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SMALL FIELD CCD ASTROMETRY WITH A LONG FOCUS REFLECTOR TELESCOPE

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RESUMEN

Se muestra la utilización del método de ajuste en bloque para la medición de posiciones astrométricas a partir de un mosaico de imágenes CCD directas tomadas con un telescopio reflector de distancia focal larga. Las observaciones cubren una región de aproximadamente $25' \times 25'$ en torno al cúmulo abierto Rup21. Como catálogo de referencia se empleó el Catálogo Tycho-2.

Se analiza el error interno de las posiciones medidas, y se estima el error externo a partir de la comparación con los catálogos Tycho-2 y USNO-A2.0. En esta comparación también se encuentra que las imágenes CCD directas tomadas con reductor focal podrían estar afectadas debido a la distorsión por curvatura de campo.

El efecto de la distorsión, supuestamente introducida por el reductor focal, se consigue eliminar mediante una corrección a las posiciones de las estrellas medidas sobre las imágenes, pero se observa un nuevo efecto sistemático en escalas de todo el campo cubierto, el cual es modelado mediante polinomios. Las posiciones finales tienen precisiones alrededor de $0\overset{''}{.}15$.

ABSTRACT

We discuss the use of the block adjustment method for determining astrometric positions from a mosaic of CCD frames taken with a long focus reflector telescope. The observations cover an area of $25' \times 25'$ around the open cluster Rup21. The source of reference positions was the Tycho-2 Catalogue.

The internal error in the measured positions is analyzed, and the external error is estimated from comparison with the Tycho-2 and USNO-A2.0 catalogs. In this comparison it is found that direct CCD images taken with a focal reducer could be distorted by severe field curvature.

The effect of distortion presumably introduced by the focal reducer is eliminated with suitable corrections to stellar positions measured on every frame, but a new systematic effect on scales of the entire field is observed. This effect is modeled with polynomials. The final positions have an accuracy of around $0\overset{''}{.}15$.

Key Words: **ASTROMETRY**

1. INTRODUCTION

The block adjustment method (Stock 1981) has been applied to a mosaic of direct CCD frames. The mosaic covers an area approximately $25'$ square around the open cluster Rup 21 ($\alpha_{2000} = 7^{\rm h}27^{\rm m}$, $\delta_{2000} = -31^\circ 12'$). This object was chosen for practical reasons: (a) it is a relatively small cluster ($10'$ diameter), so it is appropriate for a study with a CCD (b) it is located in the zone covered by Carte du Ciel plates taken in Córdoba, which will allow a future measurement of proper motions for some stars, and (c) there are no very bright stars, which could complicate the determination of the centroids

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of the stellar images, with the brightest ones having magnitudes below the limiting magnitude of the aforementioned plates ($m_{\rm pg} \approx 14$).

According to previous numerical simulations (Bustos Fierro & Calderón 2000), if optical distortions do not exceed one pixel at the edge of the frame, the smallest achievable errors in positions determined with this technique could be between 0″.10 and 0″.20. The main objective of this experimental measurement was to apply the method already tested on simulations to observational data, and to investigate what could be improved for the determination of positions with astrometric quality.

2. OBSERVATIONS

Observations were performed the night of 1997 December 8 with a CCD TEK1024, that is, a thinned back-illuminated square CCD with 1024 × 1024 24 μm square pixels, without binning. The CCD was attached to the 2.15-m Ritchey-Chrétien Jorge Sahade Telescope in CASLEO (Complejo Astronómico El Leoncito, San Juan, Argentina) with focal reducer, which gives on the frames a nominal scale factor of 0″.033/μm or 0″.792/pixel. Due to the presence of the focal reducer, the useful field on the CCD is circular, approximately 10′ diameter.

Sixteen frames (Figure 1) were taken with a V filter, following a center-edge arrangement as in the simulations mentioned above.

3. MEASUREMENT OF STELLAR IMAGES

The identification of stars present on every frame was made by means of task DAOFIND in package DAOPHOT of the program IRAF.[4] To do this, it is necesary to provide some instrumental parameters (gain and readout noise) and some characteristics of the images such as background noise, minimum good pixel value, FWHM and roundness of stellar images, as well as some parameters for the identification itself: signal-to-noise ratio and minimum and maximum values allowed for roundness and sharpness of stellar images.

After inspection of the images, a slightly elliptical model, with axis ratio of 0.9 elongated along the y-axis, and FWHM = 3 pixels (2″.4) was adopted for stellar images. The parameters for the identification were determined by trial and error, until most stars could be identified without spurious identifications.

Positions of objects found by DAOFIND were used as input coordinates for task CENTER in the same package. Finally, more than four thousand stellar images were identified and centered in the whole mosaic.

[4]IRAF is distributed by NOAO, which is operated by AURA, under cooperative agreement with NSF.

4. BLOCK ADJUSTMENT

The Tycho-2 Catalogue (Høg et al. 2000) supplies a dense enough set of high precision coordinates and proper motions. Twenty-five stars from this catalog were found in the field covered by the mosaic (Fig. 1), which were used as reference stars.

In order to apply the block adjustment technique, it is necessary to determine whether two or more stellar images found on different frames belong to a single star in the sky. This multiple-image identification was carried out in two stages. In the first stage, identification of reference stars and of multiple images of a single star was made "by hand" on a subset of about 500 stellar images, assigning an identification number to every star. This subset was used to block-adjust preliminary coefficients by least squares. The second stage consisted of automatic identification of all stars measured on every frame.

For the automatic identification, preliminary coefficients were used to transform all the positions (x, y) of centroids on every frame to (α, δ) on the sky. When two celestial positions were closer than 1″ it was assumed that they were two images of the same star. In this way, 2508 stars were found in the whole field. Only centroids with null error code provided by the task CENTER were used to perform the block adjustment of definitive coefficients by least squares. All least squares fittings were performed using the routine LFIT in Press et al. (1989).

The coefficients provided by block adjustment are the elements of sixteen transformation matrices between a three-dimensional cartesian axis system fixed to every frame and one fixed to the equatorial system (ξ, η, ζ) defined by

$$\xi = \cos\alpha\cos\delta, \quad \eta = \sin\alpha\cos\delta, \quad \zeta = \sin\delta\,.$$

The following dispersions were found on every axis from residuals in the block-adjustment equations

$$\sigma_\xi = 0.8\times10^{-6}, \quad \sigma_\eta = 1.4\times10^{-6}, \quad \sigma_\zeta = 1.2\times10^{-6}\,.$$

On the celestial sphere these dispersions are equivalent to $\sigma_{\alpha_\star,\delta} \approx 0''.25$, with $\alpha_\star = \alpha\cos\delta$. This is our estimation of the internal error of the fitting.

4.1. *Adjusted Coefficients*

In the error-free ideal case, the transformation matrices are orthogonal. In order to make the determinants of the adjusted matrices as close to unity

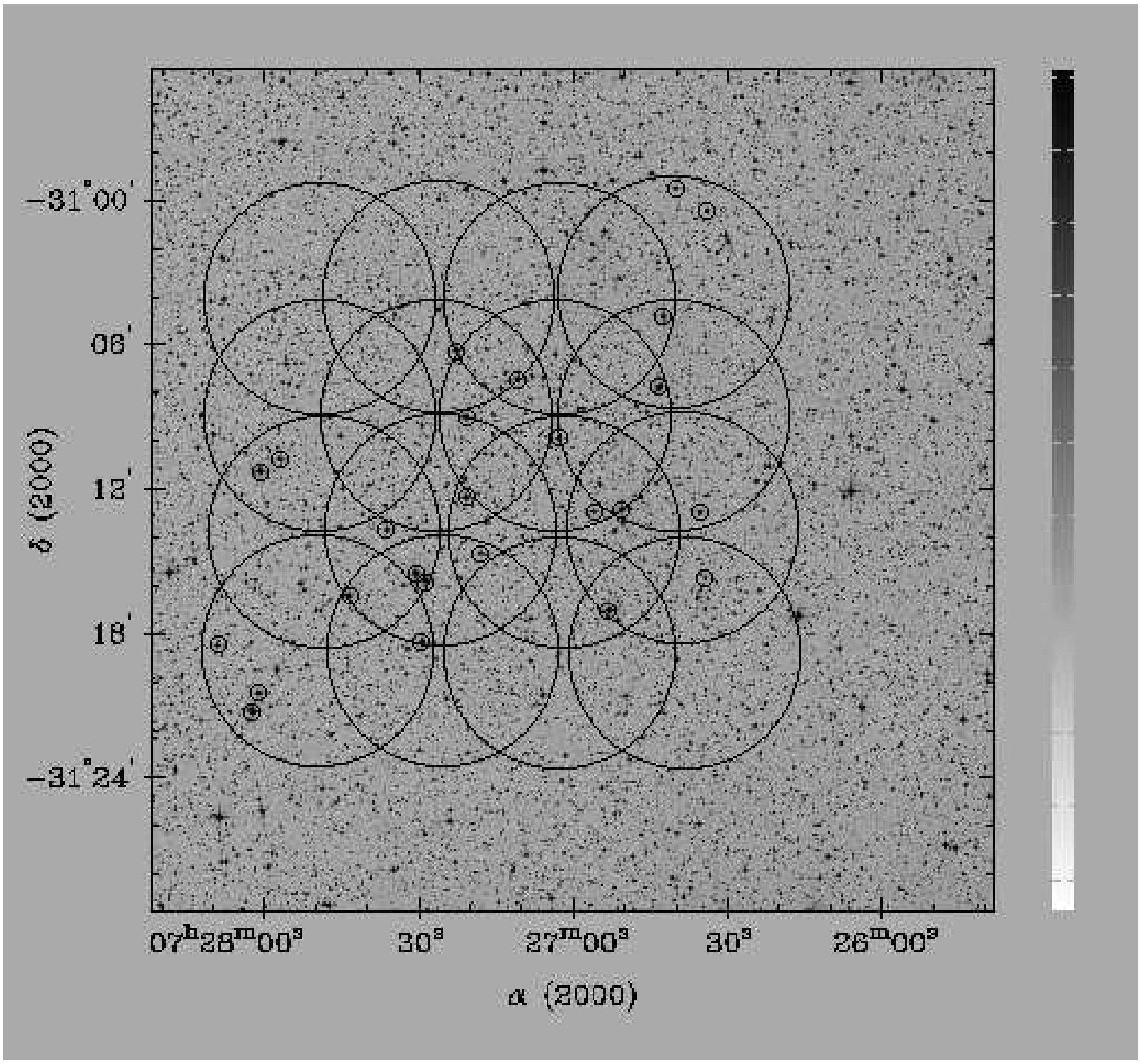

Fig. 1. Image of the entire field measured, as obtained from the Digitized Sky Survey. Large circles are the areas covered by every frame. Reference stars from the Tycho-2 Catalogue are in small circles.

as possible, the effective focal length adopted was 6.117 m, slightly smaller than the nominal one of 6.250 m.

A test made in order to check how similar the adjusted matrices were to orthogonal ones consisted of summing along rows and columns the squares of their elements. It was found that the determinants of the adjusted transformation matrices were close to unity, as well as the sums over rows and columns of their elements squared. This indicates that the matrices are quite similar to orthogonal ones, even though this condition was not imposed in the fitting.

5. THE CCD CATALOG

Definitive coefficients were employed to transform positions on the CCD frames to celestial positions in the coordinate system of the Tycho-2 Catalogue, i.e., ICRS. Due to the overlapping frames, for many stars it was possible to determine more than one celestial position, one for each image of that star present in the frames. Therefore, the arithmetic mean of the celestial coordinates calculated from all the images of a single star was adopted as its measured celestial coordinates.

Finally, a catalog was constructed containing (α, δ) in the ICRS, epoch 1997.9, of 2508 stars in an approximately $25'$ square area around the open cluster Rup 21 ($\alpha_{2000} = 7^{\rm h}27^{\rm m}$, $\delta_{2000} = -31°12'$): the *CCD Catalog* hereafter.

6. EXTERNAL COMPARISON

Measured positions were compared with the ones given in Tycho-2, though it is not a statistically significant set since it is very small and it consists of the reference stars.

In order to have a statistically significant set of differences, the comparison catalog should be sufficiently dense: 2500 stars in an area smaller than 0.2 square degrees. One catalog with the required number of objects is USNO-A2.0 (Monet et al. 1998, USNO-A2 hereafter). Therefore, measured positions were compared with the ones given in that catalog.

6.1. *Comparison with Tycho-2*

Table 1 shows the differences between the coordinates of Tycho-2 stars in the CCD catalog and in the reference catalog. The resulting differences do not allow the determination of the error in the measured positions, but they are consistent with an internal error $\sigma_{\alpha_\star,\delta} \approx 0\farcs25$.

TABLE 1

DIFFERENCES BETWEEN COORDINATES OF TYCHO-2 STARS IN CCD CATALOG[a]

Tycho-2 Star	$\Delta\alpha_\star$	$\Delta\delta$
7104 03122 1	0.016	0.025
7104 00944 1	−0.314	−0.075
7104 03878 1	−0.220	0.117
7104 00734 1	0.268	−0.003
7104 00457 1	0.213	0.123
7104 00376 1	−0.114	0.042
7104 00810 1	−0.093	0.070
7104 03014 1	−0.238	−0.510
7104 03773 1	−0.196	0.168
7104 02863 1	0.122	0.069
7104 02514 1	0.367	0.192
7104 00671 1	−0.115	−0.236
7104 00861 1	−0.247	−0.189
7104 00950 1	0.021	−0.201
7104 00914 1	−0.091	−0.001
7104 03877 1	0.323	0.053
7104 03424 1	0.076	−0.054
7104 03757 1	0.057	0.350
7104 03393 1	−0.021	0.317
7104 03352 1	−0.500	−0.604
7104 00025 1	0.179	−0.355
7104 00995 1	0.036	0.082
7104 00532 1	0.242	0.258
7104 00160 1	−0.573	−0.050
7104 00148 1	0.059	0.038
Average	−0.030	−0.024
σ	0.240	0.232

[a]Differences in arcseconds between positions of reference stars given in the CCD catalog and in Tycho-2.

6.2. *Comparison with USNO-A2*

In order to perform this comparison, it was necessary to identify in the USNO-A2 catalog the stars measured in the CCD frames. The routine developed to make this identification reads positions in USNO-A2 sequentially and, for stars within the range of right ascension and declination covered by the mosaic, they are compared with the positions in the CCD catalog. If there is a CCD position closer than $1\farcs5$ from the USNO-A2 position, it is assumed that it is the same star and the differences in both coordinates are computed and stored for further analysis.

A subset of 2190 stars from 2385 present in the CCD catalog were found within $1\farcs5$ around a position from USNO-A2. Figure 2 shows the differences found, and Figure 3 their histograms for both coordinates. The distribution of residuals shows a slight asymmetry towards negative values of $\Delta\alpha_\star$ and $\Delta\delta$.

The averages and dispersions of these residuals are

$$\overline{\Delta\alpha_\star} = (-0.18 \pm 0.47)'',$$

$$\overline{\Delta\delta} = (-0.13 \pm 0.51)''.$$

6.2.1. *Differences as a Function of Coordinates*

With the aim of determining if these differences are dependent on position, they were plotted against

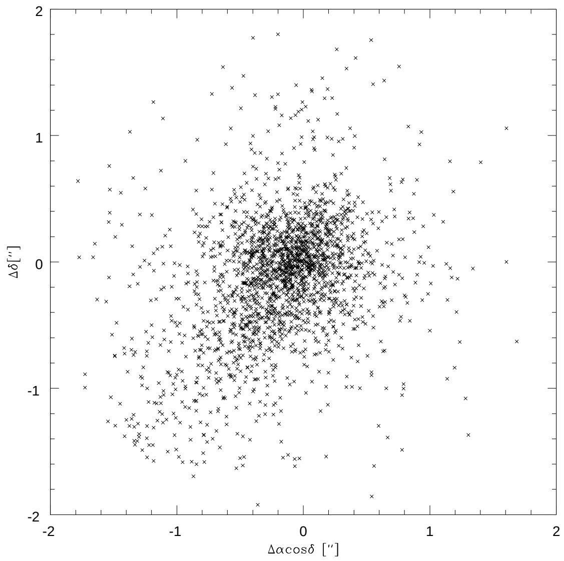

Fig. 2. Differences between positions measured with the CCD and the ones given in USNO-A2.

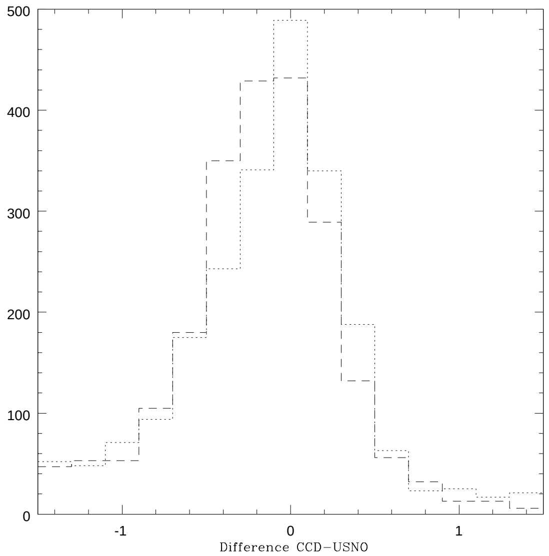

Fig. 3. Histograms of the differences between the CCD catalog and USNO-A2. The dashed line is $\Delta\alpha_\star$ and the dotted line is $\Delta\delta$.

right ascension and declination. Plots of $\Delta\alpha_\star$ versus α and $\Delta\delta$ versus δ in Figures 4 and 5 show noticeable periodic patterns that follow the locations of frames in the field, which suggest the presence of some systematic error on the scale of the frames. In order to verify this hypothesis, a vector diagram with differences $(\Delta\alpha_\star, \Delta\delta)$ as function of rectangular coordinates (x, y) on the frames was constructed.

Since there are more than 4000 stellar images with measured (x, y) coordinates, from which (α, δ) can be derived by means of the adjusted coefficients, there are more than 4000 differences $(\Delta\alpha_\star, \Delta\delta)$ between these positions and the ones given in USNO-A2. A diagram with more than 4000 individual vectors is confusing and noisy and, in addition, the interest is in the average systematic behavior not in individual differences. Therefore, they were averaged within a grid of square cells $\approx$ 60 pixels on a side, thus including approximately 30 stellar images per grid cell. Figure 6 shows the resulting vector diagram, which exhibits a clear radial pattern plus a tendency for the residuals to be higher towards the lower left corner of the frame.

6.2.2. *Dispersion of Residuals as a Function of Magnitude*

Photometric reduction of CCD images was not performed, but B and R magnitudes were obtained from USNO-A2 for all the stars identified in this catalog, which were divided in ten groups of 219 stars

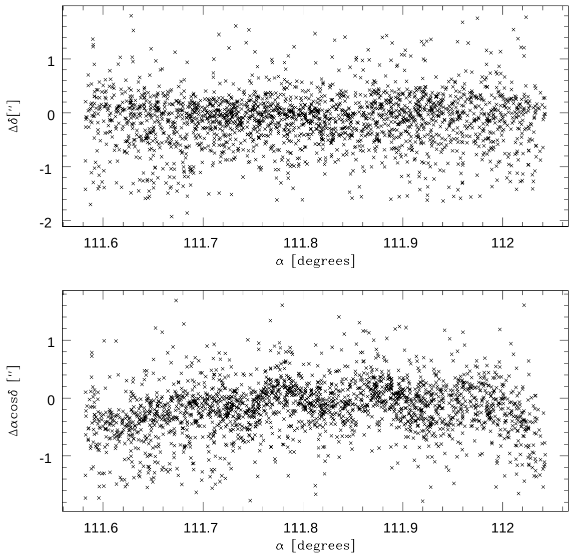

Fig. 4. Differences between positions measured with the CCD and the ones given in USNO-A2 against right ascension.

each one, with B increasing from the first to the tenth group. Table 2 shows dispersions of differences in both coordinates between positions in CCD catalog and USNO-A2 for every group. The residuals show a slight trend to be smaller for fainter magnitude intervals, which may be due to the decreasing accuracy of positions in USNO-A2 for bright stars.

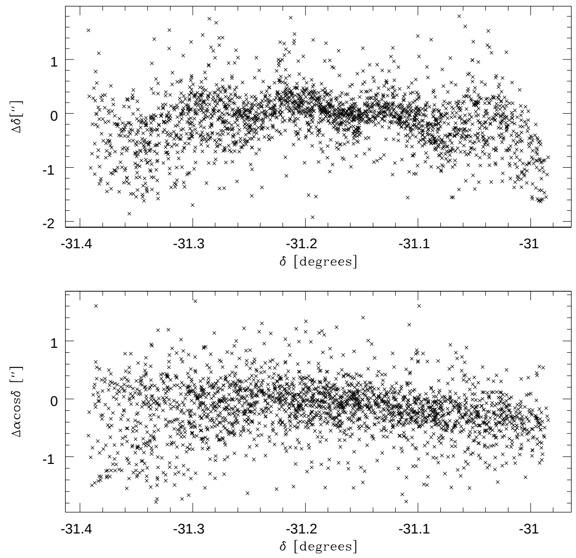

Fig. 5. Differences between positions measured with the CCD and the ones given in USNO-A2 against declination.

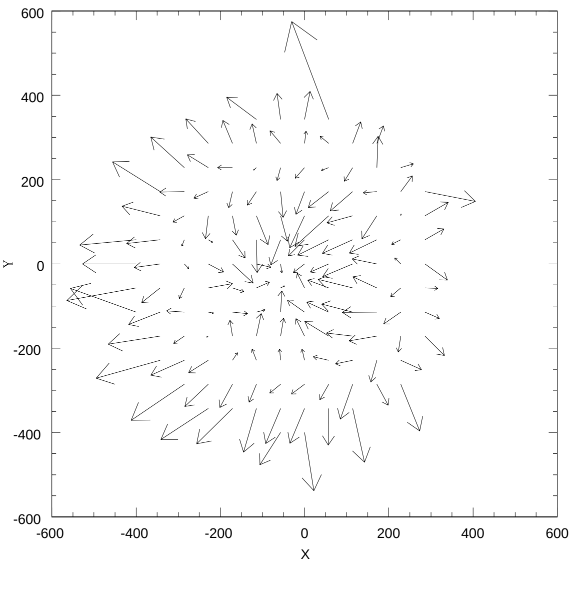

Fig. 6. Differences between CCD-based positions and USNO-A2 averaged within a grid of square cells ≈ 60 pixels on a side. The coordinates x and y of grid cells on the frame are given in pixels. Vectors have been enlarged 250 times, so the biggest ones represent approximately $1''$ (≈ 1.2 pixel).

6.2.3. *Dispersion of Residuals as a Function of Color*

Stars found in USNO-A2 were divided into ten groups with increasing color $B-R$. Table 3 shows the dispersion of differences in both coordinates between positions in the CCD catalog and USNO-A2 for each group. No dependence of residuals on color was observed.

TABLE 2

DISPERSIONS OF DIFFERENCES BETWEEN COORDINATES IN CCD CATALOG AND USNO-A2[a]

B mag	$\sigma_{\Delta\alpha_*}$ $''$	$\sigma_{\Delta\delta}$ $''$
10.7–14.6	0.582	0.578
14.6–15.5	0.561	0.604
15.5–16.0	0.468	0.556
16.0–16.4	0.470	0.520
16.4–16.7	0.437	0.534
16.7–17.0	0.418	0.466
17.0–17.2	0.442	0.489
17.2–17.5	0.418	0.477
17.5–17.7	0.402	0.442
17.7–18.6	0.433	0.404

[a]For different intervals of apparent magnitude B.

TABLE 3

DISPERSIONS OF DIFFERENCES BETWEEN COORDINATES IN CCD CATALOG AND USNO-A2[a]

$B-R$ mag	$\sigma_{\Delta\alpha_*}$ $''$	$\sigma_{\Delta\delta}$ $''$
(-1.1)–0.1	0.567	0.570
0.1–0.2	0.461	0.535
0.2–0.3	0.453	0.535
0.3–0.4	0.465	0.529
0.4–0.5	0.375	0.477
0.5–0.6	0.459	0.467
0.6–0.7	0.367	0.487
0.7–0.8	0.514	0.500
0.8–1.1	0.464	0.471
1.1–2.5	0.485	0.497

[a]For different intervals of color $B-R$.

7. CORRECTION OF OPTICAL EFFECTS

In order to remove the systematic pattern observed in Figure 6, it is necessary to model the distortion field, which must be previously constructed.

There are at least two approaches to achieve this construction: the comparison of individual positions in every frame with the average position in the CCD catalog, as in Abad (1993,1995), and comparison with an external catalog. We chose the second approach, comparing with USNO-A2 for the following reasons: (a) the average number of images per star is less than two, therefore, considerable undercorrected systematic residuals could remain (b) the number of images per star is strongly dependent on the position of the images on the frames, and due to the regular pattern of frames it could lead to a regular pattern of undercorrected residuals, too (c) since the USNO-A2 and CCD frames are very deep, most of the stars should be field stars, therefore, the lack of proper motions is not a big problem (d) in case of considerable proper motions of the cluster stars, they will not correlate with the positions of frames as in Figs. 4 and 5, and they could only leave a constant trend in some direction when averaged over all the frames. Consequently, we model the pattern in Fig. 6 that was built from a comparison with USNO-A2.

7.1. *Modeling*

An appropriate model for the radial pattern in Fig. 6 consists of supposing the existence of field curvature as in Figure 7. It can be shown that for this model the resulting radial displacements are given by

$$\Delta r(r) = ar + br^3 + cr^5 + \dots .$$

By retaining only three terms this model can be expressed for every celestial coordinate as

$$\Delta\alpha_\star(x,y) = ax + bx(x^2+y^2) + cx(x^2+y^2)^2 \; , \quad (1)$$

$$\Delta\delta(x,y) = a'y + b'y(x^2+y^2) + c'y(x^2+y^2)^2 \; . \quad (2)$$

In the case of strictly radial displacements, the coefficients should satisfy $a = a'$, $b = b'$ and $c = c'$. However, this condition was not imposed in order to allow for asymmetry, and both equations were independently fitted by least squares to the average differences plotted in the distortion field. Table 4 gives the results of the fits to $\Delta\alpha_\star$ and $\Delta\delta$.

7.2. *Correction of Rectangular Coordinates*

The Adjusted polynomials 1 and 2 were employed to correct measured rectangular coordinates (x, y) of every stellar image on every frame by assuming that $\Delta x = f\Delta\alpha_\star$ and $\Delta y = f\Delta\delta$, where f is the effective focal length.

The corrected measured positions were used to perform a new block adjustment. The coefficients

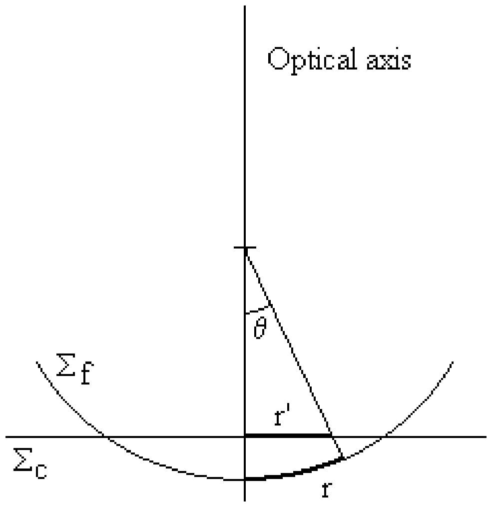

Fig. 7. Field curvature model for the pattern in Fig. 6. Σ_f is the focal surface and Σ_c is the CCD surface. r and r' are the radial coordinates on the focal surface and the one measured on the CCD, respectively.

resulting from this adjustment were used to recalculate celestial coordinates in the CCD catalog. Differences between these and the ones in USNO-A2 have the following mean values and dispersions:

$$\overline{\Delta\alpha_\star} = (-0.15 \pm 0.41)'' , \quad \overline{\Delta\delta} = (-0.09 \pm 0.43)'' \; .$$

The vector diagram in Fig. 6 was rebuilt with the new measured positions, resulting in the one shown in Figure 8. The radial pattern has been successfully eliminated, however, the trend of residuals directed towards the lower left corner appears more clearly. As expected, the periodic structure in Figs. 4 and 5 has been eliminated, too, but a new pattern on the scale of the entire field appears, as seen in Figures 9 and 10.

In order to get a more detailed picture of this new pattern, the differences with USNO-A2 were averaged within a grid of about 200 square cells approximately $1\overset{'}{.}7$ on a side in $\alpha_\star$ and δ. These average differences (Figure 11) increase rapidly in the north and south borders of the field, mainly in the north-east and south-west corners.

8. CORRECTION OF THE FIELD-SCALE SYSTEMATIC EFFECT

Although the random component dominates the distributions of residuals in both coordinates, the

TABLE 4

FITTED POLYNOMIAL COEFFICIENTS

	a [$''$/pixel]	b [$''$/pixel^2]	c [$''$/pixel^3]
	Polynomial 1		
Coefficient	-2.81×10^{-3}	4.48×10^{-8}	-0.98×10^{-13}
Error	0.29×10^{-3}	0.60×10^{-8}	0.29×10^{-13}
R^2	0.730	...	...
σ	$0\farcs124$	...	...
	Polynomial 2		
Coefficient	-3.25×10^{-3}	6.29×10^{-8}	-2.17×10^{-13}
Error	0.30×10^{-3}	0.65×10^{-8}	0.33×10^{-13}
R^2	0.699	...	...
σ	$0\farcs131$	...	...

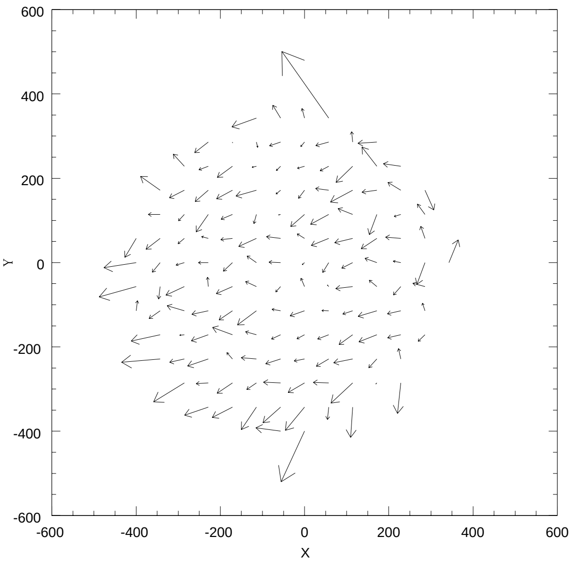

Fig. 8. Idem Fig. 6 constructed after correction of measured rectangular coordinates.

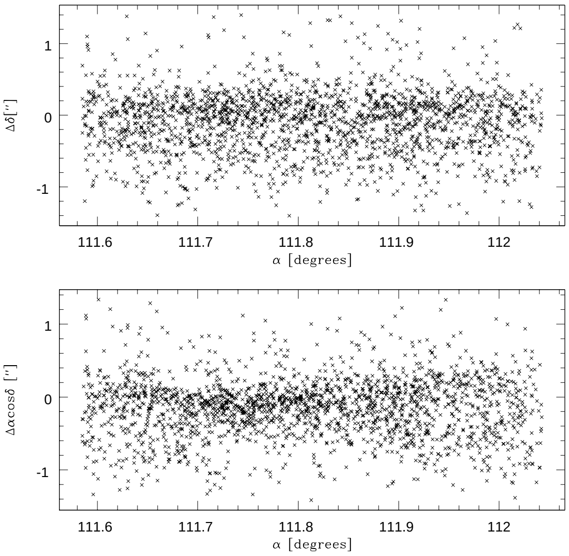

Fig. 9. Idem Fig. 4 constructed after correction of measured rectangular coordinates.

systematic effect observed in Fig. 11 could be reduced by some ad hoc treatment of the data. We attempted modeling and further subtraction from the coordinates α and δ in the CCD catalog, but unlike in § 7.1, there is no theoretical model to explain the observed behavior, and the subtraction was not satisfactory enough over the whole field.

Given the lack of a simple mathematical model, we adopted the interpolation scheme of Stock & Abad (1988), which makes use of a bidimensional polynomial and a weighting function dependent on the distance to the interpolating point $w(r) = \{[1 - (r/r_0)^2]^{1/2}\}^n$. The polynomial was chosen to be constant for every star, the interpolation radius was $r_0 = 2'$, and the exponent $n = 1$, which assures the continuity of the interpolating function and its derivative.

8.1. *Correction of Celestial Coordinates*

The differences with USNO-A2 were employed to correct the celestial coordinates (α, δ) of every star in the CCD catalog. Differences between these corrected coordinates and the ones in USNO-A2 have the following mean values and dispersions

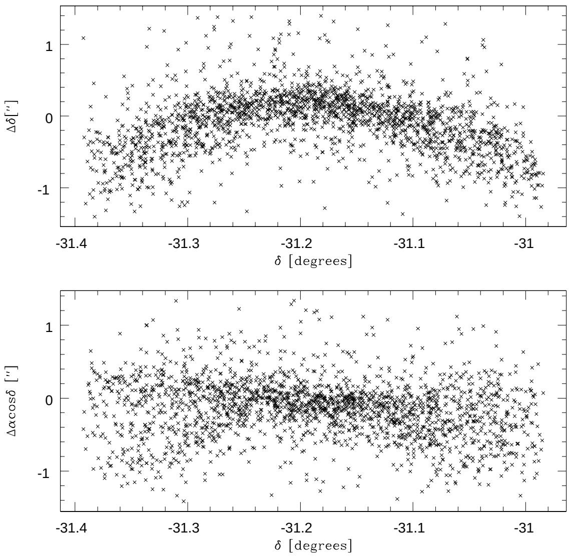

Fig. 10. Idem Fig. 5 constructed after correction of measured rectangular coordinates.

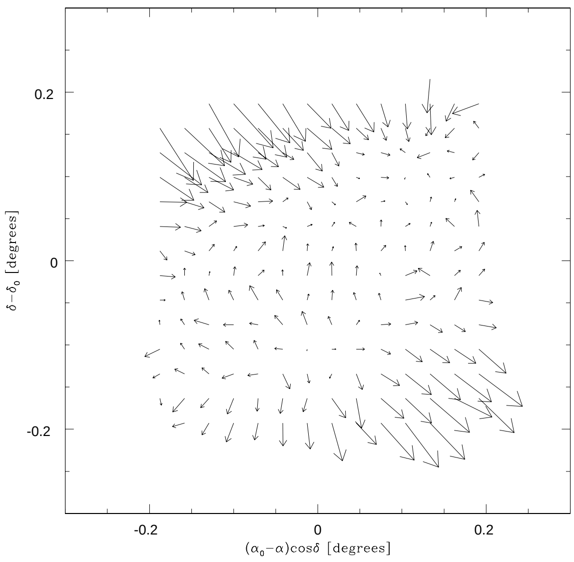

Fig. 11. Differences between CCD-based positions and USNO-A2 averaged within a grid of square cells 1′.7 on a side. (α, δ) are coordinates of grid cells and (α_0, δ_0) is the approximate center of the field. Vectors have been enlarged 180 times, so the biggest ones represent approximately 1″.5.

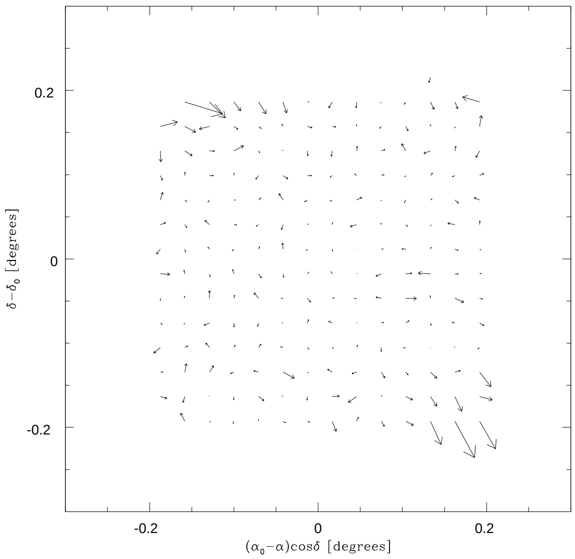

Fig. 12. Idem Fig. 11 constructed after correction of celestial coordinates in the CCD catalog.

$$\overline{\Delta\alpha_\star} = (-0.02 \pm 0.32)'', \quad \overline{\Delta\delta} = (-0.03 \pm 0.33)'' .$$

The vector diagram in Fig. 11 was rebuilt with the corrected CCD catalog, resulting in the one shown in Figure 12, which shows an almost complete elimination of the systematic effect.

The comparison with Tycho-2 was again performed. After all the corrections, the new calculated differences are

$$\overline{\Delta\alpha_\star} = (0.018 \pm 0.110)'', \quad \overline{\Delta\delta} = (-0.029 \pm 0.145)'' .$$

9. CONCLUSIONS

The Tycho-2 Catalogue provided enough reference stars for measurement of the $25' \times 25'$ field around the center of the galactic cluster Rup 21 using the block-adjustment technique. The estimation of the internal error of the measured positions prior to any correction is $\sigma_{\alpha_\star,\delta} \approx 0''.25$.

The only estimation of the external error of measured positions is provided by comparison with USNO-A2. This comparison shows that this error would amount approximately 0″.5 and that important systematic effects are present, but no effects of color or magnitud on the residuals are observed, at least not above 0″.5.

Systematic effects, presumably due to field curvature introduced by the focal reducer, can be successfully modeled and corrected. After such correction, the differences with USNO-A2 reduce their dispersion to $0\rlap{.}''43$, but they show a systematic behavior in the whole field.

A sliding constant polynomial interpolating within a radius of $2'$ with a suitable weighting function is able to correct the systematic effect observed over the entire field. After application of these corrections to positions in the CCD catalog, the dispersion of differences with USNO-A2 is reduced to $0\rlap{.}''33$ in the entire field.

Because of the lack of proper motions in USNO-A2, the error in their positions is estimated to amount to $0\rlap{.}''30$ for the epoch of the CCD observations, therefore, dispersions of around $0\rlap{.}''33$ found in differences between measured positions and the ones given by USNO-A2 indicate that the error of positions in the CCD catalog is $\sigma_{\alpha,\delta} \approx 0\rlap{.}''15$, in good agreement with the last comparison with Tycho-2.

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