Revista Mexicana de Astronomía y Astrofísica, **38**, 193–198 (2002)

EFFECT OF SCINTILLATION ON ADAPTIVE OPTICS SYSTEMS

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Received 2002 March 18; accepted 2002 July 25

RESUMEN

En los sistemas de corrección por medio de óptica adaptativa utilizados en aplicaciones astronómicas, las medidas del centroide de una imagen se utilizan ampliamente. En observaciones reales, estas medidas son afectadas por el centelleo, por lo que la reconstrucción del frente de onda contiene errores debido al efecto del centelleo. En este artículo, investigamos la influencia del centelleo en la determinación del centroide de una imagen por medio de simulaciones numéricas. La comparación de resultados para los casos de propagación vertical y horizontal muestra que no hay una fuerte dependencia de la estructura detallada del perfil de C_n^2. Mostramos resultados de esta dependencia a través de dos parámetros integrados del C_n^2: la intensidad de la turbulencia (parámetro de Fried) y el nivel del centelleo (varianza del logaritmo de la amplitud).

ABSTRACT

Image centroid measurements are the most used information in astronomical adaptive optics systems for the wavefront reconstruction. However, because in real observations these measurements are affected by scintillations, the reconstructed wavefront always contains some errors related to the scintillation effect. In this paper we investigate the influence of scintillations on the image centroid by means of computer simulations. The simulations have been performed for the case of weak-turbulence conditions for both varying and constant C_n^2 profile. The comparison of the results shows that there is no strong dependence on the form of the C_n^2 profile: rather the magnitude of the effect is determined mainly by two integral parameters of a C_n^2 profile: the integral turbulence strength (Fried parameter) and the scintillation level (intensity variance).

Key Words: **ATMOSPHERIC EFFECTS — INSTRUMENTATION: ADAPTIVE OPTICS — METHODS: NUMERICAL**

1. INTRODUCTION

Image centroid measurements are often used in Hartmann-like wavefront sensors for a reconstruction of turbulence-induced phase distortions (Voitsekhovich, Gubin, & Mikulich 1988; Rigaut et al. 1991; Jiang et al. 1993; Li, Xian, & Jiang 1993; Colucci et al. 1994; Rigaut, Ellerbroek, & Northcott 1997; Voitsekhovich 1996). This approach is applied to both fundamental research on atmospheric turbulence and for practical purposes (such as atmospheric adaptive optics) as well. The method is quite popular in applications because it provides a direct and simple relation between the measurements and phase gradients: it is assumed that the phase gradient averaged over the subaperture of a Hartmann mask is proportional to the corresponding image centroid offset. However, this simple relationship is only valid in the limit in which the effect of amplitude fluctuations (scintillations) is not important. It is widely accepted (Roddier 1981) that, under the weak-turbulence conditions, the effect of scintillations on the image centroid is negligible but this assumption has never been supported by quantitative calculations.

In this paper we calculate the magnitude of the scintillation effect using computer simulations. The simulations are based on the recently proposed method of random wave vectors (RWV) (Kouznetsov, Voitsekhovich, & Ortega-Martínez 1997; Voitsekhovich et al. 1999) that allows us to simulate the amplitude and phase samples with the desired statistics and cross-statistics.

2. DEFINITION OF THE SCINTILLATION-INDUCED ERROR.

Let the wave $\Psi(\rho)$ pass through a thin lens of diameter d and focal length f. The image centroid ρ_c of the image formed by this wave at the lens focal plane can be written as (Tatarski 1969):

$$\boldsymbol{\rho}_{\mathrm{c}} = \{x_{\mathrm{c}}, y_{\mathrm{c}}\} = -\frac{f}{k} \frac{\int_{G_d} d^2\boldsymbol{\rho} \, \exp\{2\chi(\boldsymbol{\rho})\} \, \boldsymbol{\nabla} S(\boldsymbol{\rho})}{\int_{G_d} d^2\boldsymbol{\rho} \, \exp\{2\chi(\rho)\}}, \tag{1}$$

where x_c, y_c denote the Cartesian coordinates of the image centroid, $\chi(\boldsymbol{\rho})$ and $S(\boldsymbol{\rho})$ are the log-amplitude and the phase of the wave Ψ, respectively, k is the wavenumber, and G_d denotes the integration over the lens aperture.

However, in experiments related to phase reconstruction from centroid measurements (for example, Hartmann-like wavefront sensors) it is always assumed that the effect of amplitude fluctuations on the image centroid is negligible. Mathematically this assumption can be written as

$$\boldsymbol{\rho}_{\mathrm{c}}^{'} = \left\{x_{\mathrm{c}}^{'}, y_{\mathrm{c}}^{'}\right\} = -\frac{f}{k\Sigma} \int_{G_d} d^2\boldsymbol{\rho} \, \boldsymbol{\nabla} S(\boldsymbol{\rho}), \tag{2}$$

where Σ denotes the lens area.

Since in the problems related to propagation through the atmospheric turbulence the quantities $\boldsymbol{\rho}_c$ and $\boldsymbol{\rho}_c^{'}$ are random, we can define the relative error σ of image centroid measurements associated with scintillations as

$$\sigma = \frac{1}{2} \left[\frac{\sqrt{\left\langle (x_{\mathrm{c}} - x_{\mathrm{c}}^{'})^2 \right\rangle}}{\sqrt{\langle x_{\mathrm{c}}^2 \rangle}} + \frac{\sqrt{\left\langle (y_{\mathrm{c}} - y_{\mathrm{c}}^{'})^2 \right\rangle}}{\sqrt{\langle y_{\mathrm{c}}^2 \rangle}} \right]. \tag{3}$$

From the physical point of view, the error σ shows how big is the relative contribution of the scintillations to the image centroid offset. In what follows we calculate this error as a function of the turbulence conditions and lens size by means of computer simulations.

3. SIMULATION METHOD

In this paper, we restrict our attention to the propagation of the initially plane wave through weak turbulence. Under these conditions we can use the method of random wave vectors (Kouznetsov et al. 1997) (RWV) that allows us to simulate the phase and log-amplitude samples with desired statistics and cross-statistics. We present below only a short description of RWV method, while the corresponding detailed derivations can be found in Kouznetsov et al. (1997).

The phase $S(\boldsymbol{\rho})$ and log-amplitude $\chi(\boldsymbol{\rho})$ samples at the aperture are simulated as

$$\begin{aligned} S(\boldsymbol{\rho}) &= \sum_{m=1}^{M} F(p_m) \cos(\mathbf{p}_m \cdot \boldsymbol{\rho} + \varphi_m), \\ \chi(\boldsymbol{\rho}) &= \sum_{m=1}^{M} G(p_m) \cos(\mathbf{p}_m \cdot \boldsymbol{\rho} + \varphi_m + \psi_m), \end{aligned} \tag{4}$$

where $\boldsymbol{\rho}$ is the 2-D position vector at the aperture plane, M denotes the number of harmonics used in the simulation, $\mathbf{p}_m$ is the 2-D random wave vector, p_m is the modulus of the vector $\mathbf{p}_m$.

The following statistical restrictions are imposed on the parameters in equation (4).

(i) The moduli p_m and the orientations θ_m of the vectors $\mathbf{p}_m$, and the quantities φ_m and ψ_m, are considered to be statistically independent quantities.

(ii) θ_m and φ_m are distributed uniformly inside the range $[-\pi, \pi]$.

With the conditions (i) and (ii) in hand, equation (4) allows a direct physical interpretation. The statistical independence of the φ_m and their uniform distribution inside the range $[-\pi, \pi]$ restrict the consideration to homogeneous processes only. Then by imposing a random uniform distribution inside the range $[-\pi, \pi]$ for the orientations θ_m of the vectors $\mathbf{p}_m$ only the isotropic processes are chosen from the class of homogenous ones. So, with these two restrictions, we simulate S and χ as isotropic random functions that correspond to the existing theory of atmospheric turbulence.

Furthermore, we choose the joint probability density function of the $\mathbf{p}_m$, $F(p_m)$, $G(p_m)$, φ_m and ψ_m so as to make the simulated spectra W^s_S, W^s_χ, and cross-spectrum $W^s_{\chi S}$ coincident with the corresponding theoretical ones W_S, W_χ, and $W_{\chi S}$.

Using the above conditions, one can express the quantities in equation (4) in terms of theoretical spectra as follows [see Kouznetsov et al. (1997) for detailed derivations]:

The amplitudes F and G

$$F(p) = \sqrt{\frac{W_S(p)}{\pi M \Omega(p)}}, \qquad G(p) = \sqrt{\frac{W_\chi(p)}{\pi M \Omega(p)}}. \tag{5}$$

We note that there is a misprint in equation (11) of Kouznetsov et al. (1997): the factor 2 has to be removed from the denominators in expressions for the functions F and G.

The probability density function (PDF) $\Omega(p)$ is:

$$\Omega(p) = 1/\left(2\pi p^2 \log(K_2/K_1)\right), \tag{6}$$

where $[K_1, K_2]$ is the interval within which the p are generated.

The joint PDF $\eta(\psi, p)$ of ψ and p is:

$$\eta(\psi, p) = \frac{1}{2\sqrt{\pi\alpha(p)}} \exp\left(-\frac{\psi^2}{4\alpha(p)}\right), \qquad \alpha(p) = \log\frac{\sqrt{W_S(p)\, W_\chi(p)}}{W_{S\chi}(p)}. \tag{7}$$

For the weak-turbulence conditions the theoretical spectra follow from the Rytov solution of the parabolic equation as (Tatarski 1969):

$$\begin{aligned} W_S(p) &= 0.651\Phi_n(p) \int dz\, C_n^2(z) \left[1 + \cos\left(\frac{z}{k}p^2\right)\right] \\ W_\chi(p) &= 0.651\Phi_n(p) \int dz\, C_n^2(z) \left[1 - \cos\left(\frac{z}{k}p^2\right)\right] \\ W_{\chi S}(p) &= 0.651\Phi_n(p) \int dz\, C_n^2(z) \sin\left(\frac{z}{k}p^2\right), \end{aligned} \tag{8}$$

where Φ_n is the refractive-index spectrum, $C_n^2(z)$ the profile of the refractive-index structure constant, and the integration is performed along the propagation path.

The step-by-step description of the simulation procedure can be found in Kouznetsov et al. (1997)

4. SIMULATION RESULTS

Using equations 1 to 3 and the samples simulated by the RWV method, we estimate the relative error σ defined by equation (3). The present simulation is performed for the case of Kolmogorov turbulence $\Phi_n(p) = p^{-11/3}$. The main simulation parameters are chosen as follows: the frequency limits: $K_1 = 10^{-3}$ m^{-1}, $K_2 = 10^3$ m^{-1}, the number of harmonics: $M = 100$.

The frequency limits have been chosen after a set of preliminary simulations which has shown that a subsequent decrease of K_1 and increase of K_2 does not significantly affect the final results. The number of

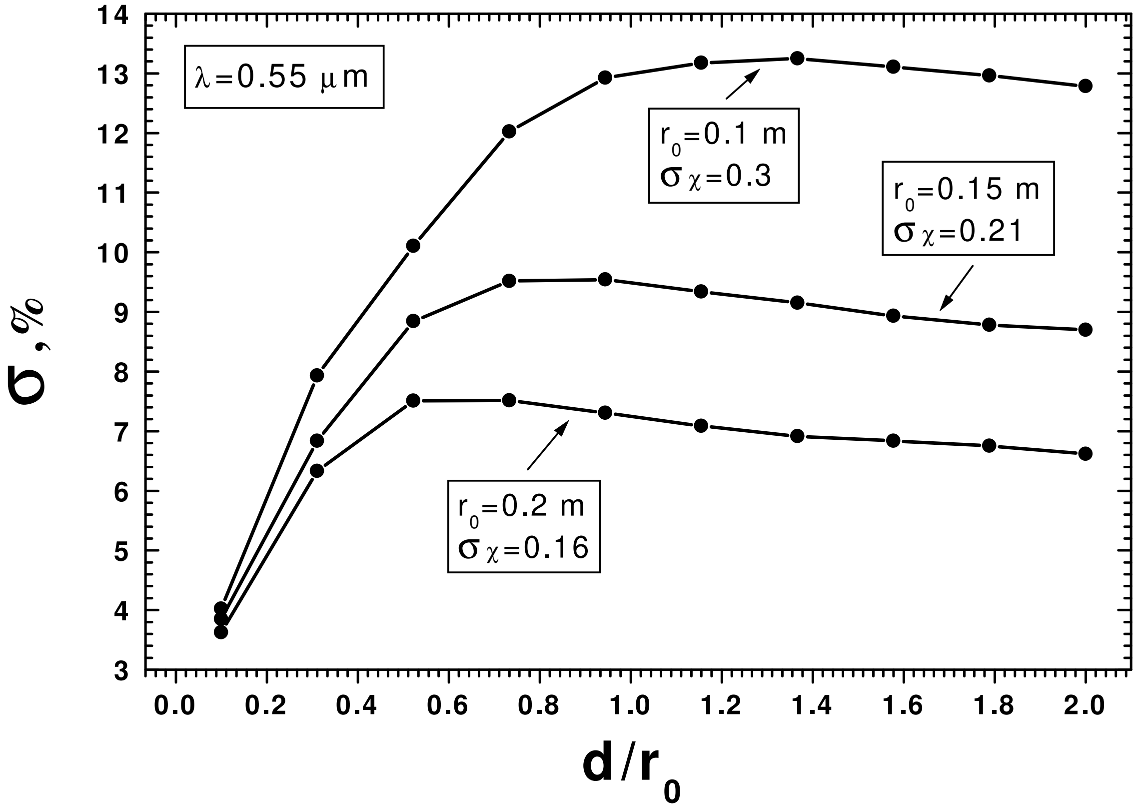

Fig. 1. Relative error σ versus the ratio of lens diameter d to the Fried parameter r_0. Vertical propagation (Hufnagel C_n^2 profile).

harmonics has been chosen in the same way. The number of samples used for the statistical averaging in equation (3) is equal to 1000. The simulations are performed for two propagation cases which are of interest for applications: varying C_n^2 (vertical path) and constant C_n^2 (horizontal path).

For the calculations with varying C_n^2 we use the Hufnagel model of $C_n^2(z)$ that is given by Hufnagel (1974):

$$C_n^2(z) = C_0 r_0^{-5/3} k^{-2} \left[\left(\frac{z}{z_0} \right)^{10} \exp\left(-\frac{z}{z_1} \right) + \exp\left(-\frac{z}{z_2} \right) \right], \tag{9}$$

where r_0 is the Fried parameter, k is the wavenumber, $C_0 = 1.027 \times 10^{-3}$ m^{-1}, $z_0 = 4.632 \times 10^3$ m, $z_1 = 10^3$ m, $z_2 = 1.5 \times 10^3$ m.

The expressions for the spectra W_S, W_χ, and $W_{\chi S}$ corresponding to this model can be found in Kouznetsov et al. (1997). The simulation results are shown in Figure 1, where the relative error is plotted versus the ratio of lens diameter d to the Fried parameter r_0. Additionally, we indicate in the graph the magnitudes of the wavelength λ, the Fried parameter r_0 and the standard of log-amplitude σ_χ.

For horizontal propagation, $C_n^2(z)$ is constant, i.e., $C_n^2(z) = C_n^2$. The theoretical spectra are expressed as

$$\begin{aligned} W_S(p) &= 1.544\, r_0^{-5/3} p^{-11/3} \left[1 + \frac{k}{p^2 L} \sin\left(\frac{p^2 L}{k} \right) \right] \\ W_\chi(p) &= 1.544\, r_0^{-5/3} p^{-11/3} \left[1 - \frac{k}{p^2 L} \sin\left(\frac{p^2 L}{k} \right) \right] \\ W_{\chi S}(p) &= 1.544\, r_0^{-5/3} p^{-11/3} \frac{k}{p^2 L} \left[1 - \cos\left(\frac{p^2 L}{k} \right) \right], \end{aligned} \tag{10}$$

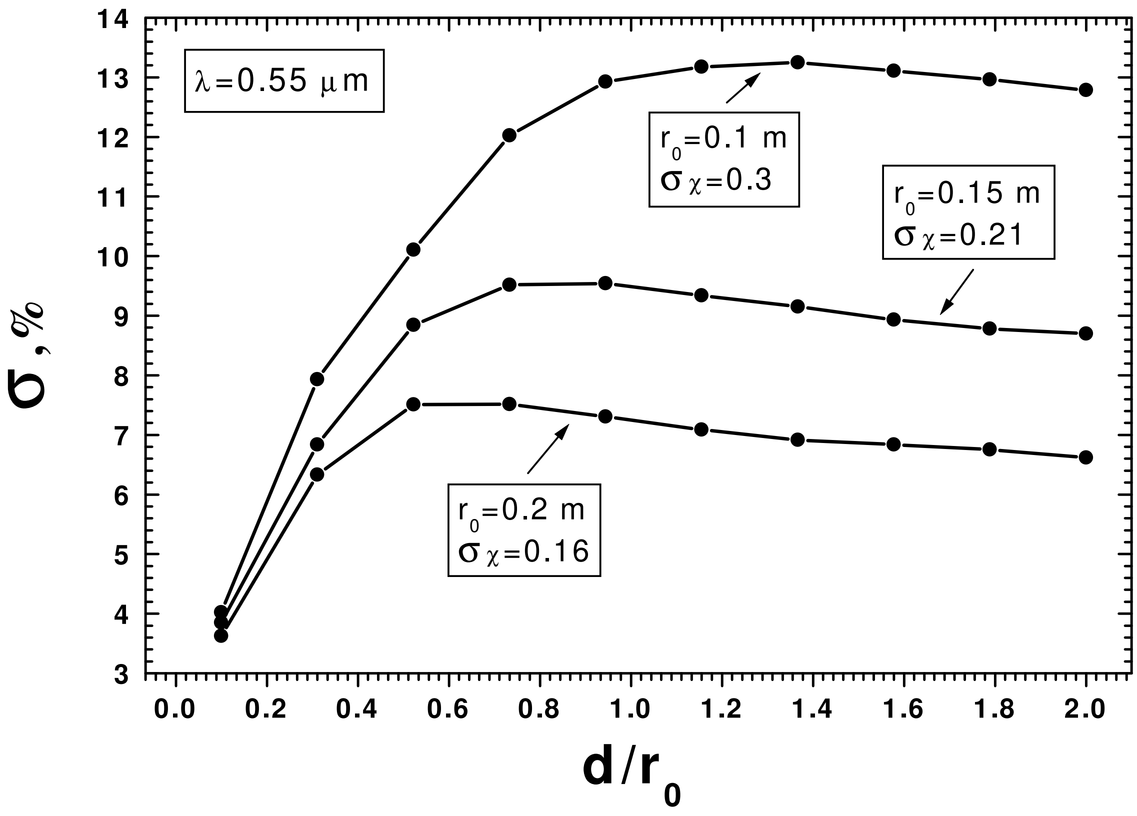

Fig. 2. Relative error σ versus the ratio of lens diameter d to the Fried parameter r_0. Horizontal propagation (constant C_n^2).

where L is the propagation length, and $r_0 = 1.68\left(k^2 C_n^2 L\right)^{-3/5}$.

The simulation results for the horizontal path are shown in Figure 2. As in the case of the vertical path, we plot the relative error versus the ratio of lens diameter to the Fried parameter. For the comparison purposes, we chose the propagation length so that it allows us to keep the same magnitudes of σ_χ as in the case of vertical propagation.

As one can see from Figs. 1 and 2, the error initially grows up to some maximum magnitude, and then it tends slowly to some asymptotic level. This behavior may have the following explanation. As it follows from equations (1–3), three different scales are involved in the problem: the amplitude correlation length, the phase gradient-amplitude cross-correlation length, and the phase gradient correlation length. Initially, when the aperture size is small compared to the correlation length of amplitude fluctuations, the main contribution to the error comes from the linear components of amplitude and phase gradient which give a nearly linear increase of the error. Then, when the aperture size becomes bigger that the amplitude correlation length, but still remains small compared to the phase gradient-amplitude cross-correlation length, the non-linear terms of the amplitude expansion start to contribute more and more to the error. In this part of the plot the error grows more slowly and in a non-linear way. The error reaches its maximum magnitude when the aperture size is of the order of the phase gradient-amplitude cross-correlation length. And finally, when the aperture size becomes bigger than the phase gradient-amplitude cross-correlation length, the main contribution to the error comes from the aperture zones which have sizes comparable to the phase gradient-amplitude cross-correlation length. In this part of the plot one can observe that the error tends slowly to its asymptotic value.

Comparing Fig. 1 and Fig. 2, one can notice that there is no big difference in the magnitude of the effect for the two propagation cases. This means that the magnitude of the effect does not strongly depend on the detailed structure of C_n^2 profile: rather this magnitude is determined mainly by two integral parameters of a C_n^2 profile: the integral turbulence strength (Fried parameter) and the scintillation level (log-amplitude variance).

5. CONCLUSIONS

We have estimated how scintillations affect measurements of the image centroid in an adaptive optics system. The results obtained show that, under weak-turbulence conditions, the magnitude of this effect does not exceed 15%. Weak-turbulence condition occur for the majority of conventional astronomical observations because outside of these conditions the turbulence affects the image quality so strongly that the extraction of astronomically-valuable information becomes practically impossible. According to the classical definition (Tatarski 1969), weak-turbulence conditions occur while the log-amplitude variance $\sigma_\chi^2 \lesssim 0.3$. However, based on recent investigations (Voitsekhovich, Kouznetsov, & Morozov 1998), it is also possible to define weak-turbulence conditions in another way. It has been shown (Voitsekhovich et al. 1998) that when weak-turbulence conditions are violated, a qualitatively different phenomenon appears: turbulence-induced phase dislocations. The phase dislocation is a special type of phase distortion where phase singularities with vortices occur. These singularities affect the observed image in a special way and they cannot be corrected by conventional adaptive optics methods. So, weak-turbulence conditions may be defined as the conditions under which the phase dislocations have not yet appeared (Voitsekhovich et al. 1998).

It has been shown in this paper that the magnitude of the scintillation effect on the image centroid increases with increasing turbulence strength. So, one can expect that under strong-turbulence conditions the influence of scintillations on the image centroid can be considerable and that, after a certain turbulence strength, it can make the phase reconstruction from image centroid measurements impossible.

The magnitude of the effect has been estimated for two propagation cases: the constant C_n^2 (horizontal propagation) and C_n^2 varying along a propagation path (vertical propagation). The comparison of the results has shown that there is some difference between two cases, but that it is rather small. Because the two considered C_n^2 profiles represent two limiting propagation cases, it is possible to conclude that the magnitude of the effect depends mainly on two integral parameters of a C_n^2 profile: the Fried parameter and the log-amplitude variance, rather than on the detailed structure of the C_n^2 profile.

This work has been partially supported by the grant number 34517-E from CONACyT (México).

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