

A NEW HALO PLANETARY NEBULA

M. Peña

Instituto de Astronomía
Universidad Nacional Autónoma de México

and

M.T. Ruiz¹, J. Maza¹ and L.E. González¹Departamento de Astronomía
Universidad de Chile

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RESUMEN

Se reporta el descubrimiento de una nebulosa planetaria en el halo galáctico; sus coordenadas galácticas son $l^{\text{II}} = 6^{\circ}165$ y $b^{\text{II}} = -41^{\circ}957$. Se ha estimado un límite inferior para la distancia al plano de $|z| \geq 1.9$ kpc con $d \geq 2.8$ kpc, aunque se considera que la distancia más probable es del orden de 7.7 kpc con $|z| \approx 5.1$ kpc. Cálculos preliminares, basados en datos espectrofotométricos, indican que la temperatura electrónica de la nebulosa es $T(\text{O III}) = 15\,000 \pm 1500$ K. Las abundancias químicas obtenidas, en unidades de $\log H = 12.00$, son $\log He = 10.96 \pm 0.05$, $\log O = 8.1 \pm 0.1$, $\log Ne = 7.5 \pm 0.2$ and $\log Ar = 5.8 \pm 0.2$. Esta nebulosa planetaria tiene una composición química comparable a las regiones H II de la Nube Menor de Magallanes y muy similar a la nebulosa planetaria de halo DDDM-1.

ABSTRACT

A new halo planetary nebula at $l^{\text{II}} = 6^{\circ}165$ and $b^{\text{II}} = -41^{\circ}957$ is presented. We have estimated a lower limit distance to the galactic plane $|z| \geq 1.9$ kpc which corresponds to $d \geq 2.8$ kpc. Nevertheless we consider that the most probable distance is $d \approx 7.7$ kpc and $|z| \approx 5.1$ kpc. Preliminary results, based on spectrophotometric data obtained for the nebula, indicate an electron temperature $T(\text{O III}) = 15\,000 \pm 1500$ K, and the chemical abundances, relative to $\log H = 12.00$, are $\log He = 10.96 \pm 0.05$, $\log O = 8.1 \pm 0.1$, $\log Ne = 7.5 \pm 0.2$ and $\log Ar = 5.8 \pm 0.2$. This planetary nebula has a chemical composition comparable to the SMC H II regions and very similar to the halo planetary nebula DDDM-1.

Key words: NEBULAE-PLANETARY – ABUNDANCES

I. INTRODUCTION

In the last few years, a wide objective prism survey has been carried out in the southern sky by astronomers of the Universidad de Chile, looking for emission line objects. The first list of Seyfert 1 galaxies has been already published (Maza *et al.* 1989a). The areas covered by the Calán-Tololo Survey and the searching method are described in that paper. Many starburst galaxies, liners, quasars, planetary nebulae and carbon stars have been discovered or rediscovered in the survey and further lists are in preparation (Maza *et al.* 1989b).

The object A06-09 in the Calán-Tololo Survey nomenclature, located in position $\alpha = 21^{\text{h}}02^{\text{m}}44.5^{\text{s}}$ and $\delta = -37^{\circ}20'18.7''$ (1950 equinox), attracted our attention due to its small and faint nebular appearance and

its high galactic latitude. It looked like a good candidate for a halo planetary nebula. The corresponding galactic coordinates are $l^{\text{II}} = 6^{\circ}165$, $b^{\text{II}} = -41^{\circ}957$ and, hereinafter, the object will be called PN 6-41.1 in agreement with the nomenclature suggested by Acker (1989). The identification chart of PN 6-41.1 is presented in Figure 1. The object appears barely resolved in the ESO Quick Blue Survey plate, which has a spatial resolution of $7''$ for stellar objects with apparent magnitude greater than 16 mag.

The four previously known halo planetaries (K 648, H 4-1, BB-1 and DDDM-1) have been the subject of a number of studies (e.g., Torres-Peimbert, Rayo, and Peimbert 1981; Adams *et al.* 1984; Barker and Cudworth 1984; Torres-Peimbert 1984; Clegg, Peimbert, and Torres-Peimbert 1987). Recently, Garnett and Dinerstein (1988) and Torres-Peimbert, Peimbert, and Peña (1988) have suggested that NGC 2242 and NGC 4361 are population II objects. The halo planetary nebulae have provided information on the abundances of ele-

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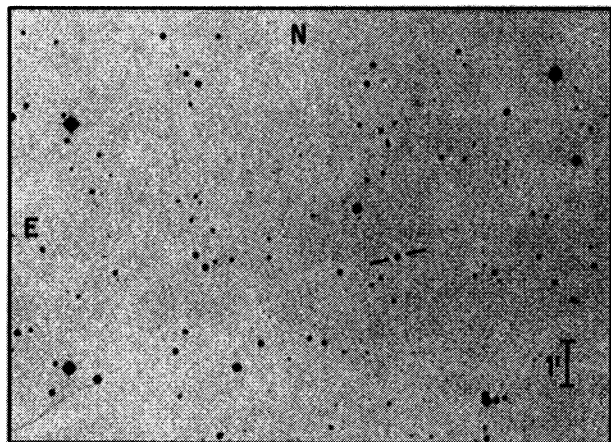


Fig. 1. Identification chart of PN 6-41.1 from the ESO Quick Blue Survey.

ments in the early history of the galaxy and on the nuclear processes and evolution of low mass stars, but their small number makes difficult to draw general conclusions. For these reasons it is important to increase the sample of known halo planetaries.

In the following sections, we present spectrophotometric data of PN 6-41.1. From this data, we have derived the chemical composition and the physical conditions of the nebula and we discuss the evidence that the object is located in the galactic halo.

II. SPECTROPHOTOMETRY

Spectrophotometric data of the object have been obtained during two observing seasons in 1988, with the CTIO 4-m telescope equipped with a R-C spectrograph. In both cases we used a long slit of $2'' \times 50''$, oriented east-west, and grating #250, in the first order.

The first observation was carried out in July 11, with the Red Air Schmidt Camera and a CCD GEC/EPI 9 detector. A tilt angle of $63^\circ.8$ was used, which permits to cover a spectral range between $\lambda\lambda 4500$ Å to 7200 Å, with a resolution of 17.1 Å. The exposure time was 30 minutes.

The second observation was carried out in October 14, with the Blue Air Schmidt Camera and a CCD GEC/EPI 11 detector. The exposure time was 25 min and a tilt angle of $54^\circ.48$ was used to cover the range $\lambda\lambda 3600$ Å to 7000 Å with 14.5 Å resolution.

Three flux standard stars, from the list of Stone and Baldwin (1983) were observed each time to calibrate the data, and a He-Ar lamp was used to calibrate the wavelength. The data reduction and analysis were performed with the IRAF software package at CTIO La Serena Computing Center Facilities. The sky was subtracted using parts of the slit that showed no evidence of emission and the extraction windows for the spectra were $2'' \times 2.5''$ and $2'' \times 8.2''$ respectively.

Both calibrated spectra agree very well, within the observational uncertainties. We also have another spectrum, observed in September 1985, which covers the $\lambda\lambda 3800$ Å to 5500 Å interval. It was obtained with the

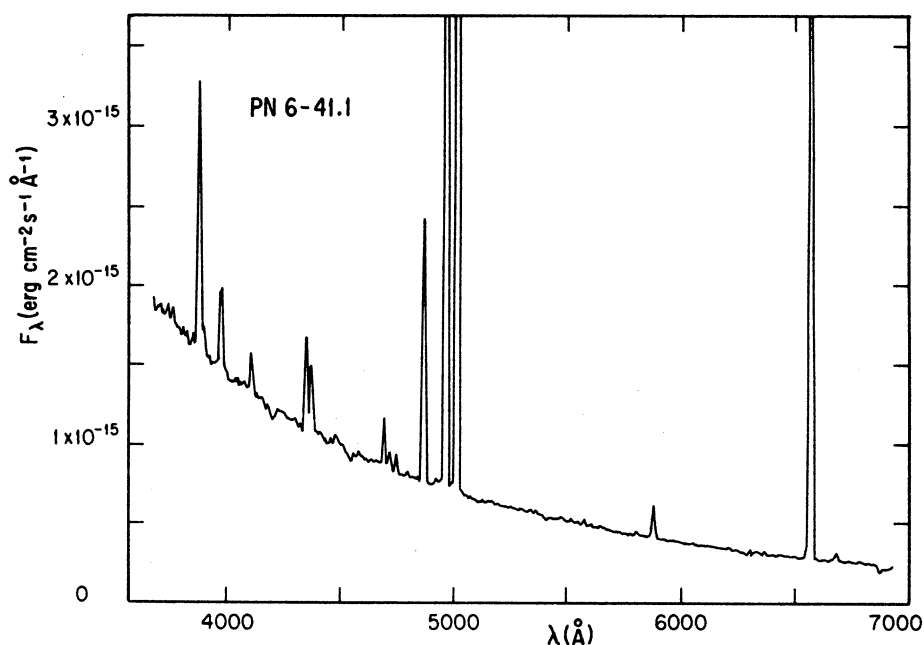


Fig. 2. CCD spectrum of PN 6-41.1, with the vertical scale expanded to show some of the faint lines.

TABLE 1
OBSERVED FLUXES AND DEREDDENED LINE INTENSITIES^a

λ	id.	$\log F(\lambda)/F(H\beta)$	$f(\lambda)^b$	$\log I(\lambda)/I(H\beta)$
3727	[O II]	< -2.41:	+0.255	< -2.36:
3869	[Ne III]	+0.00	+0.233	+0.03
3968	[Ne III] + H7	-0.50	+0.208	-0.47
4101	H δ	-0.82	+0.172	-0.79:
4340	H γ	-0.40	+0.125	-0.38
4363	[O III]	-0.66	+0.124	-0.64
4471	He I	-1.40:	+0.080	-1.39:
4686	He II	-0.89	+0.042	-0.87
4711+13	[Ar IV] + He I	-1.08:	+0.039	-1.07:
4740	[Ar IV]	-0.90:	+0.031	-0.89:
4861	H β	+0.00	+0.000	+0.00
4959	[O III]	+0.61	-0.022	+0.61
5007	[O III]	+1.06	-0.033	+1.06
5876	He I	-0.90	-0.208	-0.94
6563	H α	+0.50	-0.322	+0.45
6584	[N II]	< -1.52:	-0.325	< -1.57:
6678	He I	-1.50	-0.339	-1.55
6717+31	[S II]	< -1.50:	-0.242	< -1.55:
7065	He I	-1.42	-0.382	-1.48
7135	[Ar III]	-1.42:	-0.390	-1.48:

a. $\log F(H\beta) (2'' \times 2.5'') = -13.80$, $\log F(H\beta) (2'' \times 8.2'') = -13.58$. The estimated errors are: less than 10% if $\log I(\lambda)/I(H\beta) \geq -0.50$, about 20% if $-0.6 > \log I(\lambda)/I(H\beta) > -1.00$, 50% or greater for those marked with a colon.
b. Reddening function from Seaton (1979).

TABLE 2
IONIC ABUNDANCES

Ion	$12 + \log N(X^{+m})/N(H^+)$
He ⁺	10.90 ± 0.03
He ⁺⁺	10.08 ± 0.08
N ⁺	< 5.31
O ⁺	< 4.70
O ²⁺	8.09 ± 0.10
Ne ²⁺	7.43 ± 0.20
S ⁺	< 4.90
Ar ²⁺	5.12 ± 0.20
Ar ³⁺	5.62 ± 0.20

CTIO 1.5-m telescope equipped with an R-C spectrograph and a VIDICON detector. Unfortunately, this last spectrum is very noisy towards the blue wavelengths; nevertheless the line intensities, in the $\lambda\lambda 4500$ Å to 5500 Å range, coincide with the CCD data.
In Figure 2, we present details of the calibrated spectrum of the nebula. PN 6-41.1 appears as a high excitation sharp emission line object. The emission lines He II $\lambda 4686$, [Ar IV] $\lambda\lambda 4711$ and 4740 are clearly detected superimposed on a stellar continuum rising towards the blue. The intensity line ratio [O III] $\lambda 5007/H\beta$ is about 12. The observed line intensity ratios discard the possibility of PN 6-41.1 being a compact, isolated H II region. Moreover, from the CCD long slit image taken on October 14, during a photometric night with a seeing image of 1.8'', we measured an angular diameter of 8.2'' for the nebula and an apparent visual magnitude, $V = 17.0$ mag, for the central star.

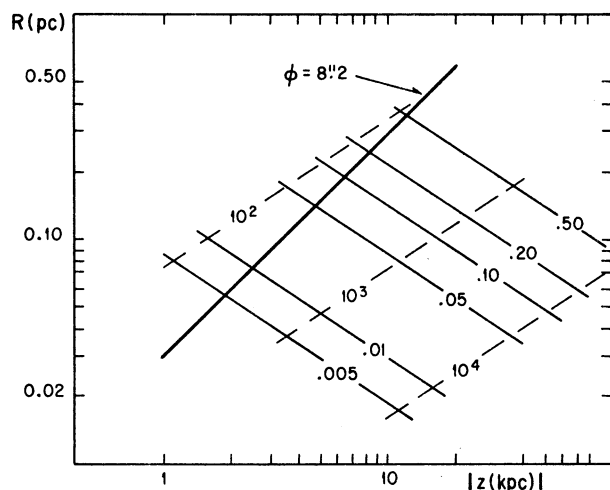


Fig. 3. Nebular radius, R , versus distance to the galactic plane, $|z|$, relation. The solid lines represent constant nebular ionized mass curves (in solar masses) calculated according to relation (1) (see the text). The broken lines represent constant rms electron density curves (in cm^{-3}). The line which corresponds to the nebular diameter $\phi = 8.2''$ is indicated by a thick solid line.

III. PHYSICAL CONDITIONS AND CHEMICAL ABUNDANCES

In Table 1 we present the observed fluxes, $F(\lambda)$, and the emission line intensities, $I(\lambda)$, relative to $H\beta$, corrected for interstellar extinction. The logarithmic reddening correction, $C(H\beta) = 0.15 \pm 0.05$ was derived by fitting the observed ratio $H\alpha/H\beta$ to the theoretical one computed by Hummer and Storey (1987) for case B. We have assumed an electron temperature $T_e = 15000 \text{ K}$ and a density of $N_e = 500 \text{ cm}^{-3}$. We have dereddened the observed fluxes with the reddening function law by Seaton (1979).

The relevant references to the atomic parameters to derive physical conditions and the chemical composition are from the compilation of Mendoza (1983).

The conventional diagnostic line ratios have been analyzed. From these, we derive $T_e(\text{O III}) = 15000 \text{ K} \pm 1500 \text{ K}$. A very uncertain electron density, $\log N_e(\text{Ar IV}) = 3.1 \pm 0.3$, is derived from the $[\text{Ar IV}] \lambda\lambda 4711/4740$ line intensity ratio, after subtracting the contribution of $\text{He I } \lambda 4713$ to the $[\text{Ar IV}] \lambda 4711$ line. Unfortunately, the $[\text{S II}] \lambda\lambda 6717, 6731$ lines are too weak and we did not obtain a reliable density estimate. However, from Figure 3 (see discussion in §IV), we can deduce that the rms electron density of PN 6-41.1 is in the range 100 to 1000 cm^{-3} . Consequently we have adopted $N_e = 500 \text{ cm}^{-3}$ for this nebula.

To derive ionic abundances we have used the $T_e(\text{O III})$ for all the available ions, and the results are presented in Table 2. The large errors in the ionic abundances are produced by the large uncertainty in the adopted electron temperature and the errors in the line intensities. The He^+ ionic abundance was derived using the $\text{He I } \lambda\lambda 5876$ and 6678 line intensities by taking into account the contribution to the observed He I line intensities resulting from the collisional excitation of the 2^3S state of He I as discussed by Peimbert and Torres-Peimbert (1987).

Total abundances can be obtained from the calculated ionic concentrations and some standard formulae for the ionization correction factors. We have employed the expressions quoted by Barker (1983) to derive the He , O , Ne and Ar abundances. The N abundance could not be derived due to the weakness of $[\text{N II}] \lambda 6584 \text{ \AA}$ line and the large uncertainty in the ionization correction factor based on the ionic oxygen abundances. Due to the high ionization degree of PN 6-41.1, we have considered that there is no He^0 in the H^+ zone. The results are presented in Table 3.

TABLE 3

TOTAL ABUNDANCES^a

Object	He	O	Ne	Ar	Ref. ^b
PN 6 - 41.1	10.96 ± 0.05	8.1 ± 0.1	7.5 ± 0.2	$5.8 \pm .2$	1
K 648	11.02	7.7	6.7	4.3	2,3
H 4-1	10.99	8.4	6.7	4.7	3,4
BB-1	10.98	7.9	8.0	4.6	3,5
DDDM-1	11.00	8.1	7.3	5.8	6
SMC H II	10.98	7.9	7.0	5.8	7,8
Sun	...	8.9	8.1	6.6	9

a. In $12 + \log X/H$.

b. References: (1) This work. (2) Adams *et al.* 1984. (3) Barker 1980. (4) Torres-Peimbert and Peimbert 1979. (5) Torres-Peimbert *et al.* 1981. (6) Clegg *et al.* 1987. (7) Peimbert *et al.* 1986. (8) Dufour 1983. (9) Lambert 1978; Lambert and Luck 1978.

IV. A HALO PLANETARY NEBULA

We have estimated the distance to PN 6-41.1 by considering that it is a uniform sphere with a rms electron density N_e [cm^{-3}] and temperature $t_e = T_e/10^4$ K. The nebular ionized mass, in terms of the absolute H β flux, $I(\text{H}\beta)$ [$\text{erg cm}^{-2} \text{s}^{-1}$], the distance d [kpc] and the He/H abundance ratio, y , is given by

$$M_i(M_\odot) = 8.07 \times 10^{11} I(\text{H}\beta) d^2 t_e^{0.88} (1+4y)/N_e, \quad (1)$$

where we have considered the H β recombination coefficient for case B by Hummer and Storey (1987).

It is possible to derive the distance from this expression if an estimate of the rms ionized mass, $M_i(\text{rms})$, is given. In Figure 3 we present a graph of the nebular radius, R , versus the distance to the galactic plane, z , where several curves of constant M_i , ranging from $0.5 M_\odot$ to $0.005 M_\odot$, have been drawn for different values of N_e . The calculations were carried out according to the expression (1) by adopting the values $t_e = 1.5$ and $y = 0.091$, as derived in §III. The absolute H β flux was taken to be $I(\text{H}\beta) = 1.043 \times 10^{-13}$ [$\text{erg cm}^{-2} \text{s}^{-1}$] by assuming that the surface flux in H β is constant in a circular area of $8.2''$ diameter. Consequently, we have multiplied the dereddened H β flux, $I(\text{H}\beta) = 3.24 \times 10^{-14}$ [$\text{erg cm}^{-2} \text{s}^{-1}$], obtained from the $2'' \times 8.2''$ slit by 3.22 to take into account the total emitted flux. The density N_e was used as a free parameter.

The chosen mass range covers the ionized mass spectrum of known galactic planetary nebulae; $0.5 M_\odot$ is the average $M_i(\text{rms})$ of the most massive optically thin PN and $0.005 M_\odot$ is the average of the $M_i(\text{rms})$ of the least massive known PN (Mallik and Peimbert 1988). The possible values for the rms ionized mass of PN 6-41.1 are located, in this graph, along the line representing the angular diameter $\phi = 8.2''$.

From Figure 3, it follows that for any value of the $M_i(\text{rms})$ between $0.005 M_\odot$ and $0.50 M_\odot$, the distance of the object to the galactic plane is in the interval $1.9 \text{ kpc} \leq |z| \leq 12 \text{ kpc}$. This $|z|$ value range implies a lower limit for the distance, d , of 2.8 kpc and indicates that PN 6-41.1 is located at the galactic halo. The lower limit for $|z|$ has been derived under the assumption that the $M_i(\text{rms})$ of PN 6-41.1 is $0.005 M_\odot$. Nevertheless, this low value of the ionized mass corresponds to planetary nebulae of extremely high density and low ionization degree, which is not the case of PN 6-41.1. On the other hand, the two halo PN with calculated ionized mass are K 648, with $M_i(\text{rms}) = 0.011 M_\odot$ (Adams *et al.* 1984), and DDDM-1, with $M_i(\text{rms}) = 0.069 M_\odot$ (Clegg *et al.* 1987); both planetaries are of lower ionization degree than PN 6-41.1. If we assume that the $M_i(\text{rms})$ of PN 6-41.1 is $0.069 M_\odot$, then the distance to the nebula would be 8.2 kpc , with $|z| = 5.4 \text{ kpc}$.

Another distance estimate can be made from the dereddened H β flux alone and the apparent diameter of the

nebula. Applying the distance scale calibration by Cudworth (1974) and his optically thick/thin criterion, as well as the lack of low ionization lines, it follows that PN 6-41.1 is optically thin and lies at a distance $d \cong 12 \text{ kpc}$. (Assuming that the nebula is optically thick leads to a gross overestimate of the distance: $d = 55 \text{ kpc}$). This value coincides with the upper limit for d , derived in Figure 3, and corresponds to a $M_i(\text{rms}) = 0.50 M_\odot$. It seems unlikely for a population II planetary nebula to have such large ionized mass and we consider that the large distance derived with this method is a consequence of the weak $I(\text{H}\beta)$ of the nebula. This could be due to a central star of extremely low mass (corresponding to an extreme population II object) or to a very low ionized nebular mass; in any case, it appears that Cudworth's calibration, based on disk galactic planetary nebulae, should not be applied to halo planetaries.

Based on central star characteristics, Méndez *et al.* (1988) have calculated the distances to some planetary nebulae by using the expression:

$$d(\text{pc}) = [3.82 \times 10^{-11} 10^{0.4 V_0} M_* F_*/g]^{0.5} \quad (2)$$

where M_* is the stellar mass, in solar units, F_* is the monochromatic atmospheric flux at 5480 \AA , in $\text{erg cm}^{-2} \text{s}^{-1} \text{cm}^{-1}$, g is the surface gravity, in cm s^{-2} , and V_0 is the apparent visual magnitude, after correcting for interstellar extinction effects.

To apply this method to PN 6-41.1, we have adopted an effective temperature $T_{\text{eff}} = 75000 \pm 5000 \text{ K}$, which is the He II Zanstra temperature calculated for the dereddened apparent He II $\lambda 4686$ flux and the net stellar flux corresponding to $V_0 = 16.6 \text{ mag}$. The ionizing radiation field has been taken from the non-LTE model atmosphere sequence by Clegg and Middlemass (1987). The adopted model has $\log g = 5.2$ and $F_* = 1.08 \times 10^{17}$ [$\text{erg cm}^{-2} \text{s}^{-1} \text{cm}^{-1}$]. The stellar mass, derived from Figure 3 of Méndez *et al.* (1988), is $0.55 M_\odot$, which is reasonable for a population II PN central star. With these data and expression (2) we derived $d = 7.7 \text{ kpc}$, corresponding to $|z| = 5.1 \text{ kpc}$. This distance is similar to that obtained by assuming that PN 6-41.1 and DDDM-1 have the same ionized mass. We will adopt 7.7 kpc as the distance to PN 6-41.1.

V. DISCUSSION AND CONCLUSIONS

The distance, obtained for PN 6-41.1, $d \geq 2.8 \text{ kpc}$, places it in the galactic halo. The derived chemical composition also indicates that it is a population II object. The O, Ne and Ar abundances, which should be very close to the initial stellar abundances, appear underabundant by a factor of approximately 5 relative to the solar values. This chemical composition is very similar to that of the SMC H II regions and other extragalactic metal-poor H II regions such as NGC 2363 (Peimbert, Peña, and Torres-Peimbert 1986).

By comparing the chemical composition of PN 6-41.1 with the other halo PN (Table 3), it appears that,

PN 6-41.1 resembles the interesting planetary nebula DDDM-1 which is the only known C-poor halo PN. On the other hand, the higher effective temperature of the central star of PN 6-41.1 could indicate a more evolved object.

The chemical composition of PN 6-41.1 and DDDM-1 are very different from the other halo PN. Clegg *et al.* (1987) have suggested that DDDM-1 formed from a fragment of the primitive halo which had low C and N and that the central star only underwent through the first dredge-up phase. I would be extremely interesting to derive the N and C abundances of PN 6-41.1 to test whether the progenitor stars behaved similarly. However, the weakness of PN 6-41.1 would require very long *IUE* and optical exposures to derive more accurate physical conditions as well as good signal-to-noise ratios in the N and C emission lines.

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Miriam Peña: Instituto de Astronomía, UNAM, Apartado Postal 70-264, 04510 México, D.F., México.

Ma. Teresa Ruíz, J. Maza, and L.E. González: Depto. de Astronomía, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Casilla 36-D, Santiago, Chile.