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INFLUENCE OF A NON-UNIFORM AXIAL MAGNETIC FIELD ON THE INSTABILITY OF A QUIESCENT PROMINENCE OVERLYING A SOLAR BIPOLAR REGION

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RESUMEN

Se investigan las propiedades de estabilidad magnetohidrodinámica (MHD) del modelo de Low (1981) de una protuberancia solar. Se ha supuesto un campo magnético no uniforme a lo largo del eje de la prominencia (B_x). Se usó un formalismo bidimensional basado en el Principio de Energía de la MHD ideal (Bernstein *et al.* 1958). Se consideraron diferentes formas funcionales de $B_x(A)$, donde A es la componente axial del potencial vectorial. Además se tomaron en cuenta tanto el rango de parámetros observados como algunos casos fuera de éste. Para campos axiales débiles y moderados, el modelo de Low puede explicar también las oscilaciones horizontales reportadas con períodos cortos hasta de 7 minutos. Tales oscilaciones son mantenidas principalmente por la fuerza de Lorentz. Intensidades altas de B_x producen la aparición de una severa inestabilidad que posiblemente pueda identificarse con una inestabilidad macroscópica de deriva. En los valores de los parámetros fuera del rango observado, el campo axial ocasiona un ligero debilitamiento de la inestabilidad con respecto al caso cuando B_x es nulo. Sin embargo, la región de estabilidad no se amplía considerablemente.

ABSTRACT

MHD-instability properties of the solar prominence model of Low (1981) assuming a non-uniform magnetic field along the prominence axis (B_x) are investigated. We used a two-dimensional formalism based on the Energy Principle of ideal MHD (Bernstein *et al.* 1958). Different functional forms for $B_x(A)$, where A is the axial component of the vector potential are considered. Observed parameter ranges as well as several cases outside these ranges are taken into account. For weak and moderate B_x, Low's model can also explain the reported short-period horizontal oscillations with periods up to 7 minutes. Such oscillations are mainly sustained by the Lorentz force. Large intensities of B_x can produce the onset of a severe instability which could be possibly identified with a macroscopic drift instability. For values of the parameters outside the observed range, B_x causes a slight weakening of the instability with respect to the case with a vanishing B_x, however a considerable enlargement of the stability region is not achieved.

Key words: INSTABILITIES – PLASMAS – SUN-CORONA – SUN-MAGNETIC FIELDS

I. INTRODUCTION

Quiescent prominences are remarkably stable plasma structures in the solar corona. They are usually located above the neutral line which separates polarities of the vertical component of the bipolar magnetic field between two sunspot regions. Possibly the most striking property of solar prominences is that they are much colder and denser than the surrounding corona by several orders of magnitude. A magnetic field is thought to be the main cause of the thermal isolation of the prominence plasma from its hostile environment.

MHD-theoretical studies consider quiescent prominences in mechanical equilibrium resulting from the balance of the Lorentz force, pressure gradients and the force of external gravity. Moreover, the appearance of the prominence resembling a very long vertical sheet allows one to assume a two-dimensional plasma configuration invariant in its longitudinal direction. Two-dimensional MHD static equilibrium models of quiescent prominences have been proposed by numerous authors (e.g., see Dungey 1953; Kippenhahn and Schlüter 1957; Lerche and Low 1980; Zweibel and Hundhausen 1982; Osherovich 1985; Ballester and Priest 1987). However, only a description of basic physical properties, like pressure, mass density, magnetic field, is not sufficient to explain the stable existence of the prominence. In this sense, the usefulness of an equilibrium model is determined above all by its stability. On the other hand, most of the proposed models merely take into account the local internal magnetic field and ignore its connection with the surrounding bipolar coronal field. In particular, Low (1981) presented an analytical model for a finite-size prominence fixed in a bipolar magnetic region which is originated in photospheric layers. This model considers the presence of a component of the magnetic field along the longitudinal axis of the prominence (B_x) and also includes non-uniform temperature.

As noted above, stability is a fundamental requisite for a realistic model. Due to the strongly inhomogeneous structure of almost any equilibrium model, stability studies for most of these configurations are up to now rather scarce. Stability of models describing a boundless prominence (i.e., without connection with the surrounding medium) has been analyzed (e.g., Brown 1958; Anzer 1968; Migliuolo 1982; Zweibel 1982). However, these approaches used very restricted perturbation classes and special methods which are valid only for a specific type of equilibrium. Galindo Trejo (1987) has extensively investigated the linear stability of four boundless prominence equilibria (Menzel 1951; Dungey 1953; Kippenhahn and Schlüter 1957; Lerche and Low 1980). According to his results, all models are stable in the observed parameter range and they are able to explain reported stable oscillations in quiescent prominences induced by perturbations emanating from the neighborhood of a solar flare (Wiehr, Stellmacher, and Balthasar 1984; Bashkirtsev and Mashnich 1984). The classical model of Kippenhahn and Schlüter is absolutely stable (see Galindo Trejo and Schindler 1984); however, all other models are destabilized for parameters outside the observed range.

Recently, Galindo Trejo (1989) has studied the stability properties of the prominence model of Low (1981). His analysis considers a two-dimensional formalism based on the Energy Principle of ideal MHD (Bernstein *et al.* 1958; Hain, Lüst, and Schlüter 1957). In the frame of this model, the prominence carries out stable horizontal oscillations with periods between 3 and 6 minutes. Such periods are consistent with reported short-period oscillations in quiescent prominences (Balthasar, Knölker, and Stellmacher 1986). On the other hand, for parameters outside the observed range, the equilibrium becomes unstable. The resulting instability is driven exclusively by compressional effects. On the contrary, the Lorentz force is stabilizing but it is insufficient to accomplish a stable state. Gravity can contribute to the dynamic state through stabilizing or destabilizing effects; however, its total influence on the instability is relatively small. Uniform longitudinal magnetic field was assumed in both parameter ranges.

The main aim of this paper is to extend the previous stability analysis of Galindo Trejo (1989), (hereafter, Paper I) for the prominence model of Low (1981) by considering explicitly a non-uniform longitudinal magnetic field which depends in general on the component of the vector potential along the longitudinal axis of the prominence. This dependence introduces a selective shearing of the magnetic field lines. In the following section, basic equations describing the MHD-equilibrium and the stability theory are set up. General results concerning stability properties of Low's configuration with a non-uniform B_x are presented in §3. Finally, our conclusions are given in §4.

II. BASIC EQUATIONS

The static equilibrium of a magnetized two-dimensional plasma, including uniform external gravity, is described in the MHD-theory by means of the non-linear elliptic equation (Low 1975; Schindler, Birn, and Janicke 1983):

$$\Delta A = -4\pi \frac{\partial \Pi(A,\phi)}{\partial A}, \quad \Pi(A,\phi) = P(A,\phi) + \frac{B_x^2(A)}{8\pi}, \tag{1}$$

where $\mathbf{A} = A(y,z)\mathbf{e}_x$ is the component of the vector potential in the x-direction (which is the direction of alignment of the longitudinal axis of the prominence and x is the Cartesian coordinate which is ignorable); ϕ defines the external gravitational potential $\phi = gz$, g being the constant gravitational acceleration. The pressure function $\Pi(A, \phi)$ is only restricted by boundary conditions, and by the requirement that the mass density $\rho = -(\partial\Pi/\partial\phi) = -(\partial P/\partial\phi)$ must remain positive. The magnetic field may be written as:

$$\mathbf{B} = \nabla A(y,z) \times \mathbf{e}_x + B_x(A)\mathbf{e}_x \quad . \tag{2}$$

Usually one assumes a prominence plasma composed by ionized hydrogen which satisfies the ideal gas equation of state:

$$P = \rho\kappa T/m \quad , \tag{3}$$

where κ is Boltzmann constant, T the temperature and m the proton mass. Note that the temperature function $T(A, \phi)$ must strictly be obtained from an additional heat transfer equation. However, for the study of mechanical equilibrium T may be considered as an arbitrary function. Therefore, any two-dimensional equilibrium model is univocally determined by prescribing the functions $P(A,\phi)$, $B_x(A)$, and appropriate boundary conditions.

An equilibrium configuration may be achievable in nature only if it is stable to small perturbations $\underline{\xi}(\mathbf{r}, t)$. The dynamic evolution of $\underline{\xi}(\mathbf{r},t)$ is described by the linearized MHD-equation:

$$\rho \frac{\partial^2 \underline{\xi}}{\partial t^2} = \mathbf{F}(\underline{\xi}(\mathbf{r},t)) \tag{4}$$

where $\mathbf{F}$ is the force density (hermitian) operator:

$$\mathbf{F}(\underline{\xi}) = \frac{1}{4\pi}(\nabla\times\mathbf{Q})\times\mathbf{Q} - \frac{1}{4\pi}\mathbf{B}\times(\nabla\times\mathbf{Q}) +$$

$$+ \nabla[\Gamma P \nabla\cdot\underline{\xi} + (\underline{\xi}\cdot\nabla)P] + \nabla\cdot(\rho\underline{\xi})\nabla\phi \ , \tag{5}$$

and $\mathbf{Q} = \nabla \times (\underline{\xi} \times \mathbf{B})$. An adiabatic energy law is assumed, and Γ denotes the ratio of specific heats. Since time does not appear explicitly in $\mathbf{F}$ one can assume:

$$\underline{\xi}(\mathbf{r},t) = \underline{\xi}(\mathbf{r}) \exp(i\omega t) \quad . \qquad (6)$$

Inserting $\underline{\xi}(\mathbf{r}, t)$ in equation (4), we obtain the generalized eigenvalue problem:

$$-\rho\omega^2 \underline{\xi}(\mathbf{r}) = \mathbf{F}(\underline{\xi}(\mathbf{r})) \quad . \qquad (7)$$

The solution of equation (7) with appropriate boundary conditions will lead to the solution of the stability problem. According to Bernstein *et al.* (1958), if ω^2 is positive, the amplitude of the perturbation oscillates and the system is stable. On the contrary, if ω^2 becomes negative, the amplitude of the perturbation grows with time and the system becomes unstable. Due to the strong inhomogeneity of most of the equilibrium configurations, the solution of equation (7) represents a formidable mathematical task. However, another convenient alternative to study the linear stability of MHD-equilibria is to use the Energy Principle of ideal MHD (Bernstein *et al.* 1958; Hain *et al.* 1957). According to this approach, the stability state of an equilibrium configuration is determined by the behavior of the change in potential energy δW resulting in applying a small perturbation $\underline{\xi}$ to the system. Explicity δW is given by (Bernstein *et al.* 1958):

$$\delta W(\underline{\xi}, \underline{\xi}^*) = -\frac{1}{2} \int \underline{\xi}^* \cdot \mathbf{F}(\underline{\xi}) d^3 r \quad ; \qquad (8)$$

i.e.,

$$\delta W(\underline{\xi}, \underline{\xi}^*) = \frac{1}{2} \int \Big\{ \frac{1}{4\pi} \, |\mathbf{Q}|^2 - $$

$$- \frac{1}{4\pi} (\nabla \times \mathbf{B}) \cdot (\mathbf{Q} \times \underline{\xi}^*) + \Gamma P |\nabla \cdot \underline{\xi}|^2 +$$

$$+ (\underline{\xi} \cdot \nabla P) \nabla \cdot \underline{\xi}^* - (\underline{\xi}^* \cdot \nabla \phi) \nabla \cdot (\rho \underline{\xi}) \Big\} \, d^3 r \, . \qquad (9)$$

The integral is taken over the volume of the system. We have used rigid boundary conditions

$$(\underline{\xi}\,|=0_{\,plasma\ boundary})$$

on the y, z-plane. The complex conjugate $\underline{\xi}^*$ appears in equations (8) and (9) since general complex perturbations are allowed. Therefore, stability is achieved if the energy functional δW is positive for all possible perturbations which satisfy the imposed boundary conditions. On the other hand, if there exists a perturbation which yields a negative δW, then the equilibrium configuration is unstable. Obviously, the MHD energy principle represents a necessary and sufficient criterion for stability.

In practice, the determination of the stability state of an equilibrium is carried out by calculating the sign of the minimum of δW. The minimization of δW is performed by using a positive definite normalization condition on $\underline{\xi}$ in order to avoid the trivial solution. In our case we choose the condition

$$\frac{1}{2} \int \rho \, |\underline{\xi}|^2 \, d^3 r = 1$$

so that the Euler-Lagrange equation of the variational principle:

$$\delta \Big[\delta W(\underline{\xi}, \underline{\xi}^*) + \lambda \frac{1}{2} \int \rho \, |\underline{\xi}|^2 \, d^3 r \Big] = 0 \, , \qquad (10)$$

with $\lambda = -\omega^2$, is precisely given by the eigenvalue equation (7). Therefore, the minimum eigenvalue ω_1^2 equals the minimum of δW (i.e., $\omega_1^2 = \mathrm{Min}\ \delta W$) and stable systems are characterized by a positive minimum of δW.

In order to analyze the stability properties of two-dimensional equilibria, we will consider the most general three-dimensional, complex perturbation given by:

$$\underline{\xi}(\mathbf{r}) = \Big[\hat{\xi}_x(y,z) \mathbf{e}_x + \hat{\underline{\xi}}_\perp(y,z) \Big] \exp(ikx)$$

$$= \Big[\hat{\xi}_x(y,z) \mathbf{e}_x + \hat{\xi}_y(y,z) \mathbf{e}_y + \hat{\xi}_z(y,z) \mathbf{e}_z \Big] \exp(ikx) \, . \qquad (11)$$

Note that periodic boundary conditions for $\underline{\xi}$ are assumed along the x-axis since equilibrium does not depend on x. Substituting $\underline{\xi}$ from equation (11) into δW, using the equilibrium equation (1) and integrating by parts, one obtains the specialized form of the potential energy functional δW for two-dimensional equilibria (see Schindler *et al.* 1983):

$$\delta W(\underline{\xi}, \underline{\xi}^*) = \frac{1}{2} \int \Big\{ \frac{1}{4\pi} \Big[|\nabla_\perp a|^2 - 4\pi \frac{\partial^2 \Pi}{\partial A^2} \, |a|^2 \Big] +$$

$$+ \frac{1}{4\pi} |\mathbf{B}_\perp \cdot \nabla_\perp \hat{\xi}_x - B_x \nabla_\perp \cdot \hat{\underline{\xi}}_\perp|^2 +$$

$$+ \frac{1}{4\pi} k^2 \, |B_x \hat{\underline{\xi}}_\perp - \hat{\xi}_x \mathbf{B}_\perp|^2 +$$

$$+ \Gamma P \Big[1 + \frac{\rho^2}{\Gamma P (\partial \rho / \partial \phi)} \Big] \, |\nabla \cdot \underline{\xi}|^2 -$$

$$- \frac{\partial \rho}{\partial \phi} | \hat{\underline{\xi}}_\perp \cdot \nabla_\perp \phi + \frac{\rho}{(\partial \rho / \partial \phi)} \quad \nabla \cdot \underline{\xi} |^2 +$$

$$+ 2kIm \left[\frac{1}{4\pi} (\nabla_\perp a \times \mathbf{e}_z) \cdot (B_z \hat{\underline{\xi}}^*_\perp - \hat{\xi}^*_z \mathbf{B}_\perp) - \frac{1}{4\pi} \Delta A \hat{\xi}_z a^* - \right.$$

$$\left. - \frac{1}{8\pi} B_z \Delta A (\hat{\underline{\xi}}_\perp \times \hat{\underline{\xi}}^*_\perp) \cdot \mathbf{e}_z \right] \Bigg\} d^3 r \quad . \qquad (12)$$

where $\nabla_\perp = \mathbf{e}_y \frac{\partial}{\partial y} + \mathbf{e}_z \frac{\partial}{\partial z}$,

$$a = - \hat{\underline{\xi}}_\perp \cdot \nabla_\perp A, \mathbf{B}_\perp = \nabla_\perp A \times \mathbf{e}_z \, .$$

The x-integration is carried out over one period. As explained above, we have considered rigid boundary conditions for $\underline{\xi}$ on the y, z-plane.

These conditions decouple the corona from the photosphere and they adequately simulate the magnetic line tying at the lower edge of the prominence region. For the other three edges, one formally looks for displacements which exactly vanish there, where then one assumes the existence of rigid, ideal conducting walls. Therefore, for a given equilibrium model described by the functions P, B_x, and the solution A, the minimization of δW will lead to definitive stability statements.

III. STABILITY ANALYSIS OF LOW'S MODEL CONSIDERING A NON-UNIFORM AXIAL MAGNETIC FIELD

Before we present our stability analysis, it is convenient to write all used variables in an adimensional form, namely:

$$\hat{r} = r/h_0, \quad \hat{A} = A/(h_0 B_0), \quad \hat{P} = P/(B_0^2/8\pi),$$

$$\hat{\mathbf{B}} = \mathbf{B}/B_0, \quad \hat{J}_x = J_x/(cB_0/4\pi h_0), \quad \hat{T} = T/T_0,$$

$$\hat{\phi} = \phi/(h_0 g), \quad \hat{\rho} = \rho/(B_0^2/8\pi h_0 g),$$

$$\hat{\underline{\xi}} = \underline{\xi}/h_0, \hat{k} = k h_0, \hat{\omega}^2 = \omega^2/(g/h_0),$$

$$\delta\widehat{W} = \delta W/(B_0^2 h_0^3/8\pi) \qquad ;$$

where $J_x = -\Delta A$ is the current density along the x-axis, B_0 a typical value of magnetic field strength, h_0 a characteristic length determining the prominence structure and T_0 a typical temperature. In terms of dimensionless variables the basic functions defining the equilibrium model of Low (1981) are given by:

$$\hat{P} = \frac{4\tilde{a}^2}{\alpha^2 \hat{z}^2} \left\{ \frac{\alpha^2 \hat{z}^2}{U^2} - \frac{2\alpha\hat{z}}{U} + ln\left[1 + \frac{2\alpha\hat{z}}{U}\right] \right\} +$$

$$+ \exp(\hat{z}_1 - \hat{z}) - \hat{B}_z^2, \qquad (13)$$

$$\hat{B}_z = G(\hat{A}) \qquad , \qquad (14)$$

∇

$$\hat{A} = ln\left\{ \frac{U}{[2\alpha\hat{z} + U]^2} \right\} \qquad (15)$$

where

$$U = \hat{y}^2 + (\hat{z} - \hat{z}_1)^2 + \tilde{a}^2, \tilde{a}^2, = \gamma^2 - \hat{z}_1^2, \; \alpha = \gamma + \hat{z}_1$$

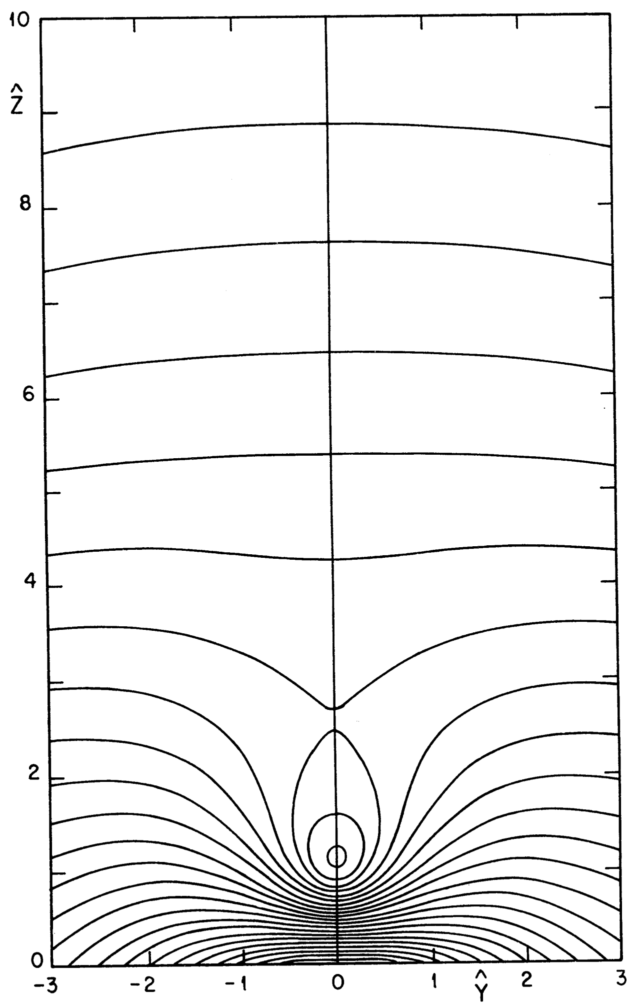
and G is an arbitrary function. The exponential term in equation (13) provides a hydrostatic background corona. The perpendicular field $\hat{\mathbf{B}}_\perp = \hat{\nabla} \hat{A} \times \mathbf{e}_x$ is constructed by superposing the field generated by the prominence current above the base of the corona to a bipolar potential field. Thus γ fixes the length scale of the potential field produced by a line current at $\hat{y} = 0$, $\hat{z} = -\gamma$. Besides, $\hat{z}_1$ locates the height above the origin where the symmetry axis of the current density of the prominence is situated. From this axis outwards the current density decreases over a characteristic length $\tilde{a}$. Figure 1 displays two examples of typical configurations of the magnetic field $\hat{\mathbf{B}}_\perp$ (defined as $\hat{A}$ = const. contours). For fixed γ, $\hat{z}_1$ may vary between $-\gamma$ and γ. Closed loops arise as soon as $\hat{z}_1 > 7\gamma/9$ otherwise the field lines are open. The case $\hat{z}_1 = -\gamma$ corresponds to a potential field. Moreover, neglecting the term $e^{(\hat{z}_1 - \hat{z})}$, the case $\hat{z}_1 = \gamma$ leads again to a global potential field except at the point $\hat{y} = 0$, $\hat{z} = \gamma$ where a singular line current flows.

In order to improve our knowledge about the stability or instability mechanism, we consider the separation of the energy contributions to the functional $\delta \hat{W}_{min}$:

$$\delta\widehat{W}_{min}(\underline{\xi}_{min}, \underline{\xi}^*_{min}) = \delta M + \delta K + \delta G \quad = \hat{\omega}_1^2, \qquad (16)$$

where δM, δK and δG express the contribution of the Lorentz force, pressure gradients, and gravitational force to the global potential energy associated with the minimizing displacement $\underline{\xi}_{min}$, respectively (details can be found in Galindo Trejo 1987).

The minimization of $\delta \hat{W}$ taking into account the general displacement (11) was carried out by using the variational method of finite elements (see e.g., Zienkiewicz 1977). This method deals with displacements defined locally on each element of the discretized region. Any single displacement is approximated by means of spline functions which depend on several free param-



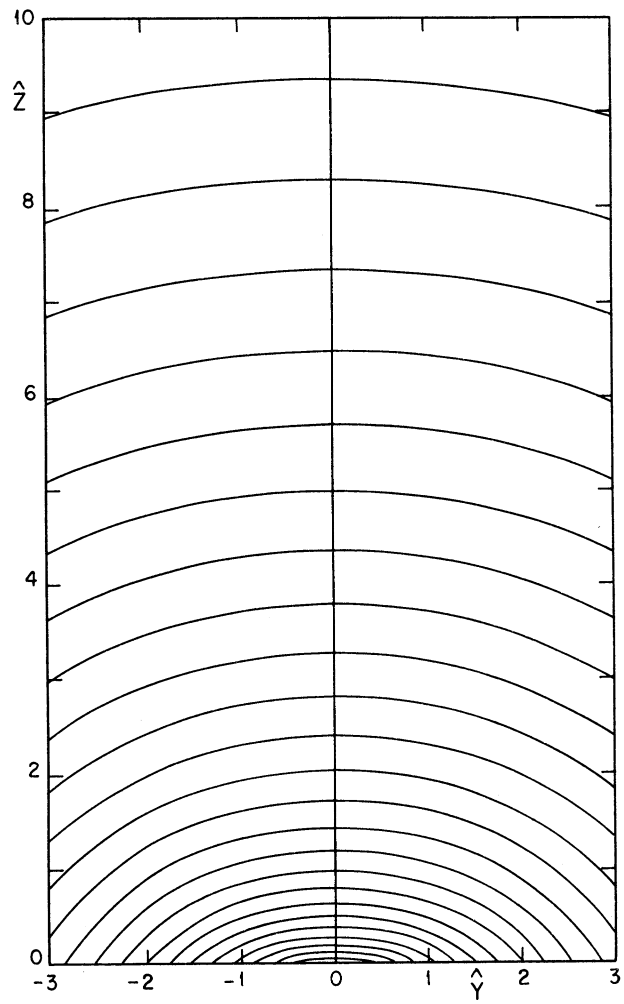


Fig. 1. Magnetic field lines of the prominence model of Low (1981): *a*) $\gamma = 1$, $\hat{z}_1 = 0.9$; *b*) $\gamma = 1$, $\hat{z}_1 = -0.3$.

eters. The variation process allows us to determine these parameters so that the minimizing mode $\hat{\underline{\xi}}_{min}$ is piecewise obtained. The quality of the approximation depends on only the discretization mesh. In the case of a singularity, one can utilize an improved mesh with smaller elements about the singular point. A computer code for numerical minimization was extensively tested by applying it to several equilibria whose dynamic behavior could be studied by other analytical methods (e.g., Alfvén waves in an homogeneous plasma, plane current sheets and acoustic gravity waves). We used a maximum of 32 triangular elements in each mesh of the plasma region on the y, z-plane, and we have tried different distributions of the elements (for details see Galindo Trejo 1987).

Previously, Galindo Trejo (1989) investigated the prominence model of Low (1981) by considering a uniform longitudinal magnetic field $\hat{B}_x$. In the case of instability, low intensities of $\hat{B}_x$ lead to a slight weakening of the instability. However, a further increase of the strength of $\hat{B}_x$ yields a drastic magnification of the instability. It is found that instabilities are driven only by compressional effects so that we are possibly dealing with macroscopic drift instabilities. It is interesting to note that the stabilizing effect obtained for low intensities of $\hat{B}_x$ seems to be consistent with the well known magnetic shear stabilization of drift instabilities (see e.g., Mikhailovskii 1967). On the other hand, the abrupt behavior of the instability seems to arise exclusively from the way by which Low's model introduces $\hat{B}_x$ in equation (13). Depending on the magnitude of $\hat{B}_x$, the initial pressure can be diminished so that the total pressure $\hat{P}(\hat{y}, \hat{z})$ becomes negative at isolated regions or even on the whole $\hat{y}$, $\hat{z}$-plane. Of course, this situation merely means that such a longitudinal magnetic field cannot stably coexist with the initial plasma pressure.

For the parameter ranges which are observed in solar quiescent prominences, the reduction of the intensity of $\hat{B}_x$ can effectively produce a stabilization of the equilibrium state. Although such a stabilization happens for intensities of $\hat{B}_x$ smaller than the mean value within

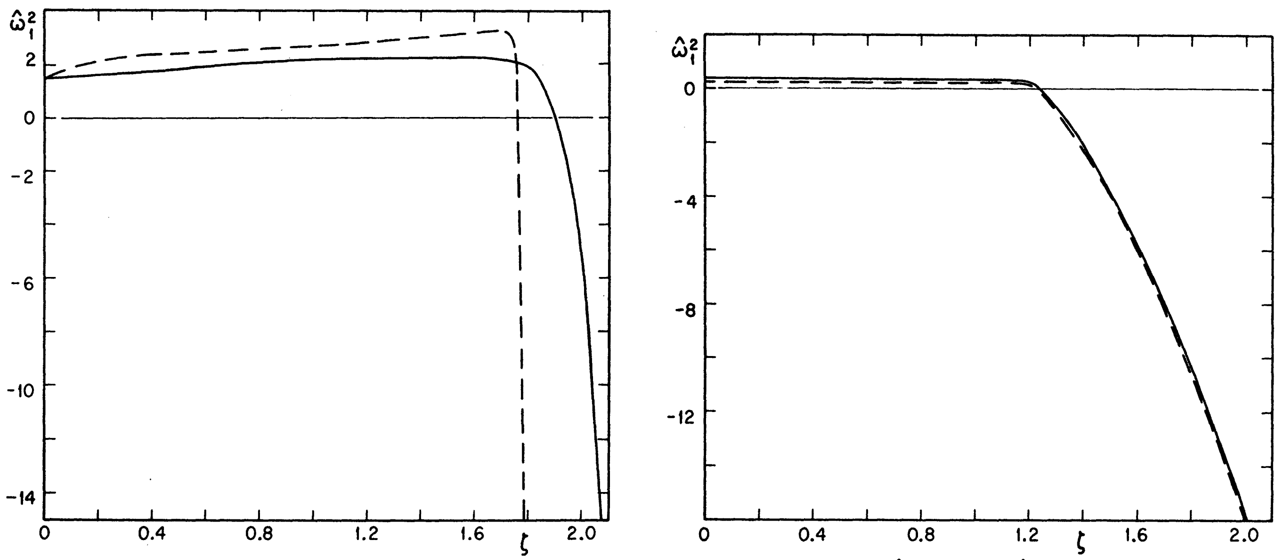


Fig. 2. Smallest value of $\hat{\omega}_1^2$ as a function of ς (maximum strength of the axial magnetic field: $\hat{B}_x = \varsigma/\cosh^2 \hat{A}$) for observed parameters: *a*) $\hat{Y}=6, \hat{Z}=10; \gamma=0.0166, \hat{z}_1=0.0166$, —— $\hat{k}=10$, - - - - $\hat{k}=50$; *b*) $\hat{Y}=24, \hat{Z}=70; \gamma=0.0035, \hat{z}_1=0.0035$, —— $\hat{k}=0.06$, - - - - $\hat{k}=0.017$.

prominences, our results may yet be applicable to the prominence class having a supporting magnetic field almost perpendicular to the longitudinal axis (Leroy 1978).

This paper provides an extension of our previous analysis of the model of Low (1981) by taking into account a non-uniform $\hat{B}_X(\hat{A})$. Note that a $\hat{B}_X(\hat{A})$ provides a selective shearing to the field lines of $\hat{\mathbf{B}}_\perp$. The first purpose of our study was to reduce the rather wide instability regions. We have assumed several functional forms of $\hat{B}_X(\hat{A})$ and performed the minimization of $\delta\hat{W}$. In doing so, we have considered parameter ranges reported for quiescent prominences and also other parameters outside the observed ranges.

We have obtained strongly increasing instability for the following functions describing $\hat{B}_X(\hat{A})$:

$$\hat{A},\ 1/\hat{A},\ 1/\hat{A}^2,\ e^{-\hat{A}}, e^{\hat{A}},\ e^{2\hat{A}},\ \ln \hat{A}^2,\ 1/\ln \hat{A}^2,\ \cos\hat{A}.$$

TABLE 1

STABILITY PROPERTIES OF THE PROMINENCE FOR LOW'S (1981) MODEL CONSIDERING A LONGITUDINAL MAGNETIC FIELD $\hat{B}_x(\hat{A}) = \varsigma/cosh^2\hat{A}$ [a]

T_p (°K)	γ	$\hat{k}$	$\hat{Y}$	$\hat{Z}$	$\hat{z}_1$	ς	$\hat{\omega}_1^2$	ν_1 ($10^{-3}s^{-1}$)	P (*min*)
7×10^3	0.0035	0.017	24.	70.	0.0035	0.3	0.2669	2.9631	5.624
7×10^3	0.0035	0.017	24.	70.	0.0035	1.0	0.2570	2.9079	5.731
7×10^3	0.0035	0.017	24.	70.	0.0035	1.2	0.1643	2.3247	7.169
7×10^3	0.0035	0.060	24.	70.	0.0035	0.3	0.3478	3.3825	4.927
7×10^3	0.0035	0.060	24.	70.	0.0035	1.0	0.3281	3.2852	5.073
7×10^3	0.0035	0.060	24.	70.	0.0035	1.2	0.2068	2.6086	6.389
5×10^4	0.0166	10.00	6.	10.	0.0166	0.3	1.6349	2.7504	6.059
5×10^4	0.0166	10.00	6.	10.	0.0166	1.0	2.1361	3.1438	5.301
5×10^4	0.0166	10.00	6.	10.	0.0166	1.2	2.1860	3.1803	5.240
5×10^4	0.0166	50.00	6.	10.	0.0166	0.3	2.2556	3.2305	5.159
5×10^4	0.0166	50.00	6.	10.	0.0166	1.0	2.6299	3.4883	4.777
5×10^4	0.0166	50.00	6.	10.	0.0166	1.2	2.7734	3.5822	4.652

a. Frequency $\nu_1 = (g/h_0)^{1/2}\hat{\omega}_1/2\pi$ and period P of stable oscillations for the parameter range observed in quiescent prominences.

TABLE 2

STABILITY PROPERTIES OF THE PROMINENCE FOR LOW'S (1981) MODEL WITH UNIFORM LONGITUDINAL FIELD $\hat{B}_x$ [a]

T_p (°K)	γ	$\hat{k}$	$\hat{Y}$	$\hat{Z}$	$\hat{z}_1$	$\hat{B}_x$	$\hat{\omega}_1^2$	ν_1 ($10^{-3} s^{-1}$)	P (*min*)
7×10^3	0.0035	0.017	24.	70.	0.0035	0.001	0.2671	2.9645	5.620
7×10^3	0.0035	0.06	24.	70.	0.0035	0.001	0.3487	3.3868	4.920
5×10^4	0.0166	10.	6.	10.	0.0166	0.020	1.5416	2.6708	6.240
5×10^4	0.0166	50.	6.	10.	0.0166	0.020	5.2060	4.9080	3.390

a. Frequency $\nu_1 = (g/h_0)^{1/2}\hat{\omega}_1/2\pi$ and period P of stable oscillations for the parameter range observed in quiescent prominences.

On the other hand, the initial ($\hat{B}_x = 0$) instability is now slightly weakened by the functional forms of $\hat{B}_x(\hat{A})$:

$$1/\cosh\hat{A},\ 1/\cosh^2\hat{A},\ 1/\cosh^3\hat{A},\ 1/\cosh^4\hat{A}.$$

In the case of observed parameters, these last forms lead to stable oscillations. Notice that such a cosh-class is the only one which shows simultaneously symmetry, no oscillations, a finite value at the origin and a moderately

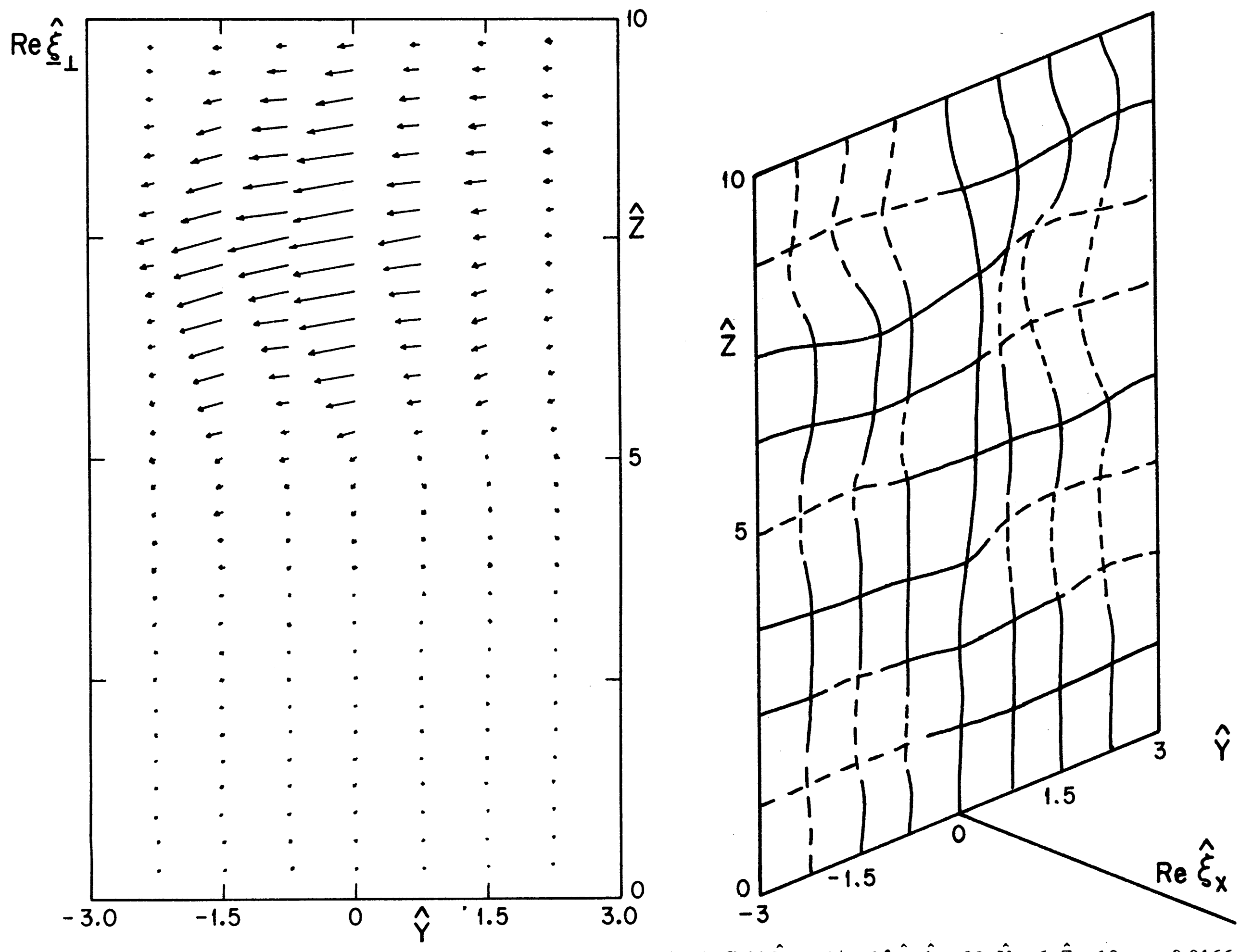


Fig. 3. Minimizing mode for observed parameters assuming an axial magnetic field $\hat{B}_x = 1/\cosh^2\hat{A}$: $\hat{k} = 20$, $\hat{Y} = 6$, $\hat{Z} = 10$; $\gamma = 0.0166$, $\hat{z}_1 = 0.0166$, $\hat{\omega}_1^2 = 2.3119$. *a*) Re$\hat{\xi}_\perp$; *b*) Re$\hat{\xi}_x$ such that $|\max \mathrm{Re}\hat{\xi}_\perp| / |\max \mathrm{Re}\hat{\xi}_x| \approx 51$; —— Re$\hat{\xi}_x > 0$, - - - - Re$\hat{\xi}_x < 0$, ... Re$\hat{\xi}_x = 0$ (This convention will be used in what follows, to visualize the longitudinal component of the mode).

decreasing behavior. Since the behavior of $\hat{\omega}_1^2$ remains practically the same for all members of this cosh-class, we have chosen the functional form $\hat{B}_x(\hat{A}) = \zeta/\cosh^2\hat{A}$ (where ζ is a positive constant) in order to investigate in detail the effect of a non-uniform $\hat{B}_x$ on the stability state of Low's equilibrium.

As an illustration of the influence of a variable $\hat{B}_x$ on the prominence equilibrium, we have considered the same observed parameter sets used in Paper I. Figures 2*a* and 2*b* show ω_1^2 as a function of parameter ζ which determines the maximum intensity of $\hat{B}_x$. Considering longitudinal fields from weak to moderate ($\zeta \sim 1.2$), the prominence carries out stable oscillations practically with constant frequency. Table 1 summarizes the obtained frequencies and periods taking into account some values of ζ which correspond to the plateau of the curves. As a comparison, the frequencies obtained in Paper I for uniform $\hat{B}_x$ are shown in Table 2. We conclude that the applied non-uniform $\hat{B}_x$ does not change substantially the period of oscillation for weak and moderate intensities. However, as soon as ζ exceeds a critical value (usually between 1.2 and 1.9), the equilibrium is rapidly destabilized with extremely large growth rate. The cause of this abrupt behavior is the same one which was briefly discussed for the case of uniform $\hat{B}_x$. It is interesting to note that in the case of stable oscillations the main energy contribution arises from the electromagnetic fields. Compressional effects are also stabilizing. Although gravity is destabilizing ($\delta G < 0$), its energy contribution is very small (i.e., $|\delta G| << \delta M$). On the other hand, instability is driven mainly by compressional effects.

Figure 3 shows a typical minimizing mode for the case of a prominence with $T = 7 \times 10^3$ °K (Landman 1983). Figure 3*a* exhibits $\text{Re}\hat{\xi}_\perp$ as a function of position on the y, z-plane. The amplitude of $\text{Re}\hat{\xi}_\perp$ is much larger than

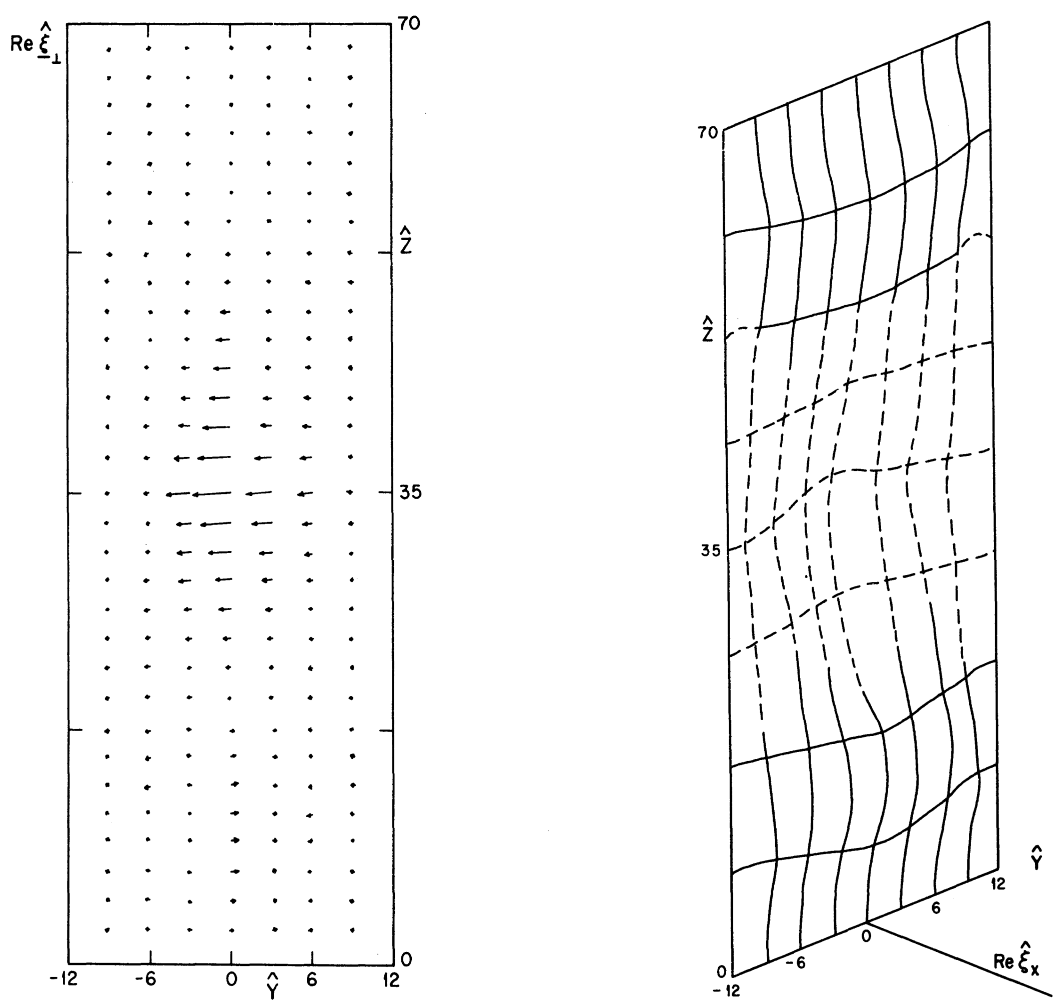


Fig. 4. Minimizing mode for observed parameters assuming an axial magnetic field $\hat{B}_x = 1/\cosh^2\hat{A}$: $\hat{k} = 0.06$, $\hat{Y} = 24$, $\hat{Z} = 70$; $\gamma = 0.0035$, $\hat{z}_1 = 0.0035$, $\hat{\omega}_1^2 = 0.3281$. *a*) $\text{Re}\hat{\xi}_\perp$; *b*) $\text{Re}\hat{\xi}_x$ such that $|\max \text{Re}\hat{\xi}_\perp| / |\max \text{Re}\hat{\xi}_x| \approx 1.2$.

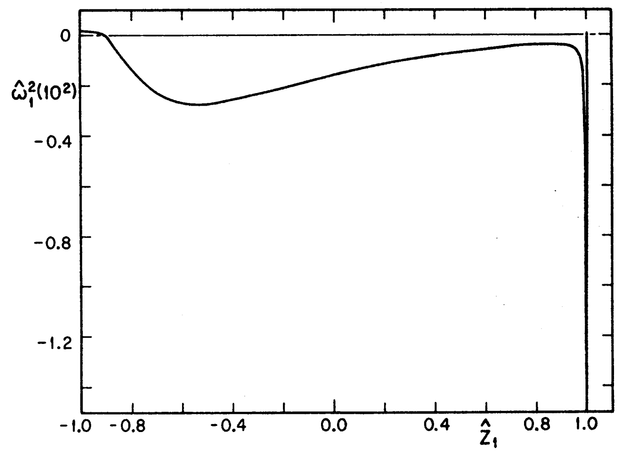


Fig. 5. Smallest value of $\hat{\omega}_1^2$ as a function of the parameter $\hat{z}_1$ assuming an axial magnetic field $\hat{B}_x = 1/\cosh^2\hat{A}$ and $\hat{k} = 1$, $\hat{Y} = 6$, $\hat{Z} = 10, \gamma = 1$.

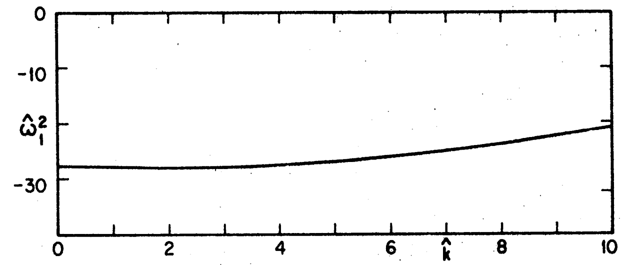


Fig. 6. Smallest value of $\hat{\omega}_1^2$ as a function of wavenumber $\hat{k}$ assuming an axial magnetic field $\hat{B}_x = 1/\cosh^2\hat{A}$ and $\hat{Y} = 6$, $\hat{Z} = 10, \gamma = 1, \hat{z}_1 = -0.55$.

that of $Re\hat{\xi}_x$ (Figure 3*b*). Therefore, the oscillations have nearly horizontal polarization which is perpendicular to the longitudinal axis and their largest amplitudes are attained in the upper portion of the prominence region. On the contrary, Figure 4 shows the mini-

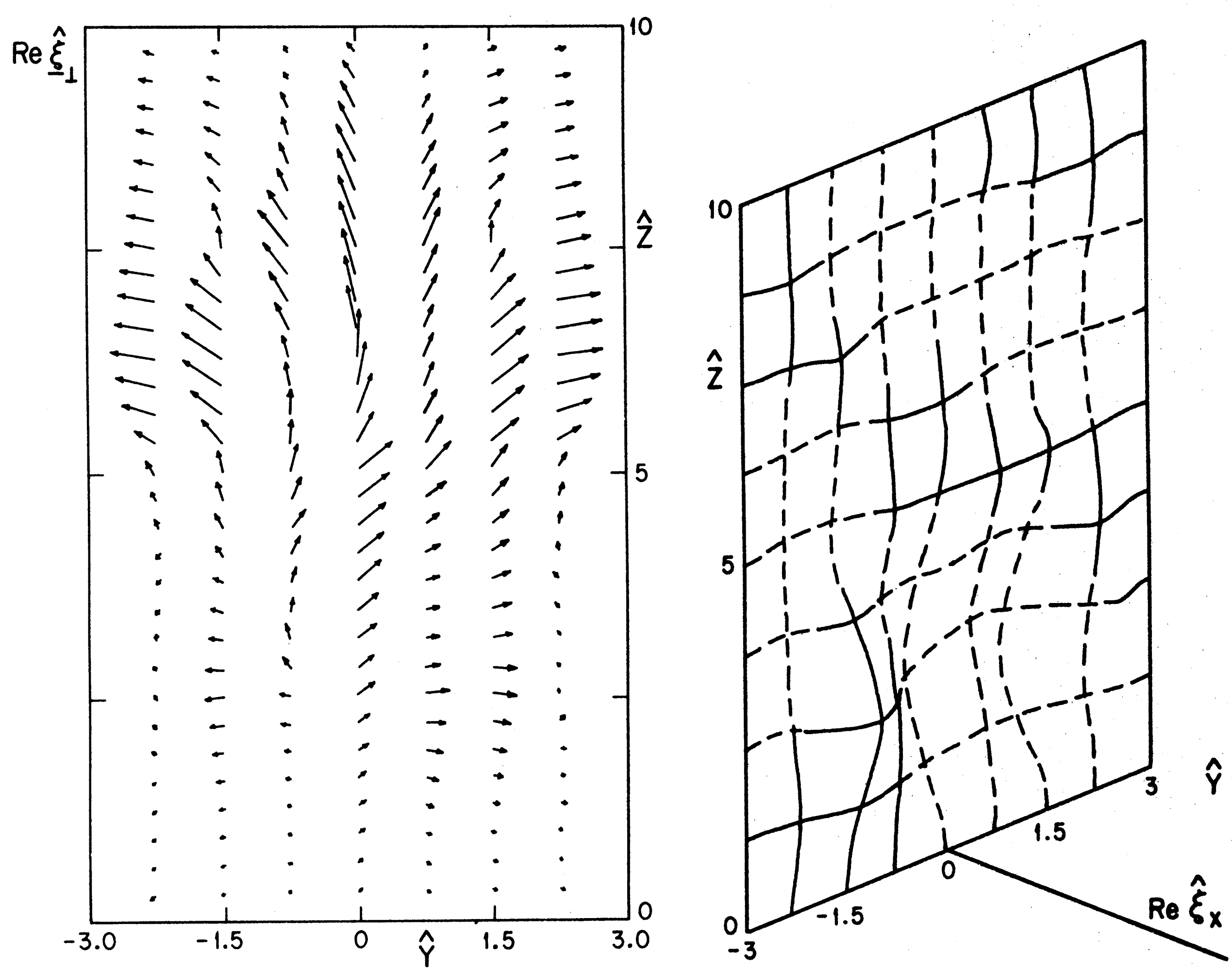
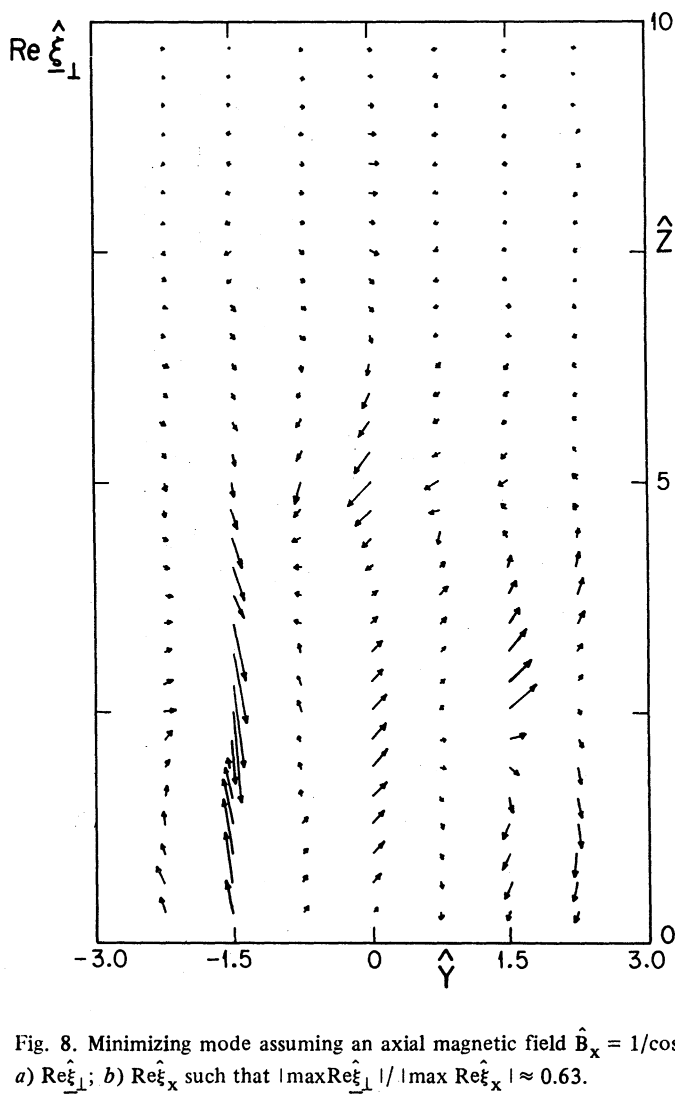


Fig. 7. Minimizing mode for observed parameters assuming an axial magnetic field $\hat{B}_x = 2/\cosh^2\hat{A}$: $\hat{k} = 10$, $\hat{Y} = 6$, $\hat{Z} = 10, \gamma = 0.0166$, $\hat{z}_1 = 0.0166$, $\hat{\omega}_1^2 = -5.5503$. *a*) $Re\hat{\xi}_\perp$; *b*) $Re\hat{\xi}_x$ such that $|\max Re\hat{\xi}_\perp| / |\max Re\hat{\xi}_x| \approx 5.8$.

mizing mode for a prominence with $T = 5 \times 10^4$ °K (Zhang and Fang 1987; Kundu, Melozzi, and Shevgaonkar 1986). On the y, z-plane, the oscillations are concentrated around the center of the plasma region (Figure 4*a*). However, the amplitude of $\mathrm{Re}\hat{\xi}_\perp$ is of the same order of magnitude than that of $\mathrm{Re}\xi_x$ (Figure 4*b*). Hence, the polarization of oscillation is horizontal, in planes which are parallel to the Sun's surface.

Large-scale oscillations of quiescent prominences are often reported after a major flare and they are predominantly horizontal. Wiehr *et al.* (1984) and Balthasar *et al.* (1986) have detected short-period oscillations with the period range 3–6.5 min. Thus, we conclude that Low's model, assuming a non-uniform axial magnetic field, may also explain such short-period oscillations with periods less than 7 minutes.

In order to illustrate stability properties in other parameter ranges which may be relevant for plasma structures in other stellar atmospheres, we have again considered an equilibrium in the region with $\hat{Y} = 6$ and $\hat{Z} = 10$. Figure 5 shows $\hat{\omega}_1^2$ as a function of the parameter $\hat{z}_1$ in the case of a moderate wavenumber, $\gamma = 1$ and axial field $\hat{B}_x = 1/\cosh^2\hat{A}$. The resulting behavior of $\hat{\omega}_1^2$ resembles that of Figure 5*a* of Paper I which assumes no axial field. In this case instability prevails again over most of the $\hat{z}_1$ interval but, as expected, the instability is now slightly weakened relative to the case $\hat{B}_x = 0$. Besides the minimum values of $\hat{\omega}_1^2$ in the regular $\hat{z}_1$-interval is shifted to $\hat{z}_1 = -0.55$. However, a substantial enlargement of the stability region is not found. Instability is again driven only by pressure forces. Gravity and the Lorentz force are stabilizing but the contribution of the former is much



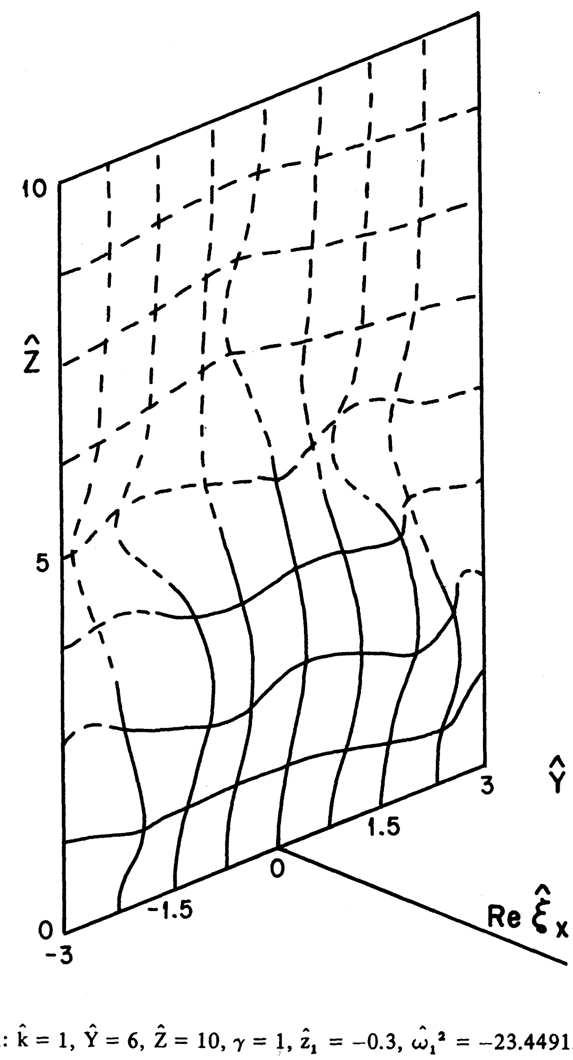


Fig. 8. Minimizing mode assuming an axial magnetic field $\hat{B}_x = 1/\cosh^2\hat{A}$: $\hat{k} = 1$, $\hat{Y} = 6$, $\hat{Z} = 10$, $\gamma = 1$, $\hat{z}_1 = -0.3$, $\hat{\omega}_1^2 = -23.4491$. *a*) $\mathrm{Re}\hat{\xi}_\perp$; *b*) $\mathrm{Re}\xi_x$ such that $|\max\mathrm{Re}\hat{\xi}_\perp| / |\max \mathrm{Re}\xi_x| \approx 0.63$.

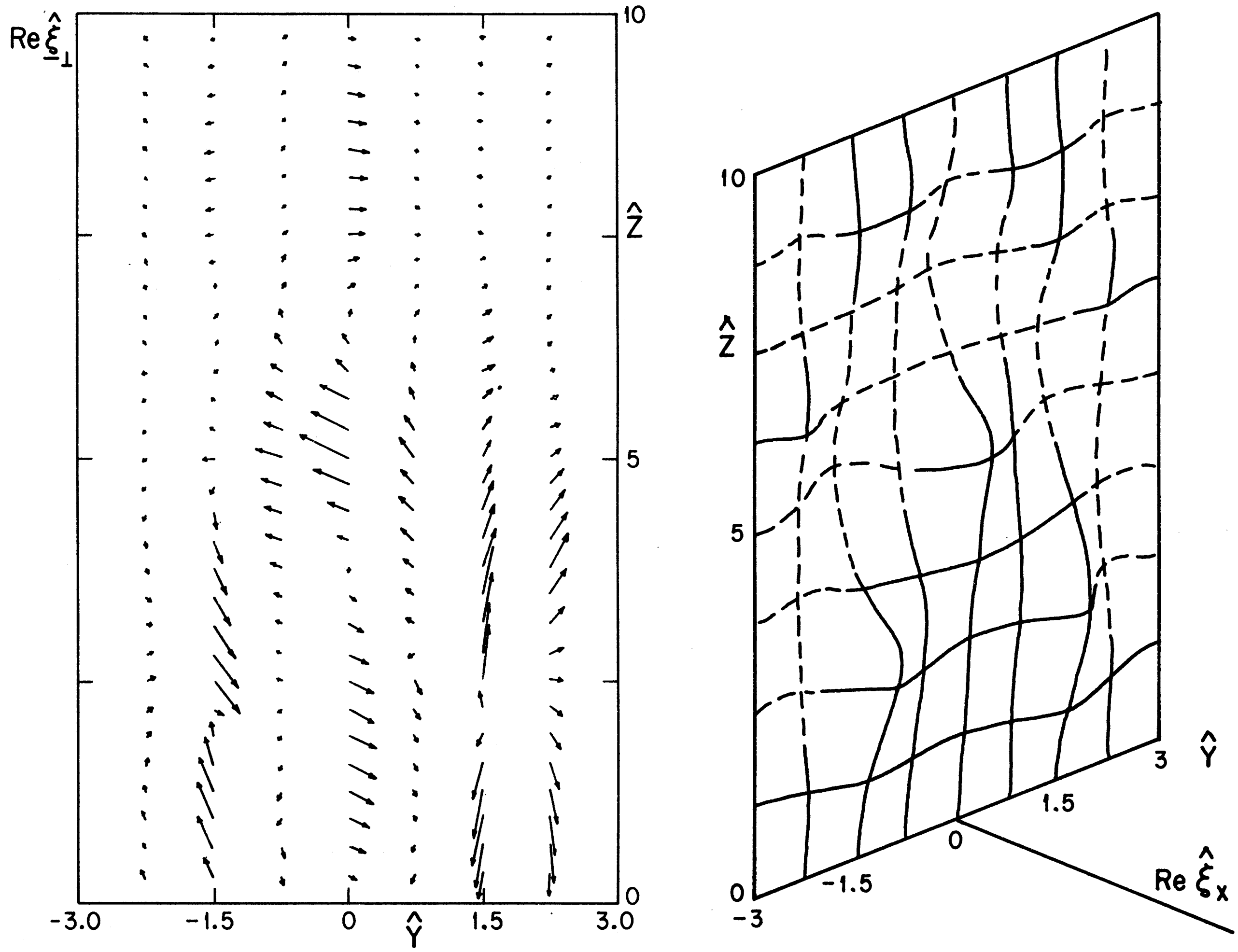


Fig. 9. Minimizing mode assuming an axial magnetic field $\hat{B}_x = 1/\cosh^2\hat{A}$: $\hat{k} = 1$, $\hat{Y} = 6$, $\hat{Z} = 10$, $\gamma = 0.75$, $\hat{z}_1 = -0.55$, $\hat{\omega}_1^2 = -26.0063$. *a*) Re$\hat{\xi}_\perp$; *b*) Re$\hat{\xi}_x$ such that $|\max \mathrm{Re}\hat{\xi}_\perp| / |\max \mathrm{Re}\hat{\xi}_x| \approx 1$.

smaller than that of the latter ($|\delta K| > \delta M >> \delta G$). On the other hand, the inadequate curvature of magnetic field lines is apparently not enough to cause electromagnetic flute or ballooning instability.

When the wavenumber $\hat{k}$ is allowed to vary, the preceding qualitative behavior of $\hat{\omega}_1^2$ remains again practically unchanged. Figure 6 illustrates the rather weak dependence of $\hat{\omega}_1^2$ on $\hat{k}$ by considering $\hat{B}_x = 1/\cosh^2\hat{A}$ and the $\hat{z}_1$-value at which the regular minimum in the former case (Figure 5) is attained. The energy contributions of gas pressure gradients, the Lorentz force and gravity keep nearly the same relation between them compared with that of the former case. The dependence of $\hat{k}$ of such contributions is also relatively weak.

To exemplify the typical appearance of minimizing modes $\underline{\xi}$ in the presence of a non-uniform axial field, we will briefly present them for some specific situations.

As shown above in the case of observed parameter ranges, an increase of the parameter ζ can cause an abrupt destabilization of the equilibrium. Resulting instability is exclusively driven by pressure gradients. Figures 7*a* and 7*b* illustrate the unstable minimizing mode for the case with $\zeta = 2$. The amplitude of Re$\hat{\xi}_\perp$ is in general much larger than that of Re$\hat{\xi}_x$. The fluid elements move on the y, z-plane along the magnetic field lines only in the lower region (neutral points are located at $\hat{z}_0 = 0.0166$ and $\hat{z}_0 = 0.0399$). On the other hand, Re$\hat{\xi}_\perp$ attains its largest amplitudes in the upper regions, showing a rather vertical pattern.

The effect of a non-uniform longitudinal field on the instability is dramatically exhibited in Figures 8*a* and 8*b* for the case $\gamma = 1$, $\hat{z}_1 = -0.3$ (see Figure 1*b*). Compared with Figure 10*a* of Paper I (case with $\hat{B}_x = 0$), Re$\hat{\xi}_\perp$ shows now a relatively complicated sheared pattern. However, the amplitude of Re$\hat{\xi}_x$ is usually larger than that of $\mathrm{Re}\hat{\xi}_{\perp}$. Like the case without $\hat{B}_x$, this instability is also driven solely by compressional effects. If one takes $\hat{z}_1 = -0.55$ (where $\hat{\omega}_1^2$ attains its regular minimum, see Figure 5), the orientation of both components of $\mathrm{Re}\hat{\xi}$ does not substantially change, but the amplitude of $\mathrm{Re}\hat{\xi}_x$ can be even much larger than that of $\mathrm{Re}\hat{\xi}_{\perp}$. Moreover, such a relation between both amplitudes can be inverted when one considers larger wavenumbers.

Finally, Figures 9*a* and 9*b* display an example of the specially complex pattern of an unstable minimizing mode. We have also assumed $\hat{B}_x = 1/\cosh^2 \hat{A}$. The magnetic field lines on the y, z-plane are similar to those of Figure 1*b*. Hence, fluid elements on this plane move along the field lines only in the upper region. On the contrary, in the lower half of the prominence region, two complicated vortices appear which resemble a turbulent pattern. The amplitudes of both components of $\mathrm{Re}\hat{\xi}$ are of the same order of magnitude so that the global motion of the fluid elements may be very complex. This instability is driven once again only by compressional effects which cancel completely the weak stabilizing effects of the Lorentz force and gravity.

IV. CONCLUSIONS

In this paper, we have analyzed the stability properties of the two-dimensional prominence model of Low (1981) by taking into account explicitly a non-uniform magnetic field $\hat{B}_x$ along the longitudinal axis of the prominence. We use a two-dimensional stability formalism based on the ideal MHD-Energy Principle of Bernstein *et al.* (1958) and Hain *et al.* (1957). We have considered several functional forms for $\hat{B}_x$. A special class of functional forms was found which leads to a stable behavior of the equilibrium for observed parameter ranges. Within the frame of the model of Low, detected short-period oscillations in quiescent prominences seem to be adequately described by assuming moderate intensity of $\hat{B}_x$. Obtained stable oscillations are mainly sustained by the electromagnetic force. On the other hand, larger intensities of $\hat{B}_x$ can produce the onset of an instability which is driven by compressional effects. Using the axial field $\hat{B}_x = 1/\cosh^2 \hat{A}$ we have studied its influence on the instability in parameter ranges outside the observed ones in detail. Although $\hat{B}_x$ causes a slight weakening of the instability with respect to the case with $\hat{B}_x = 0$, a considerable enlargement of the stability region is not found. The growth rate remains relatively unchanged when wavenumber $\hat{k}$ varies. In any case, resulting instabilities are driven exclusively by pressure gradients. This suggests that we are possibly dealing with macroscopic drift instabilities. Moreover, the slight stabilization caused by a non-uniform $\hat{B}_x(\hat{A})$ seems to be consistent with the magnetic shear stabilization of drift instabilities. However, our approach cannot identify the resulting instability more exactly.

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