \alpha}{6c^3}$

Spin-down law:  $\dot{E} = \dot{E}_{md} \Rightarrow \dot{\Omega} = - \frac{R^6 B_p^2}{6c^3 I} \Omega^3 = -K \Omega^3$

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$$\Rightarrow P\dot{P} = -4\pi^2 \frac{\dot{\Omega}}{\Omega^3} = -\frac{2\pi^2}{3} \frac{R^6 B_p^2}{c^3 I}$$

$$\Rightarrow B_p = 3 \times 10^{13} \left( P\dot{P}_{-13} \right)^{1/2} \text{ G}$$

$$\dot{\Omega} = -K \Omega^3 \Rightarrow \frac{d\Omega}{\Omega^3} = -K dt \Rightarrow \frac{1}{2} \left( \frac{1}{\Omega_0^2} - \frac{1}{\Omega^2} \right) = Kt$$

$$\Rightarrow \tau_{sd} = \frac{P}{2\dot{P}}$$

Rough  
estimates

## Pulsar spin-down

Spin-down power:  $\dot{E} \equiv \frac{d}{dt} \left[ \frac{1}{2} I \Omega^2 \right] = I \Omega \dot{\Omega}$

Magneto-dipolar power:  $\dot{E}_{md} = - \frac{R^6 B_p^2 \Omega^4 \sin^2 \alpha}{6c^3}$

It is essentially a dimensional analysis estimate:

At the light cylinder:

$$r_{LC} \equiv \frac{c}{\Omega}$$

Magnetic field:

$$B_{LC} = B_p \times \left( \frac{R}{r_{LC}} \right)^3 = \frac{B_p R^3 \Omega^3}{c^3}$$

Magnetic energy density:

$$E_{m(LC)} = \frac{B_{LC}^2}{8\pi} = \frac{B_p^2 R^6 \Omega^6}{c^6}$$

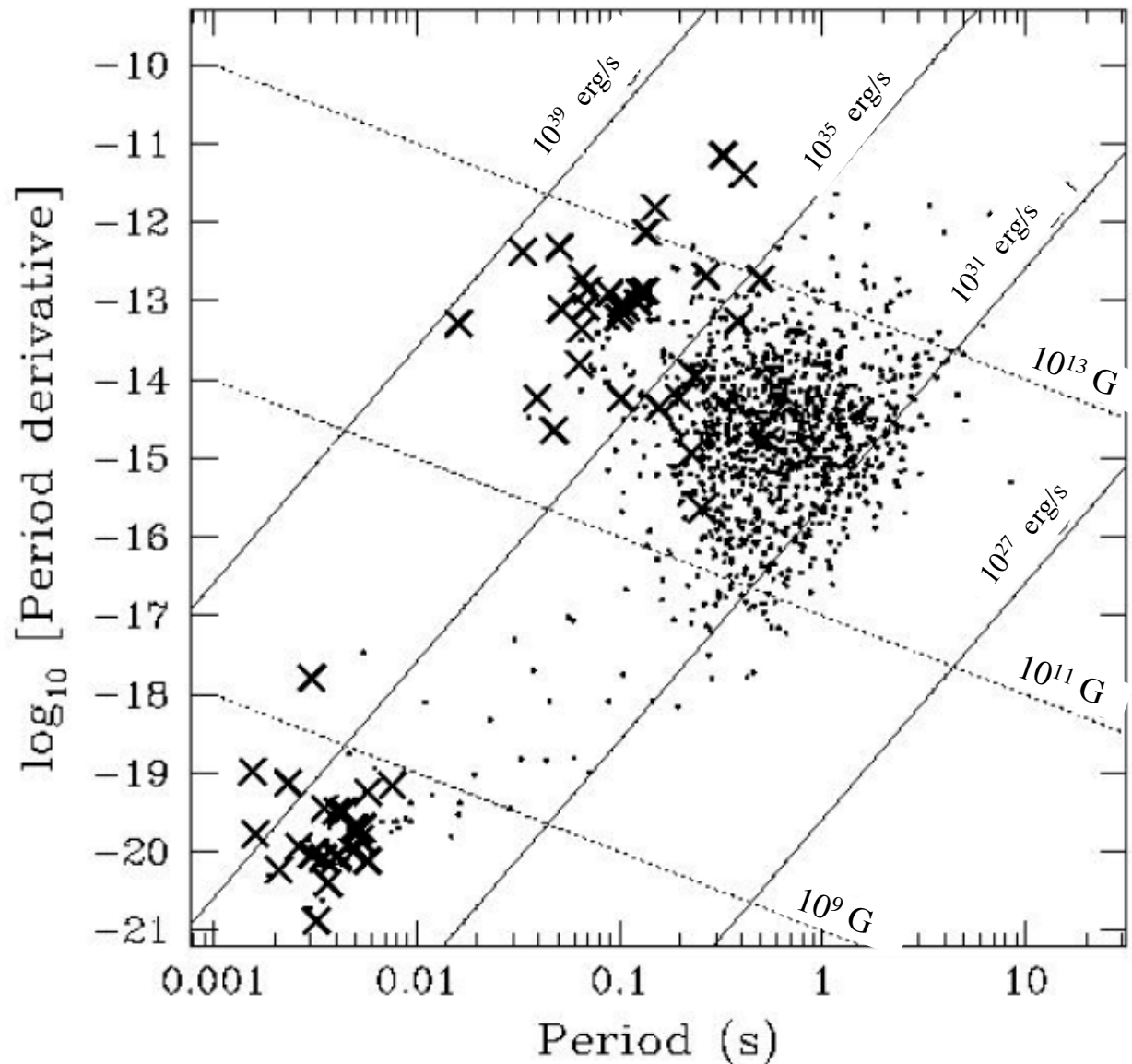
$$\dot{E}_{md} \sim -c E_{m(LC)} \cdot 4\pi r_{LC}^2 = -c \frac{B_p^2 R^6 \Omega^6}{8\pi c^6} 4\pi \frac{c^2}{\Omega^2}$$

# Pulsar P- $\dot{P}$ diagram and X-ray detected pulsars

1403 known  
rotation-powered  
pulsars:

66 detected in X-  
rays:

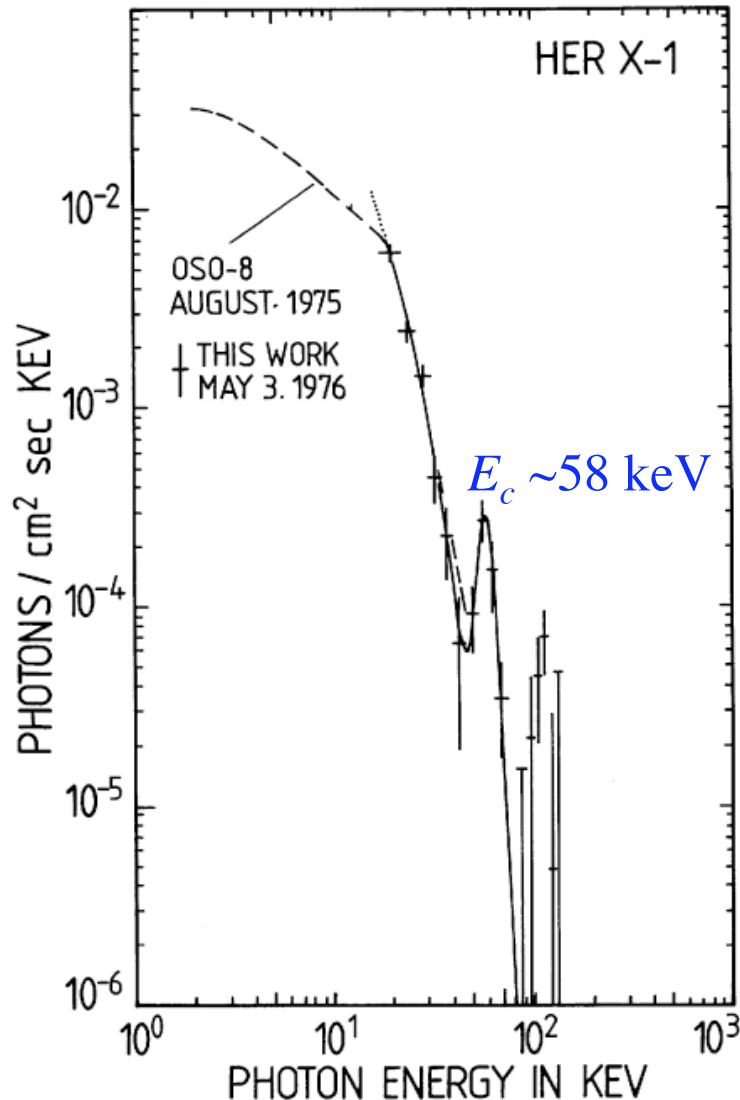
pulsed, unpulsed or  
nebular emission,  
thermal and/or  
non-thermal



*Isolated Neutron Stars,*  
V.M Kaspi, M.S.E. Roberts & A.K. Harding  
astro-ph/0402136



## Discovery of a cyclotron line in Her X-1



$$E_c = \hbar \frac{eB}{m_e c} = 11.2 B_{12} \text{ keV}$$

Was interpreted as an emission line.

Present interpretation favors an  
absorption line at  $\sim 40 \text{ keV}$

*Evidence for strong cyclotron line emission in the hard X-ray spectrum of Hercules X-1*  
Truemper, J.; Pietsch, W.; Reppin, C.; Voges, W.; Staubert, R.; Kendziorra, E.  
1978ApJ 219, L105

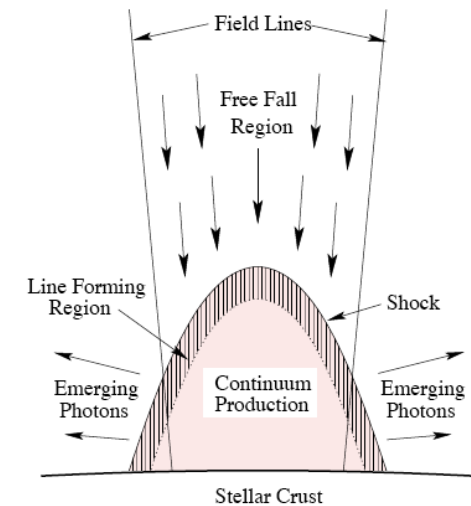
# Cyclotron lines in accreting X-ray pulsars

**TABLE 1.** List of pulsars with securely detected cyclotron lines. The discovery instrument is listed along with the discovery reference and whether the line has been observed with *RXTE*.

Source	Energy (keV)	Discovery Instrument	<i>RXTE</i> ?
4U 0115+63 <sup>†</sup>	12	<i>HEAO-1</i>	Y
4U 1907+09 <sup>†‡</sup>	18	<i>Ginga</i>	Y
4U 1538–52 <sup>‡</sup>	20	<i>Ginga</i>	Y
Vela X–1 <sup>†‡</sup>	25	HEXE	Y
V 0332+53	27	<i>Ginga</i>	N
Cep X–4	28	<i>Ginga</i>	Y
Cen X–3 <sup>‡</sup>	28.5	<i>RXTE/BSAX</i>	Y
X Per	29	<i>RXTE</i>	Y
XTE J1946+274	36	<i>RXTE/BSAX</i>	Y
MX0656–072	36	<i>RXTE</i>	Y
4U 1626–67	37	<i>RXTE/BSAX</i>	Y
GX 301–2 <sup>‡</sup>	37	<i>Ginga</i>	Y
Her X–1 <sup>‡</sup>	41	Balloon	Y
A 0535+26	50?, 110	HEXE	N

<sup>†</sup> objects with  $> 1$  harmonic observed

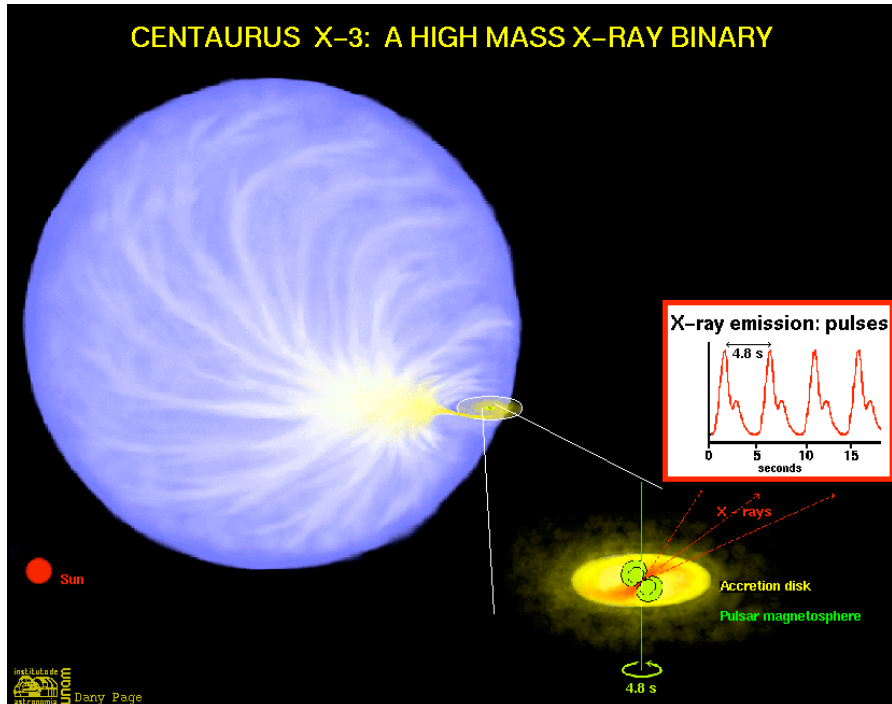
<sup>‡</sup> high inclination system



**FIGURE 1.** Schematic diagram of the “accretion mound” showing the line-forming region as a discrete layer covering the continuum production zone.

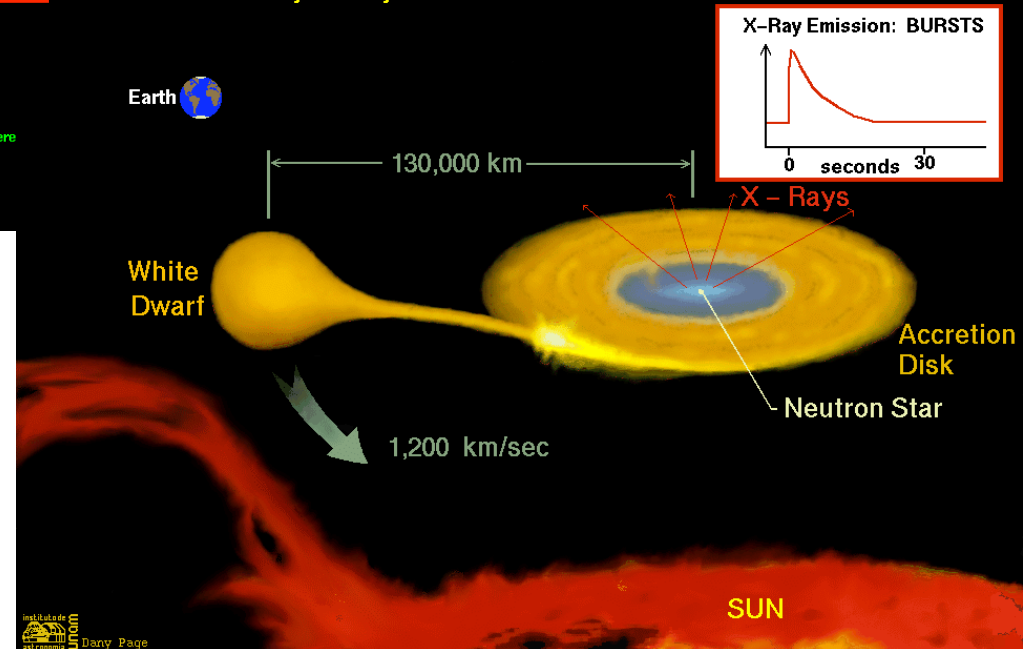
Timing and Spectroscopy of Accreting X-ray Pulsars: the State of Cyclotron Line Studies  
Heindl, W. A et al. (2004)  
AIP, .714 p. 323 [astro-ph/0403197]

# Neutron stars in X-ray binaries



- ← High mass companion:
- short lived ( $\sim 10^6$  yrs)
  - little mass accretion ( $\sim 0.01 M_0$ )

**A Low Mass X-Ray Binary: 4U 1820-30**



Low mass companion →

long lived ( $\sim 10^9$  yrs)

large mass accretion ( $\sim 0.5 M_0$ )

## Neutron stars do not exist !

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Do blackholes exist ?

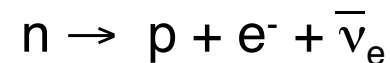
Simple question because blackhole are simple objects

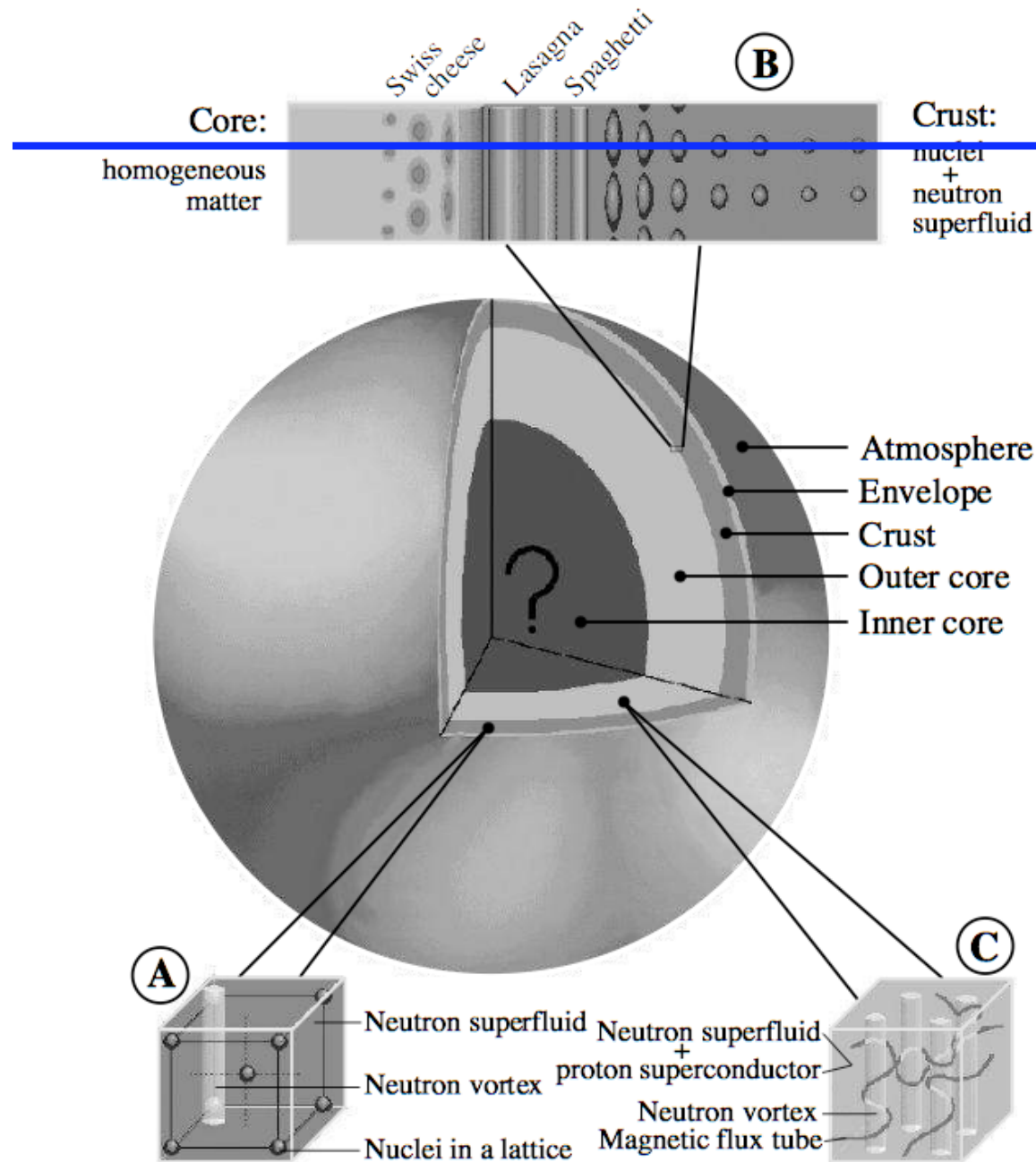
Do "neutron" stars exist ?

No doubt that compact (or whatever name you choose) stars exist.

Neutron stars, as originally conceived by Landau, Baade & Zwicky, Oppenheimer & Volkoff **do not** exist:

having a one solar mass ball of neutrons, the neutrons will immediately start to decay into protons





*Dense Matter in Compact Stars: Theoretical Developments and Observational Constraints*, Page, D., & Reddy, S. 2006, *Annu. Rev. Nucl. Part. Sci.*, 56, p. 327

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## Building models of dense matter

Simplest case of n-p-e- $\mu$  matter:

- Baryon number density:  $n_B = n_p + n_n$
- Charge neutrality:  $n_p = n_e + n_\mu$
- Chemical equilibrium:  $\mu_n = \mu_p + \mu_e$  &  $\mu_e = \mu_\mu$

$$\mu_e = p_F(e)c$$

$$\mu_\mu = \sqrt{m_\mu^2 c^4 + p_F(\mu)^2 c^2}$$

$$\mu_p = m_p c^2 + \frac{p_F^2(p)}{2m_p} + V_p \quad \mu_n = m_n c^2 + \frac{p_F^2(n)}{2m_n} + V_n$$

$\Rightarrow$  strong model dependence on how  $V_n$  and  $V_p$  are calculated

## Walecka type model of dense matter

The model proposes that in dense matter, nucleons interact with effective short-range forces. The Lagrangian is given by

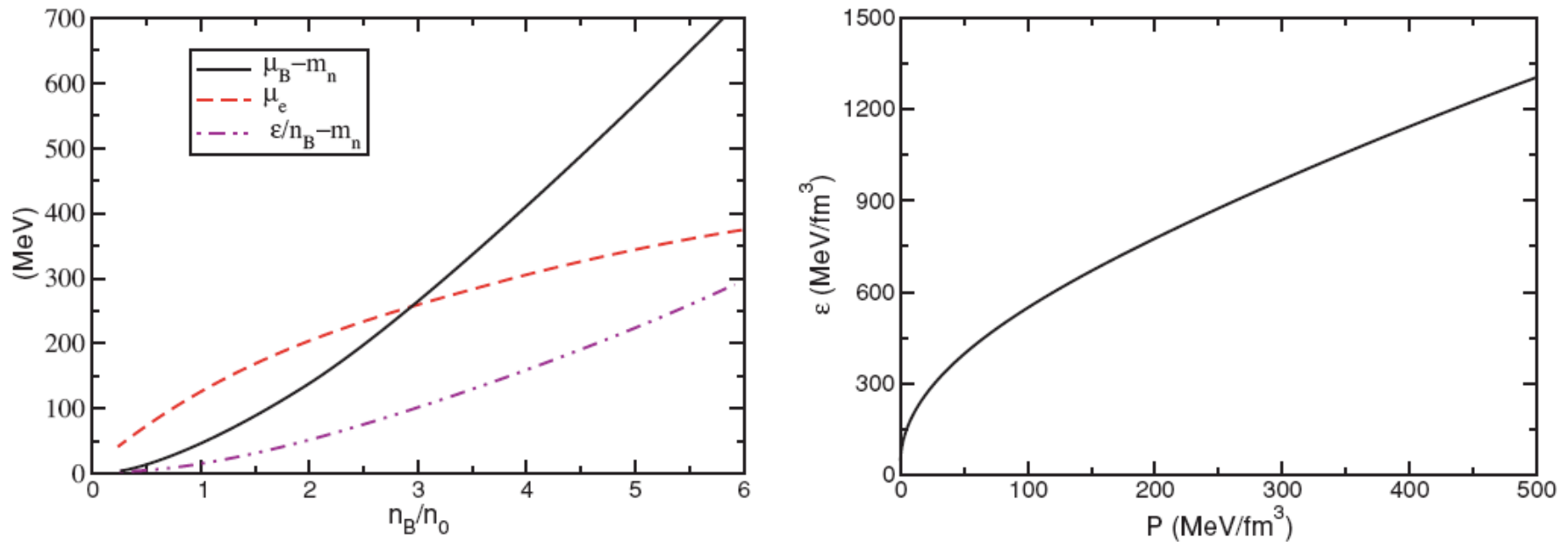
$$\begin{aligned}
 \mathcal{L}_N = & \bar{\Psi}_N (i\gamma^\mu \partial_\mu - m_N^* - g_{\omega N} \gamma^\mu V_\mu - g_{\rho N} \gamma^\mu \vec{\tau}_N \times \vec{R}_\mu) \Psi_N \\
 & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} \\
 & + \frac{1}{2} m_\omega^2 V_\mu V^\mu - \frac{1}{4} \vec{R}_{\mu\nu} \times \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{R}_\mu \times \vec{R}^\mu,
 \end{aligned} \tag{7}$$

where  $m_N^* = m_N - g_{\sigma N} \sigma$  is the nucleon effective mass, which is reduced in comparison to the free nucleon mass  $m_N$  owing to the scalar field  $\sigma$ , taken to have  $m_\sigma = 600$  MeV. The vector fields corresponding to the isoscalar omega and isovector rho mesons are given by  $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$  and  $\vec{R}_{\mu\nu} = \partial_\mu \vec{R}_\nu - \partial_\nu \vec{R}_\mu$ , respectively. The exchange of these mesons mimics the short-range forces between nucleons. In addition to the coupling between nucleons and mesons, a self-interaction between scalar mesons given by

$$U(\sigma) = \frac{b}{3} m_N (g_{\sigma N} \sigma)^3 + \frac{c}{4} (g_{\sigma N} \sigma)^4, \tag{8}$$



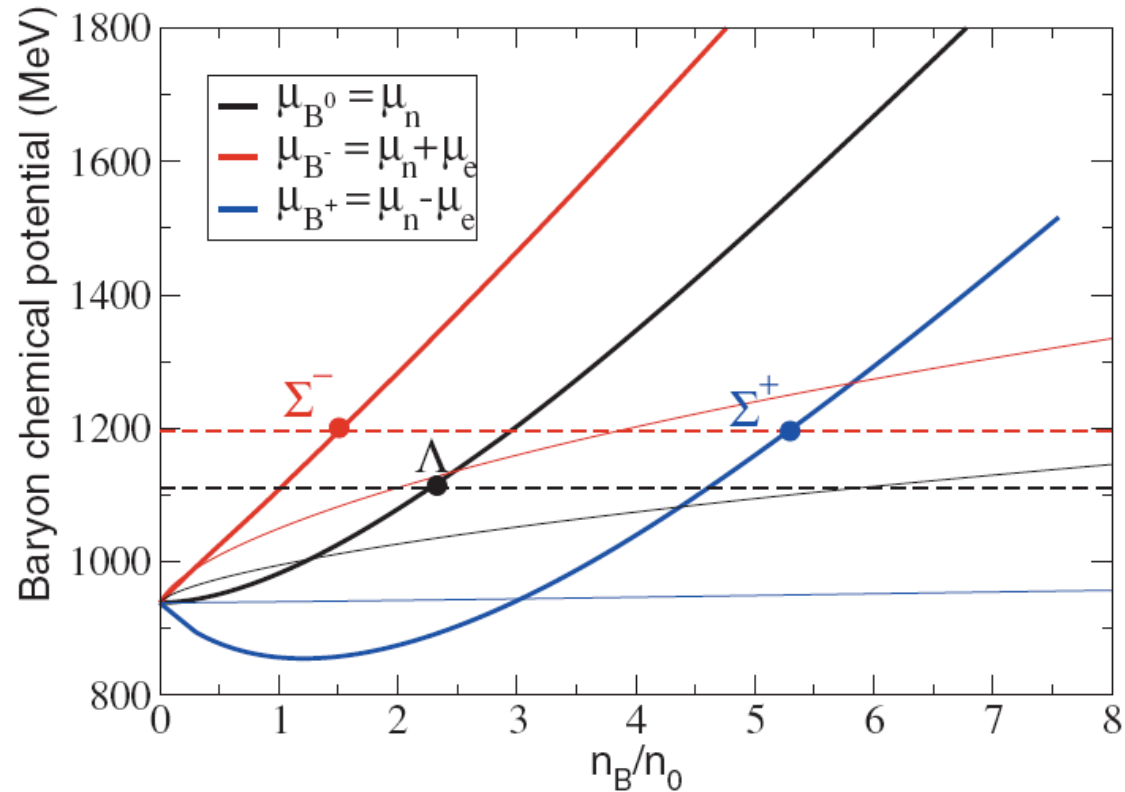
# Walecka type model of dense matter



**Figure 9** The nuclear equation of state. (*Left*) The density dependence of the baryon chemical potential, the electron chemical potential, and the energy per baryon. (*Right*) The relation between energy density and pressure.

$\mu_B \equiv \mu_n = m_n + \text{hundreds of MeVs: rapidly reaches } \Lambda \text{ and } \Sigma \text{ masses !}$

# Hyperons in dense matter

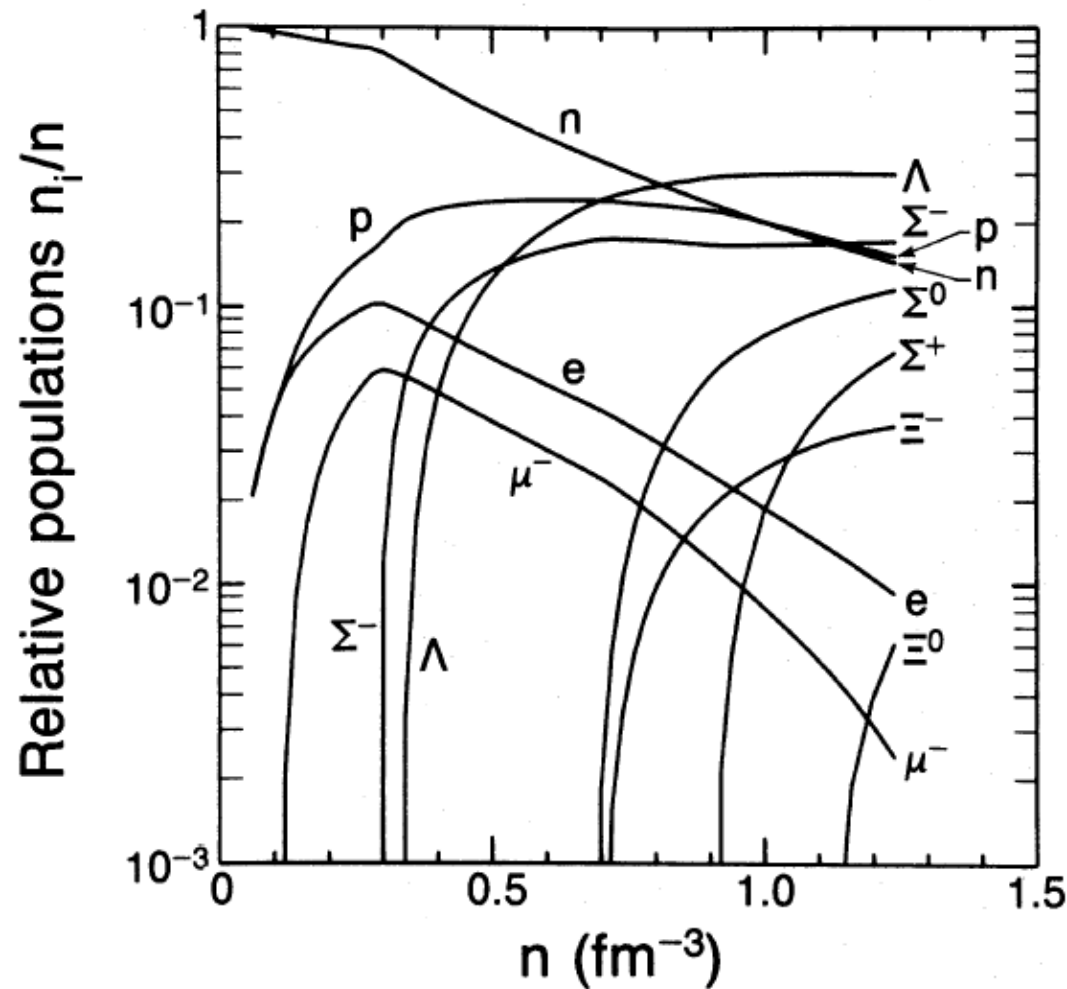


**Figure 10** Baryon chemical potentials in dense stellar matter.

Thick lines: Walecka type model

Thin lines: free gases

## An example of hyperonic matter



*Neutron stars are giant hypernuclei ?*, N.K. Glendenning, ApJ 293, 470 (1985)

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# How to find "exotic" matter: neutrino cooling

SOME CORE NEUTRINO EMISSION PROCESSES AND THEIR EMISSIVITIES

Process Name	Process	Emissivity $Q_\nu^a$ (ergs s <sup>-1</sup> cm <sup>-3</sup> )	Emissivity References
Modified Urca .....	$\begin{cases} n + n' \rightarrow n' + p + e^- + \bar{\nu}_e \\ n' + p + e^- \rightarrow n' + n + \nu_e \end{cases}$	$\sim 10^{20} T_9^8$	Friman & Maxwell 1979
Kaon condensate .....	$\begin{cases} n + K^- \rightarrow n + e^- + \bar{\nu}_e \\ n + e^- \rightarrow n + K^- + \nu_e \end{cases}$	$\sim 10^{24} T_9^6$	Brown et al. 1988
Pion condensate .....	$\begin{cases} n + \pi^- \rightarrow n + e^- + \bar{\nu}_e \\ n + e^- \rightarrow n + \pi^- + \nu_e \end{cases}$	$\sim 10^{26} T_9^6$	Maxwell et al. 1977
Direct Urca .....	$\begin{cases} n \rightarrow p + e^- + \bar{\nu}_e \\ p + e^- \rightarrow n + \nu_e \end{cases}$	$\sim 10^{27} T_9^6$	Lattimer et al. 1991
Quark Urca .....	$\begin{cases} d \rightarrow u + e^- + \bar{\nu}_e \\ u + e^- \rightarrow d + \nu_e \end{cases}$	$\sim 10^{26} \alpha_c T_9^6$	Iwamoto 1980

<sup>a</sup>  $T_9$  is the temperature in units of  $10^9$  K.

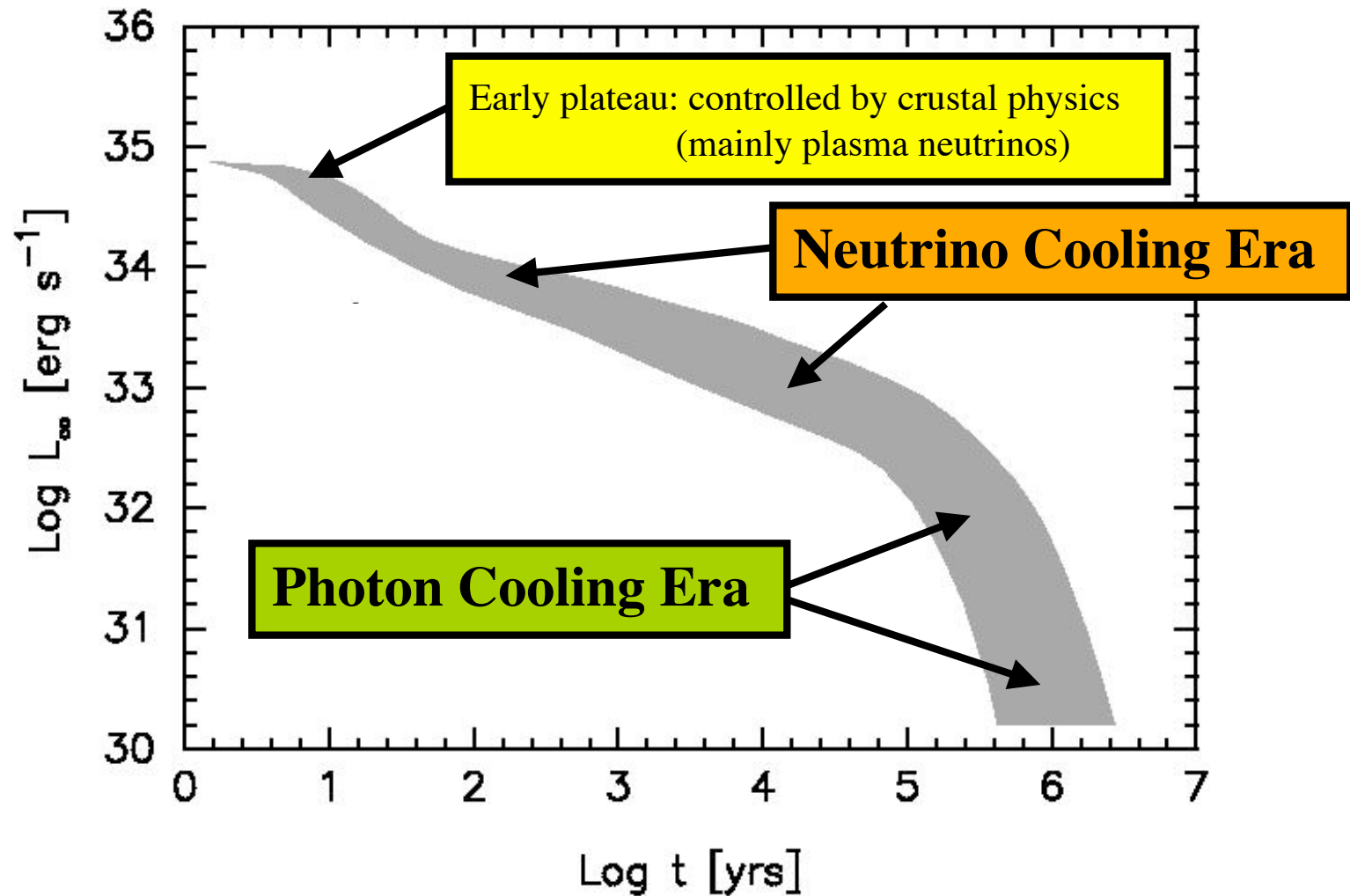
# How to find "exotic" matter: neutrino cooling

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Kaon condensate .....	$\begin{cases} n + K^- \rightarrow n + e^- + \bar{\nu}_e \\ n + e^- \rightarrow n + K^- + \nu_e \end{cases}$	$\sim 10^{24} T_9^6$	
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<sup>a</sup>  $T_9$  is the temperature in units of 10<sup>9</sup> K.

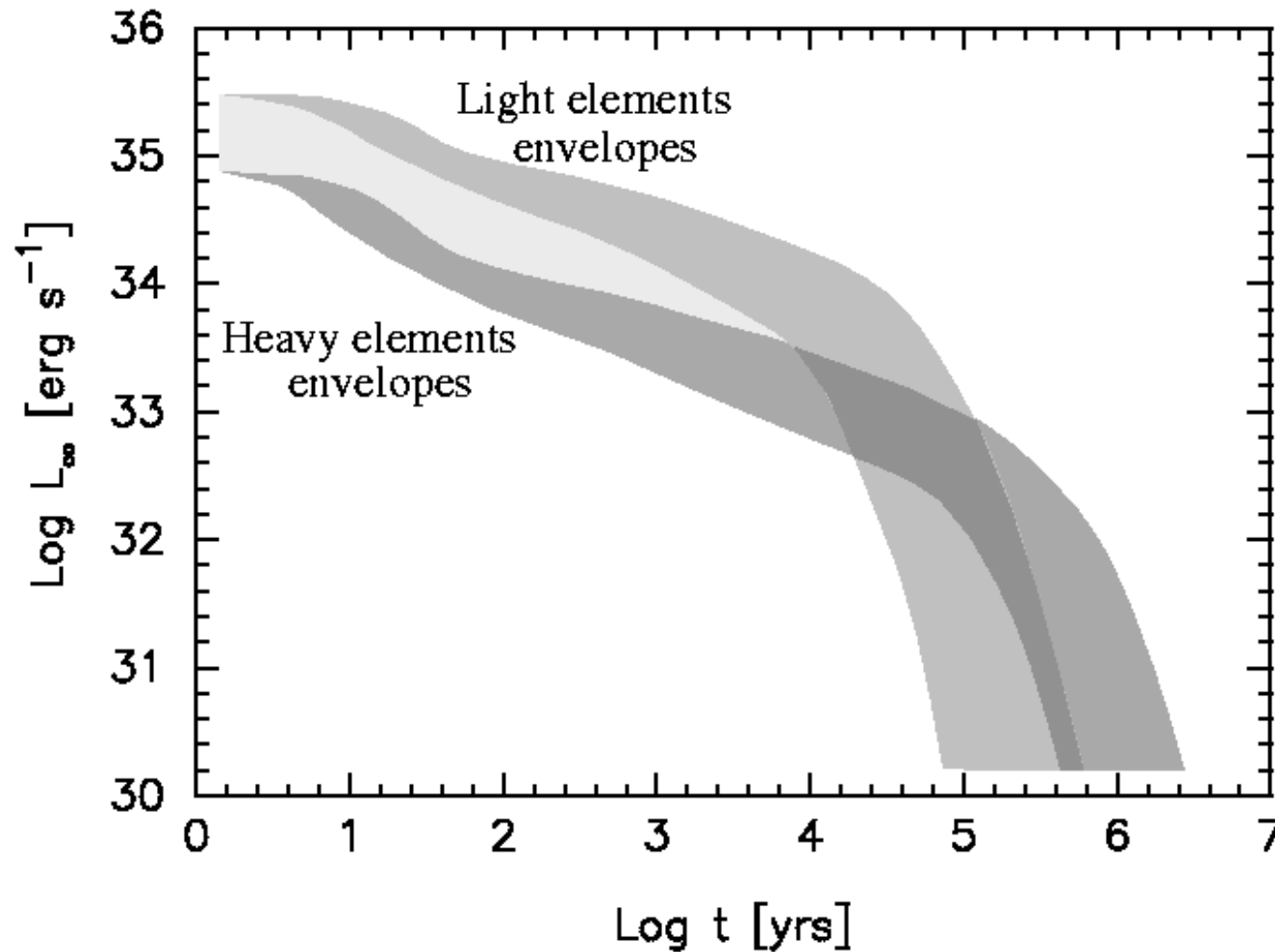
# Minimal cooling of neutron stars



*Minimal Cooling of Neutron Stars: A New Paradigm*

D. Page, J.M. Lattimer, M. Prakash & A.W. Steiner, 2004, ApJS 155, p. 623

# Minimal cooling of neutron stars

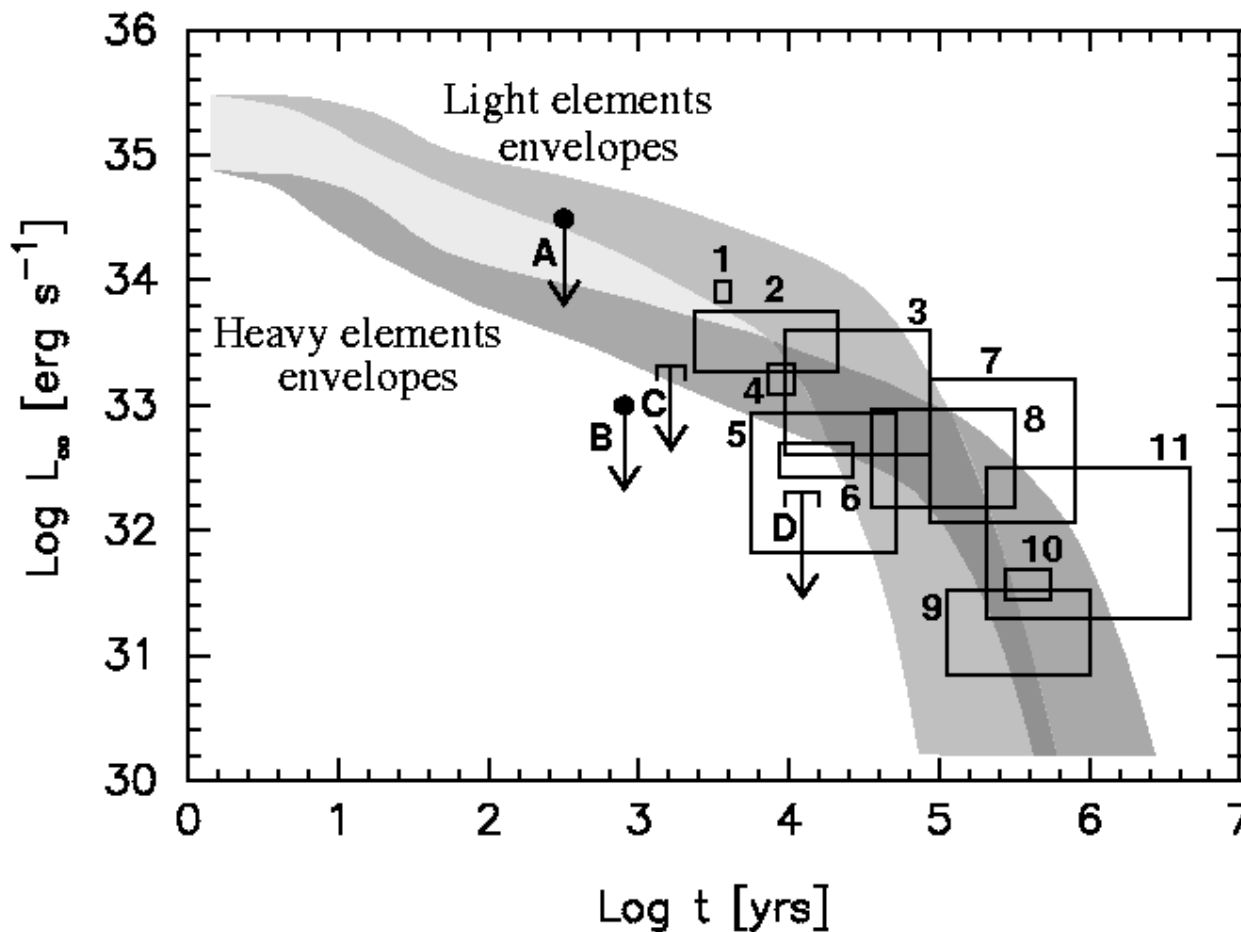


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# Minimal cooling: comparison with data



## Mag H fits:

- 1) RX J0822-4247 (in Puppis A)
- 2) 1E 1207.4-5209 (in PKS 1209-52)
- 3) PSR 0538+2817
- 4) RX J0002+6246 (in CTB 1)
- 5) PSR 1706-44
- 6) PSR 0933-45 (in Vela)

## BB fits:

- 7) PSR 1055-52
- 8) PSR 0656+14
- 9) PSR 0633+1748 "Geminga"
- 10) RX J1856.5-3754
- 11) RX J0720.4-3125

## Upper limits:

- A) CXO J232327.8+584842 (in Cas A)
- B) PSR J0205+6449 (in 3C58)
- C) PSR J1124-5916 (in G292.0+1.8)
- D) RX J0007.0+7302 (in CTA 1)

*Minimal Cooling of Neutron Stars: A New Paradigm*

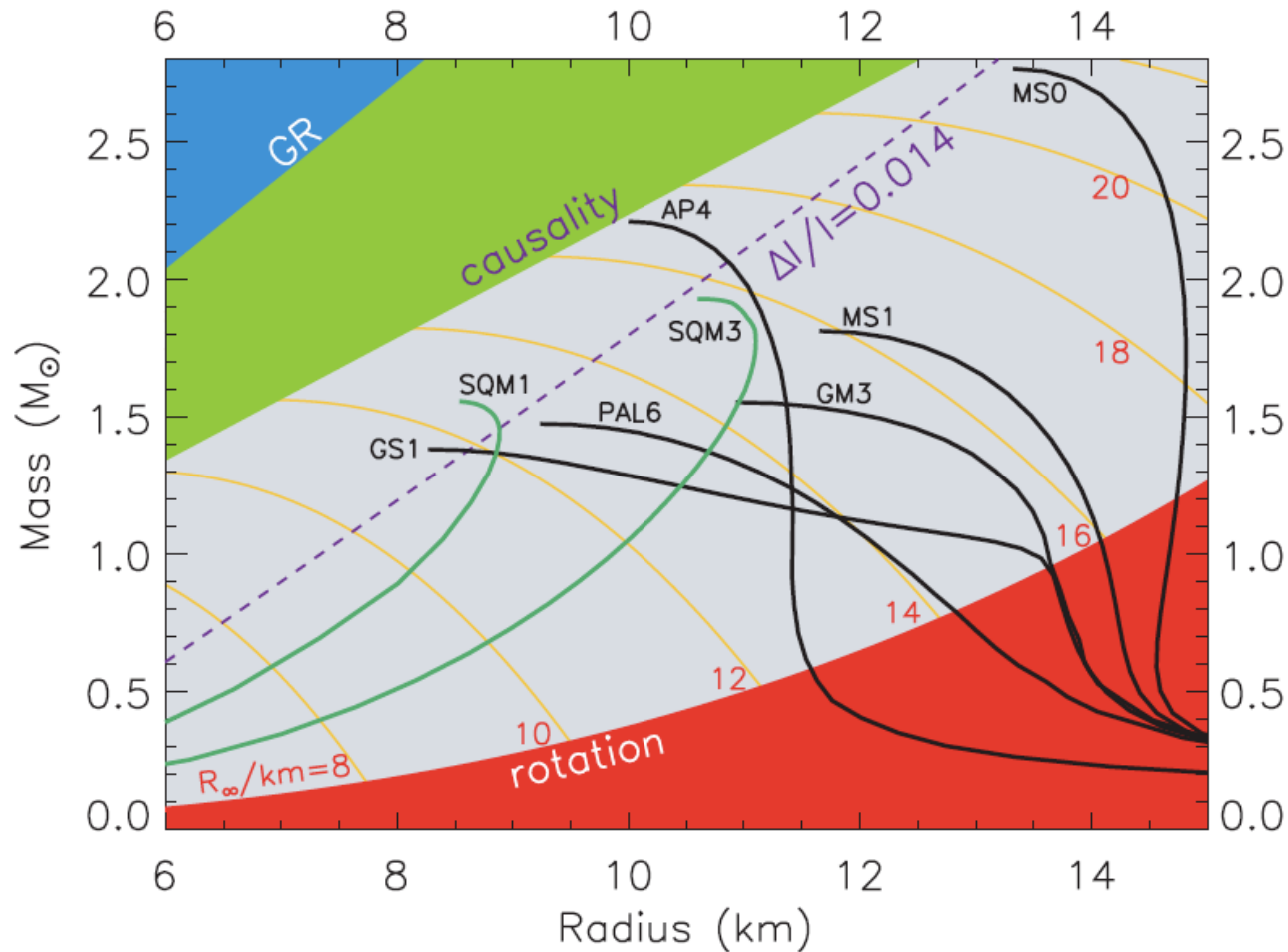
D. Page, J.M. Lattimer, M. Prakash & A.W. Steiner, 2004, *ApJS* 155, p. 623

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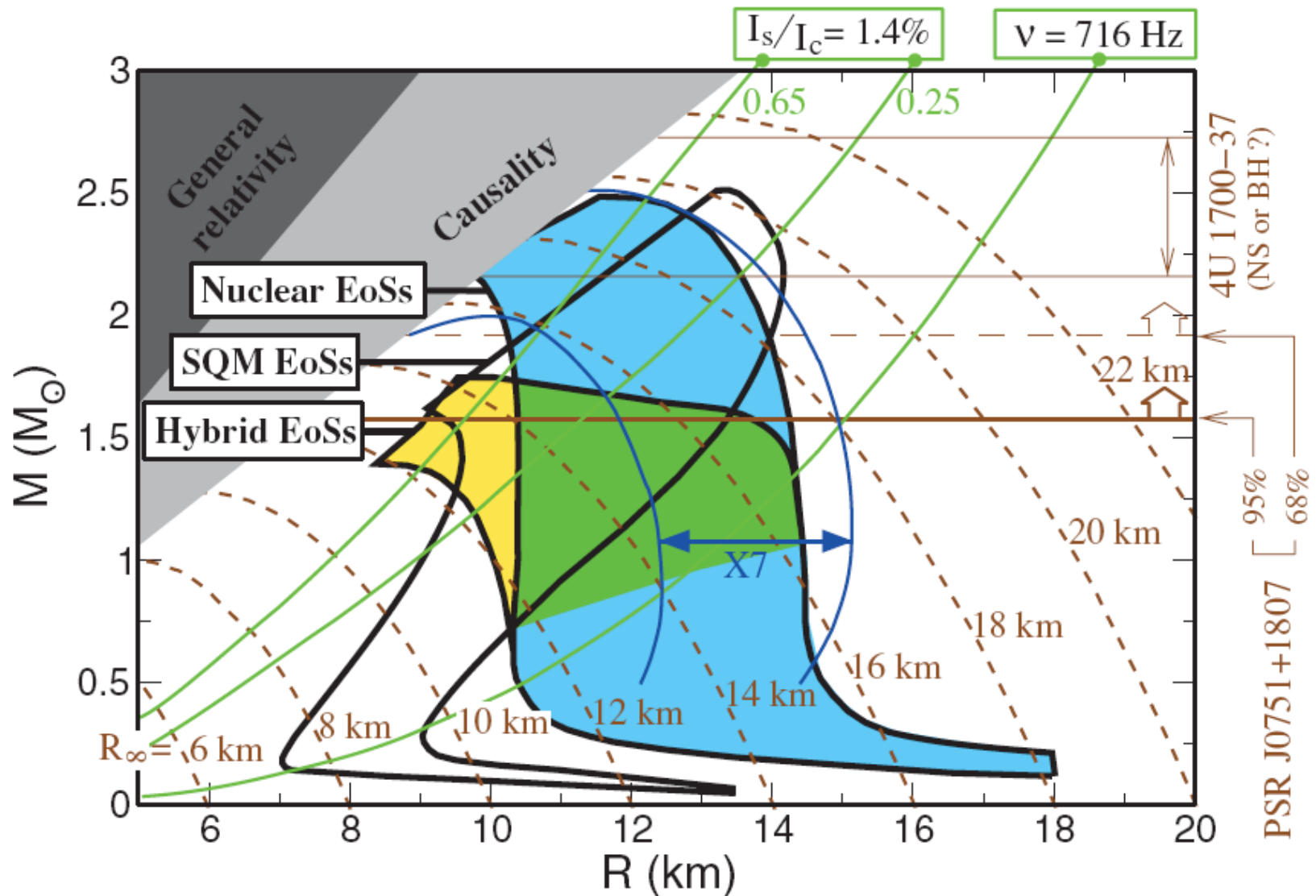
## How to find "exotic" matter: stellar radii



**Fig. 2.** Mass-radius diagram for neutron stars. Black (green) curves are for normal matter (SQM) equations of state [for definitions of the labels, see (27)]. Regions excluded by general relativity (GR), causality, and rotation constraints are indicated. Contours of radiation radii  $R_{\infty}$  are given by the orange curves. The dashed line labeled  $\Delta/I = 0.014$  is a radius limit estimated from Vela pulsar glitches (27).

*The Physics of Neutron Stars*, Lattimer, J. M.; Prakash, M. 2004, Science 304, p. 536

# How to find "exotic" matter: stellar radii



*Dense Matter in Compact Stars: Theoretical Developments and Observational Constraints*, Page, D., & Reddy, S. 2006, *Annu. Rev. Nucl. Part. Sci.*, 56, p. 327

# Measuring the radius of an isolated neutron star

## Quark-Matter Stars Said Found

By Jack Lucentini



The sizes of a neutron star and a quark star compared to the Grand Canyon. The smallest, most massive, most compressed neutron star possible is about 17 kilometers in diameter. A quark star can be smaller than 11 kilometers diameter. The canyon is 29 kilometers from rim to rim. In reality this scene couldn't exist; the entire Earth would collapse almost instantly to a thin layer coating the surface of either superdense star. Illustration by D. Berry / Chandra X-ray Center.

April 11, 2002 | Astronomers say they have likely found a bizarre new type of superdense star made of a weird form of matter like nothing else in the universe.

$$L_{\gamma} = 4\pi R^2 \sigma_{\text{SB}} T^4$$

$$\Rightarrow F_{\text{rec}} = \left(\frac{R}{D}\right)^2 \sigma_{\text{SB}} T^4$$

$F_{\text{rec}}$  observed "directly"

$T$  "measured" from the shape of the spectrum

$D$  measured (HST parallax)

$\Rightarrow R$  "measured"

**RX J1856.5-3754:**

**Drake *et al.* (2002) found**

**$R = 4\text{-}6 \text{ km} \Rightarrow \text{Quark Star} !$**

**"Better" spectral models +**

**new parallax distance**

**$\Rightarrow R = 24 \text{ km} !!!!!$**

**"Anti Quark Star" ?**

# Thermal spectra and interstellar absorption

$$F_{\text{Obs}}(E) = e^{-N_H \sigma_{\text{eff}}(E)} F(E)$$

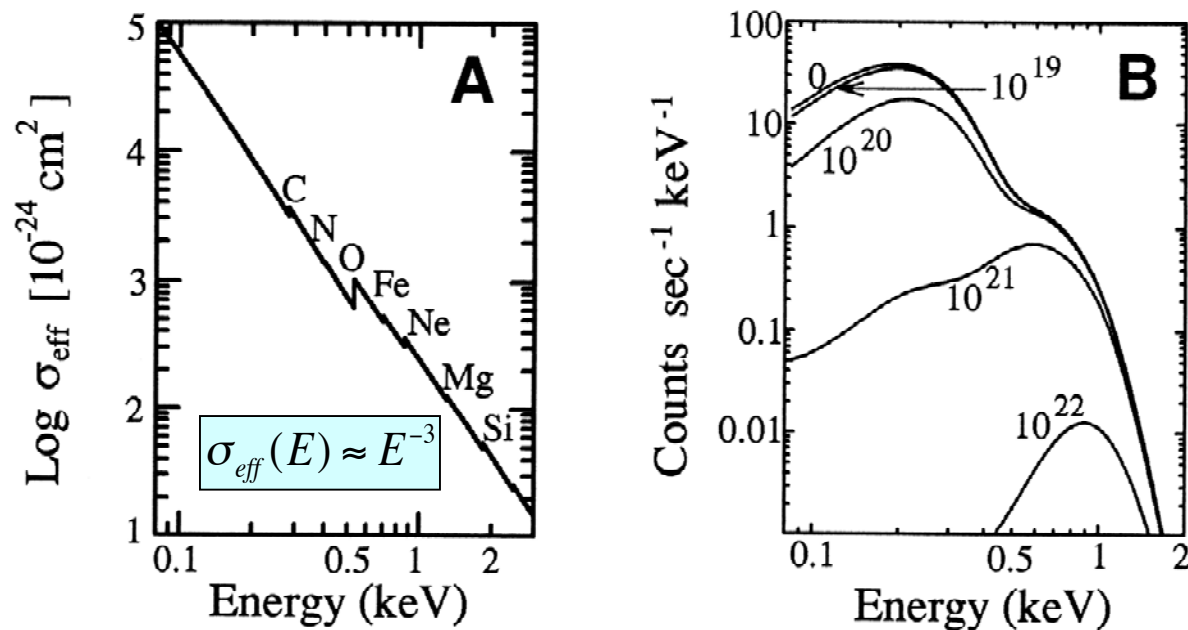
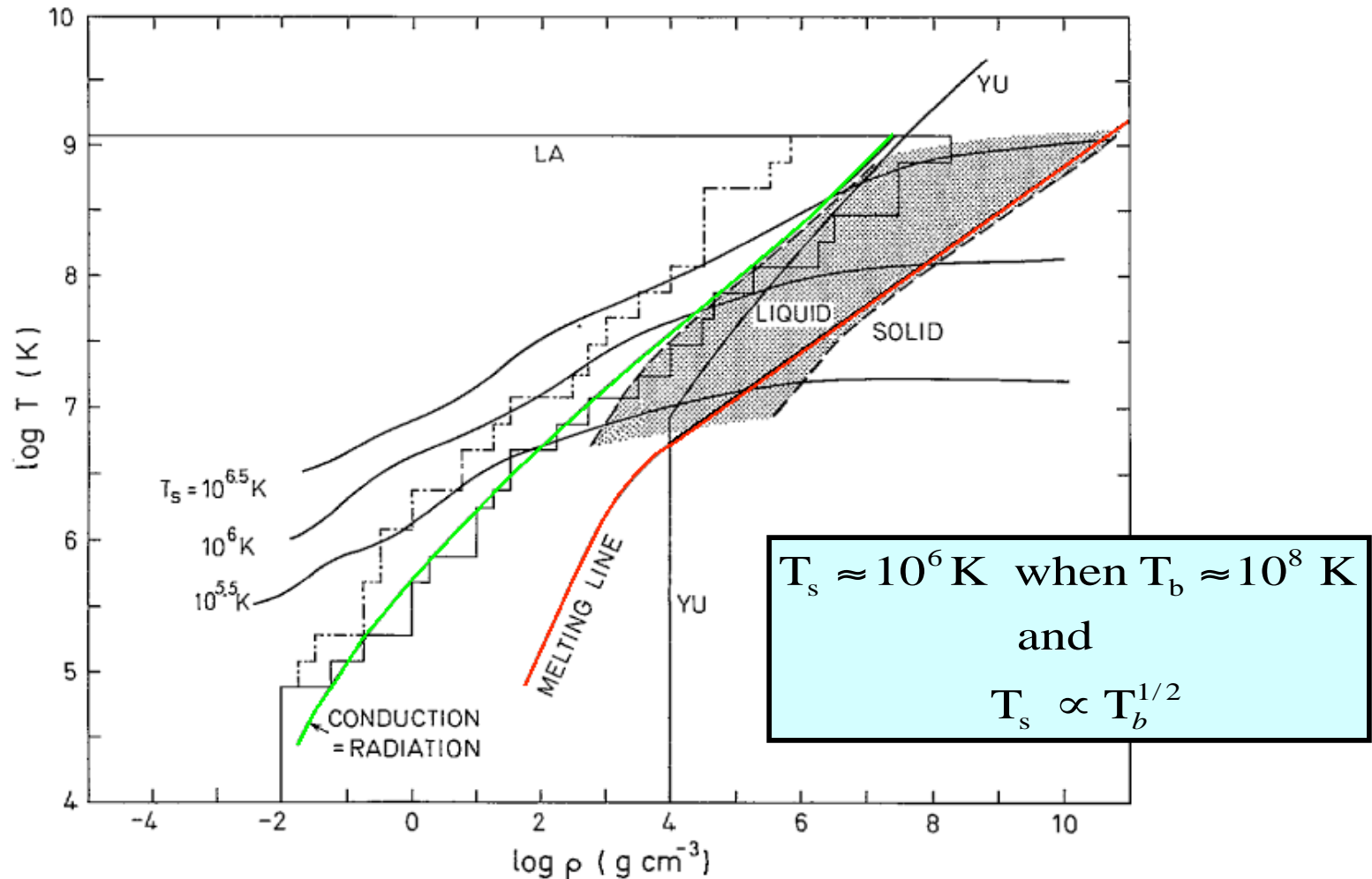


Fig. 1. **A:** Effective cross section for interstellar absorption, taking into account a standard chemical composition of the interstellar medium. Absorption edges from metals are indicated. (From Morrison & McCammon, 1983).

**B:** Blackbody spectrum ( $R_\infty = 13 \text{ km}$ ,  $M = 1.4 M_\odot$  neutron star at 500 pc with  $T_e^\infty = 10^6 \text{ K}$ ) and interstellar absorption with various values of  $N_H$  as indicated. These spectra take into account the *ROSAT* PSPC response.

# Temperature drop in the neutron star envelope



Gudmundsson, Pethick & Epstein, Ap. J. 259 (1982), L19 and Ap. J. 272 (1983) 286

# Anisotropic heat transport with magnetic fields

## Electron thermal conductivity

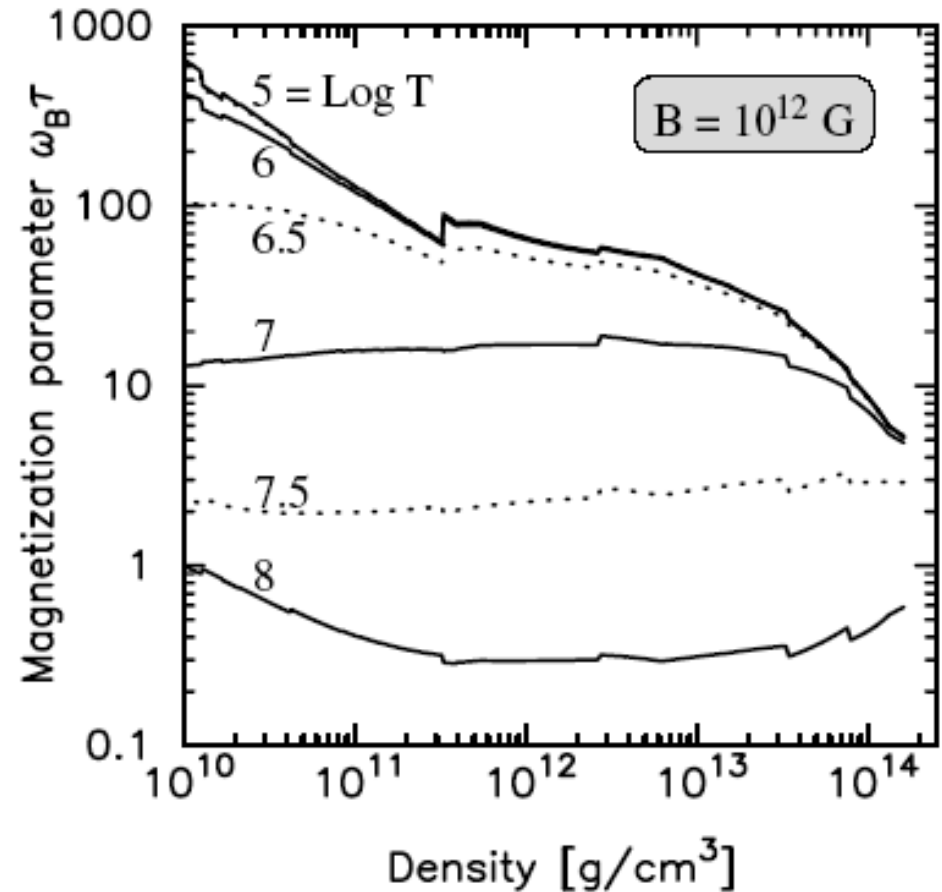
$$\vec{\kappa} = \begin{pmatrix} \kappa_{\parallel} & 0 & 0 \\ 0 & \kappa_{\perp} & \kappa_{\wedge} \\ 0 & -\kappa_{\wedge} & \kappa_{\perp} \end{pmatrix}$$

$$\kappa_{\parallel} = \kappa_0$$

$$\kappa_{\perp} = \frac{\kappa_0}{1 + (\omega_B \tau)^2}$$

$$\kappa_{\wedge} = \frac{\kappa_0 (\omega_B \tau)}{1 + (\omega_B \tau)^2}$$

$$\omega_B = \frac{eB}{m^*c} \quad \text{electron gyrofrequency}$$





# Anisotropic heat transport in a magnetized envelope

$$\kappa(\Theta_B) = \kappa_{\parallel} \cdot \cos^2 \Theta_B + \kappa_{\perp} \cdot \sin^2 \Theta_B \quad \Theta_B = \text{angle}(\mathbf{B}, \mathbf{r})$$

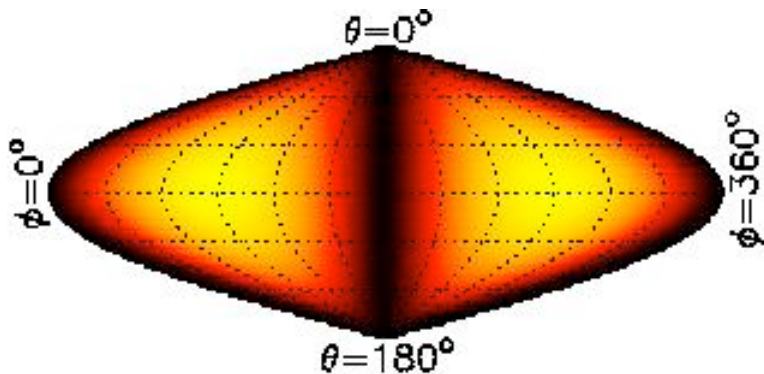
Considering the effect of the magnetic field only in the envelope:

$$T_s^4(\Theta_B) = T_s^4(\Theta_B = 0) \cos^2 \Theta_B + T_s^4(\Theta_B = 90) \sin^2 \Theta_B$$

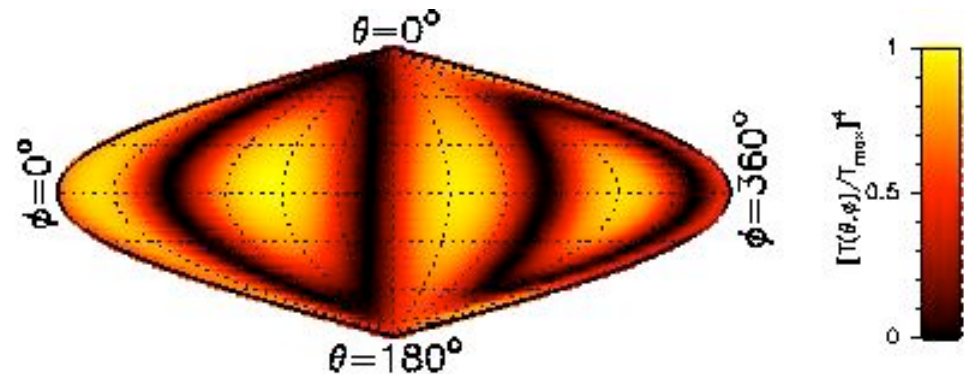
**Greenstein & Hartke, 1983**

Best present version: Potekhin & Yakovlev, 2001

Purely Dipolar Field



Dipole + Quadrupole Field

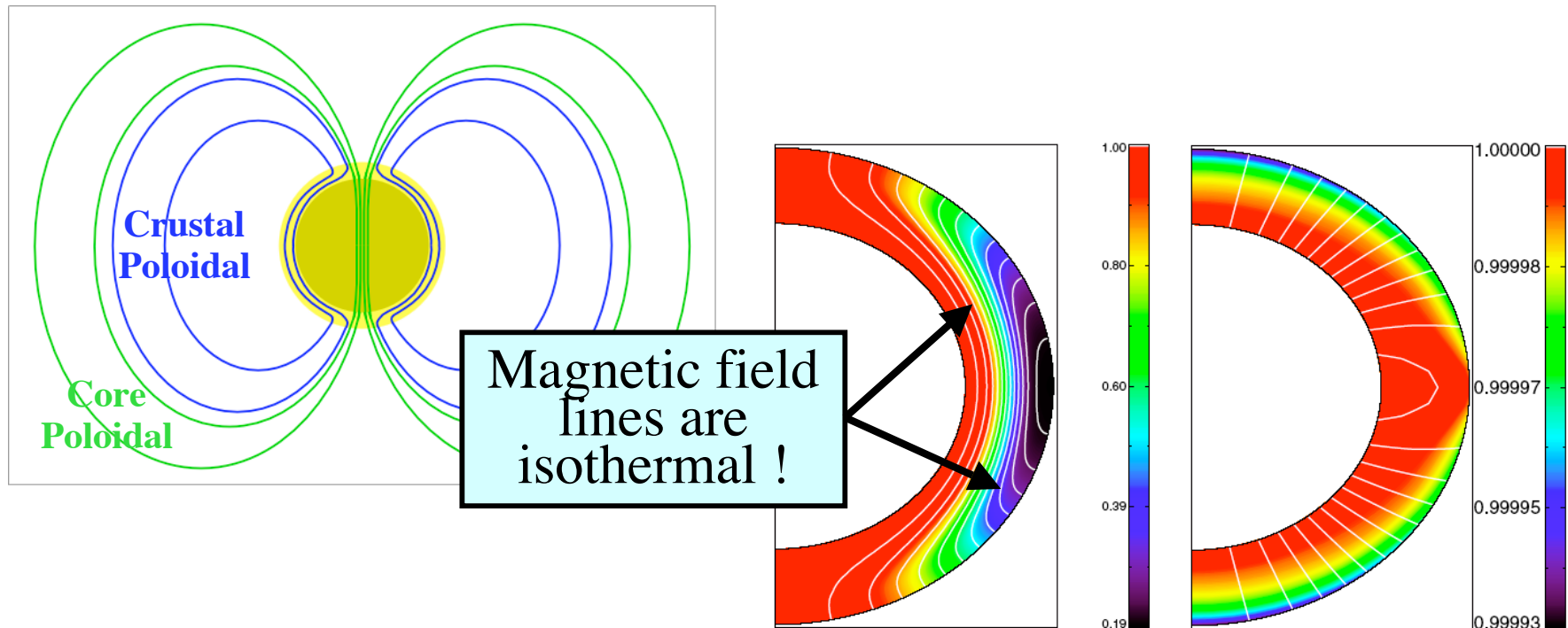


D. Page "Surface temperature distribution in magnetized neutron stars. I. dipolar fields", 1995

D. Page & A. Sarmiento, "Surface temperature distribution in magnetized neutron stars. II" 1996

# Anisotropic heat transport in a magnetized crust

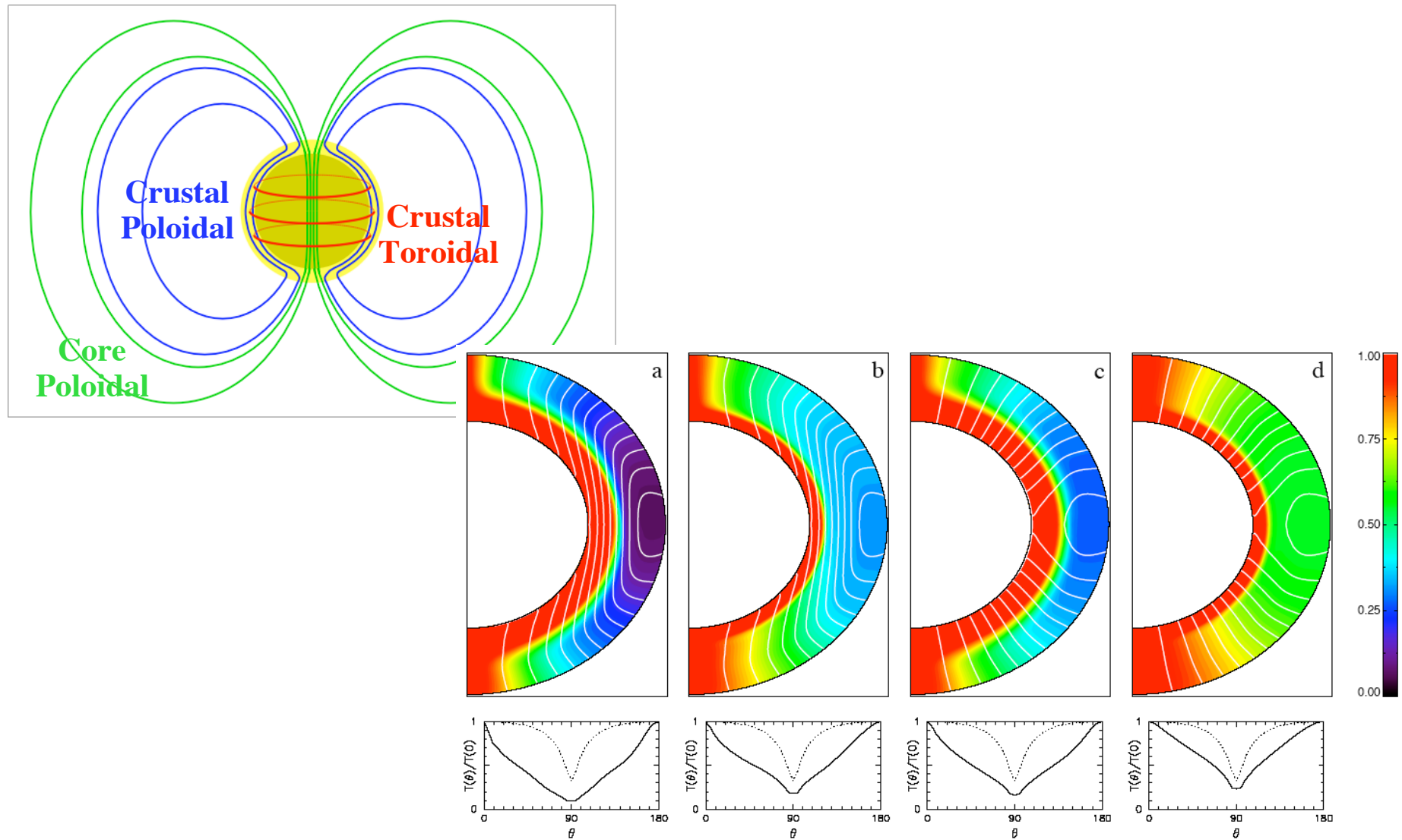
Need to model  $B$  in the crust  $\Leftrightarrow$  Choose currents locations



**Fig. 7.** Representation of both field lines and temperature distribution in the crust whose radial scale ( $r(\rho_n) \leq r \leq r(\rho_b)$ ) is stretched by a factor of 5, assuming  $B_0 = 3 \times 10^{12}$  G and  $T_{\text{core}} = 10^6$  K. Left panel corresponds to a crustal field, right panel to a star-centered core field. Bars show the temperature scales in units of  $T_{\text{core}}$ .

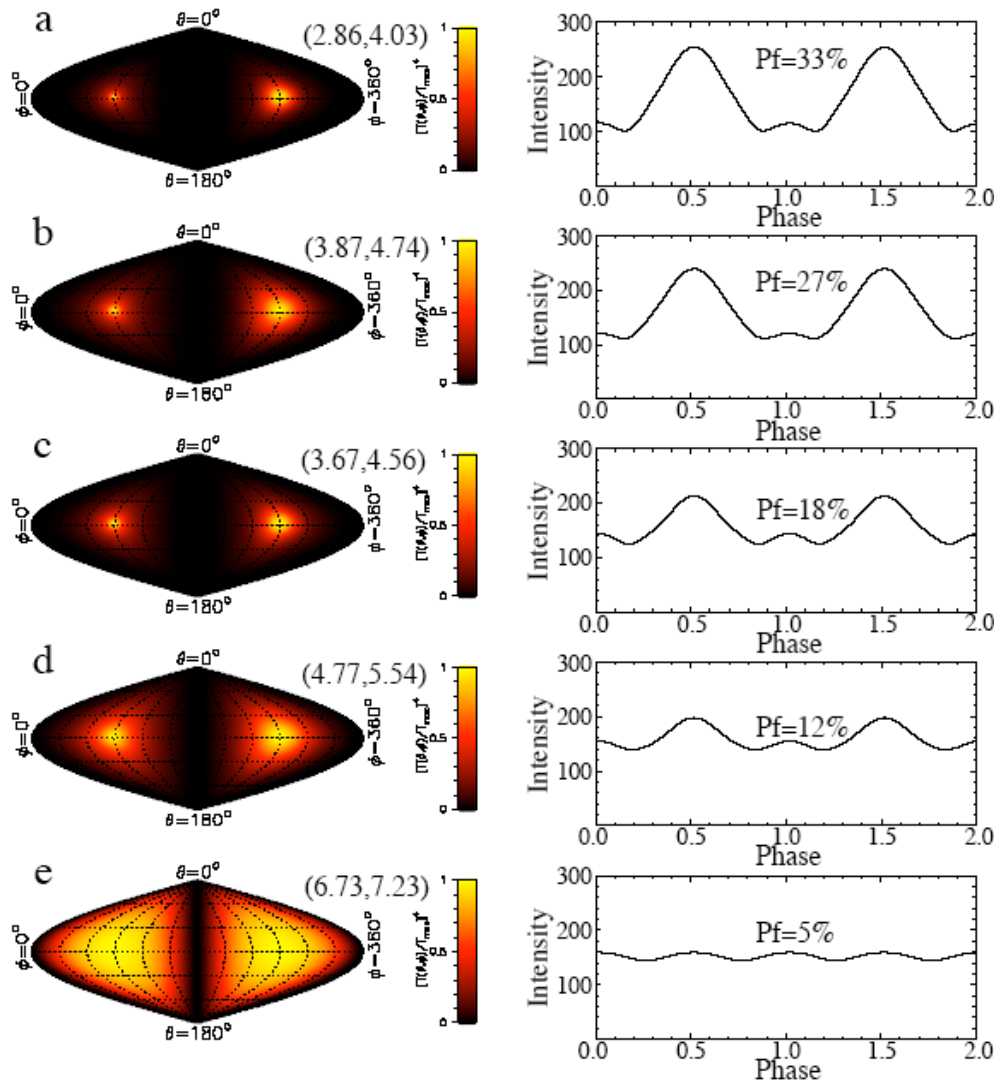
Geppert, Kueker & Page, 2004

# Anisotropic heat transport in a magnetized crust

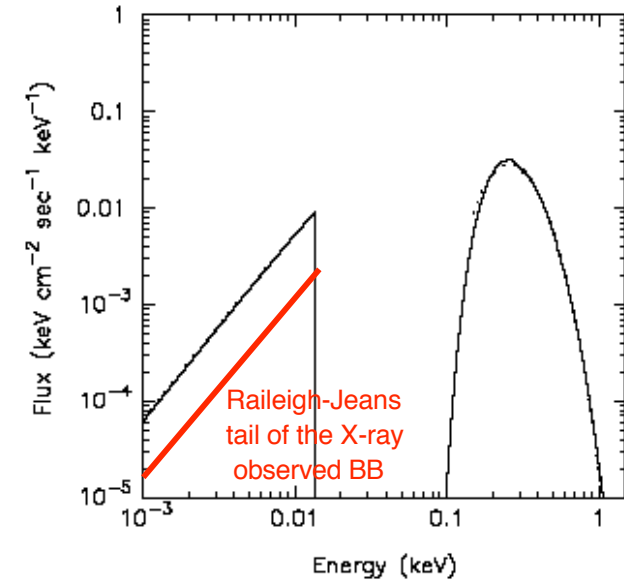


Geppert, Kueker & Page, 2006

# Surface temperatures, pulse profiles, BB spectra



## Fit of RX J1856.5-3754 optical and X-ray spectrum



**Fig. 10.** Fit of the spectrum of RX J1856.5-3754. Dotted lines show the two blackbodies fit to the data from Trümper *et al.* (2004). The continuous line shows our results: the star has a radius  $R = 14.4$  km and  $R_\infty = 17.06$  km for a  $1.4 M_\odot$ , at a distance of 122 pcs ( $N_H = 1.6 \times 10^{20}$  cm<sup>-2</sup> for interstellar absorption) and the observer is assumed to be aligned with the rotation axis. The magnetic field structure corresponds to model c of Figure 6 adjusted to the 14.4 km radius with  $T_b = 6.8 \times 10^7$  K, resulting in  $T_{\text{eff}}^\infty = 4.62 \times 10^5$  K and  $T_{\text{max}}^\infty = 8.54 \times 10^5$  K

Geppert, Kueker & Page, 2006

# CONCLUSIONS

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**we need to understand the thermal emission from the surface of a neutron star in order to have a chance to understand what is happening inside a "neutron" star.**

**⇒ we need to understand the structure of matter in strong magnetic fields.**

