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A SIMPLE METHOD TO COMPENSATE SOFTENING EFFECTS IN N-BODY SIMULATIONS OF SPHERICAL STELLAR SYSTEMS

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RESUMEN

La simulación de sistemas estelares requiere, primero, crear un modelo y, luego, seguir su evolución mediante la integración numérica de las ecuaciones del movimiento. Los modelos pueden generarse a partir de distribuciones teóricas obtenidas del potencial Newtoniano, pero la integración requiere recurrir a potenciales ablandados para reducir los efectos de relajamiento. Habitualmente, la diferencia entre ambos potenciales sólo causa alteraciones insignificantes en el modelo, pero en ciertos casos (por ejemplo, distribuciones muy concentradas) puede alterarlos en forma substancial. Aquí presentamos un método simple, independiente del modelo, que corrige este problema en simulaciones de sistemas estelares esféricos.

ABSTRACT

Simulations of stellar systems involve, first, creating a model and, subsequently, following its evolution through numerical integration of the equations of motion. The models can be generated from theoretical distributions obtained from the Newtonian potential, but the integration demands resorting to softened potentials in order to reduce relaxation effects. Usually, the difference between both potentials causes only negligible alterations in the model, but in some cases (e.g., cuspy distributions) it can substantially alter it. Here we present a model-independent simple method to correct this problem in simulations of spherical stellar systems.

Key Words: **GALAXIES: SPHERICAL — GLOBULAR CLUSTERS — METHODS: N-BODY**

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1. INTRODUCTION

Numerical simulations are a powerful tool to understand the dynamics of stellar systems. The usual procedure is to begin by generating an N-body distribution that represents the system under study (which, of course, may include subsystems, as in a cluster of galaxies), then to follow its dynamical evolution through the integration of the equations of motion and, finally, to analyze the results.

Except in special cases (e.g., open clusters), the number of bodies that enter in the simulation is much smaller than the number of stars in the true stellar system (10^5 to 10^7 times smaller when simulating a galaxy) and, as a result, collisional relaxation effects in the model would be much larger than in the real world [see, e.g., Binney & Tremaine (1987)]. Softened potentials that mimic the Newtonian potential at long distances but drastically reduce the short-distance forces are thus used in the simulations to increase the relaxation time and to reduce those effects [see, e.g., Huang, Dubinsky, & Carlberg (1993)]. Initial conditions for N-body simulations are generated from theoretical equilibrium models obtained using the Newtonian potential so that, when they are placed in the softened potential used for the integrations, they are no longer in equilibrium. The departure from equilibrium is usually small, however, so that it is enough to let the system evolve in isolation for a few crossing times to reach a new equilibrium very similar to the theoretical one [see, e.g., Vergne & Muzzio (1995)].

Nevertheless, in the course of an investigation we are doing on cuspy stellar systems, the simple procedure just described failed to produce acceptable models: models generated from the (Newton-based) theoretical distributions strongly reduced their cen-

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tral cusps after they were let to evolve for a few crossing times in the softened potential.

What to do? The analytical derivation of the distribution function for a spherical stellar system [see, e.g., Dehnen (1993)] usually starts by choosing a density distribution whose gravitational potential is subsequently computed using Poisson's equation and, finally, the distribution function is obtained via Eddington's (1916) formula. Nevertheless, when a softened potential is used, Poisson's equation is lost because one no longer has the Dirac delta function that makes it possible to relate the potential at a certain point with the mass density *at that same point*. Softening introduces contributions to the potential from points other than the point where the potential is evaluated and one ends up with an integro-differential equation much more difficult to deal with than Poisson's equation. Numerical solutions are always possible, but they are less simple to use than analytical formulae and their obtention may be complicated by the presence of singularities that, although integrable, cause numerical problems. Besides, this approach demands a special derivation, either analytical or numerical, for each distribution of interest. Finally, as softening only partially compensates relaxation effects and essentially all N-body codes are collisional [see, e.g., Hernquist & Barnes (1990)], some artificial evolution will always be present, so that a theoretically exact solution is not mandatory: an approximate one that preserves the shape of the initial distribution is all that is needed.

In the present paper, we present a simple method to solve the problem in the case of spherical systems which, together with disks, are the ones whose theoretical distribution functions are usually known. Besides its simplicity, the method has the advantage of being model-independent, so that the same routine can be used for different distribution functions.

2. THEORY

As the cusps we were trying to model in our investigation of cuspy stellar systems became flatter in the softened potential, we reasoned that the velocities of the bodies derived from the Newtonian potential were too large for the softened potential and thought that reducing them might offer the solution to our problem. Thus, we decided to start generating the N-body distributions from the theoretical (Newtonian-based) models and, thereafter, correcting the velocities of the bodies in some systematic way.

Our first idea was to use the escape velocity: we computed the escape velocity for every body using separately the theoretical and the softened potential, and then we corrected all the velocities multiplying them by the ratio of softened to Newtonian escape velocities. Nevertheless, that simple approach failed to yield acceptable results. In the example of Figure 1 (see description below), after a $10T_{\rm cr}$ evolution, the radius that contained the innermost 1% of the mass increased by 220% in the case of the original distribution and by 190% in that of the escape velocity-corrected distribution; an improvement, indeed, but a very meager one. Thus, we decided to use instead the velocity dispersion to make the correction.

The Jeans equation for spherical stellar systems in equilibrium, with isotropic velocity distribution and no systematic motions is:

$$\frac{d(\rho(r)\sigma^2)}{dr} = -\rho\Big(\frac{d\Phi}{dr}\Big), \tag{1}$$

where ρ is the mass density, σ the one-dimensional velocity dispersion, r the radius and Φ the potential.

Since ρ and σ are zero when $r \to \infty$, we can integrate from ∞ to r and obtain:

$$\rho(r)\sigma^2(r) = -\int_\infty^{\rm r} \rho(r)\frac{d\Phi}{dr}dr. \tag{2}$$

Let us apply now this equation to a system of N particles of masses $m_{\rm i}$ at distances $r_{\rm i}$ from the center of the system, numbered so that $r_{\rm i}$ increases as i increases. If G is the constant of gravitation, the force can be approximated simply by the monopole as:

$$\Big(\frac{d\Phi}{dr}\Big)_{\rm r_j} = \frac{G}{r_{\rm j}^2}\sum_{i<j} m_{\rm i}, \tag{3}$$

for the Newtonian potential, and as:

$$\Big(\frac{d\Phi}{dr}\Big)_{\rm r_j} = \frac{Gr_{\rm j}}{(r_{\rm j}^2+\epsilon^2)^{3/2}}\sum_{i<j} m_{\rm i}, \tag{4}$$

for the softened potential. Any additional force (say, one due to a central black hole) should be added to the two previous expresions but, of course, it will be the same for both of them. Although one might use the true forces, rather than their monopole approximations, the latter work exceedingly well in the case of spherical systems modeled with the large numbers of particles that modern computers allow us to use, and pose no limitation, as we will show below.

Expressing the integral as a summation, we finally have:

$$4\pi\rho(r_{\rm i})\sigma^2(r_{\rm i}) = \sum_{\rm j>i}\frac{m_{\rm j}}{r_{\rm j}^2}\Big(\frac{d\Phi}{dr}\Big)_{\rm r_j}. \tag{5}$$

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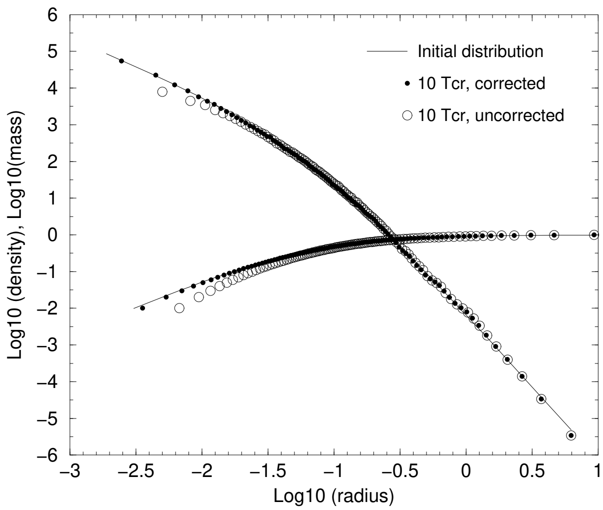

Fig. 1. Density (decreasing curve) and mass (increasing curve) profiles for a Dehnen model as is generated and after letting it evolve $10T_{\rm cr}$; the softening parameter was adopted as $\epsilon = 0.010$ in this case.

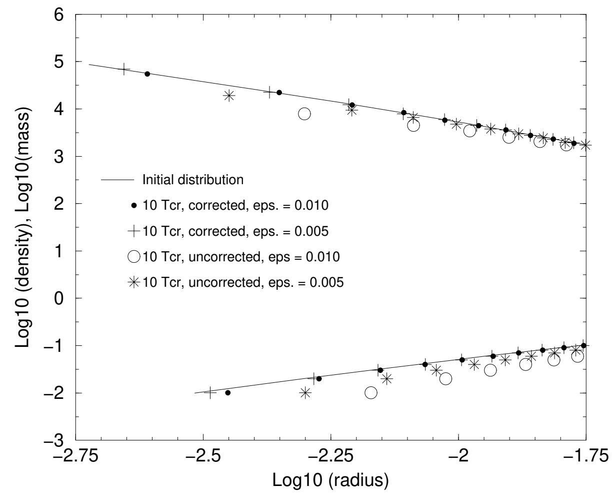

Fig. 2. Same as Fig. 1 but showing the innermost region only and showing two models with different softening.

The idea is to follow now the subsequent steps: (1) Generate the N-body distribution from the theoretical model derived from the Newtonian potential; (2) Compute the right-hand side of the previous equation for each one of the N particles twice, once using the softened potential and once using the Newtonian potential; (3) For every particle compute the ratio between the results obtained with each potential which, as π is constant and $\rho(r_{\rm i})$ is the same in both cases, gives the ratio of the corresponding velocity variances; (4) Multiply the velocity components of every particle by the square root of the corresponding ratio and take these values as the new velocities for the distribution.

3. RESULTS

As an example, we present here the results obtained for a Dehnen (1993) distribution with $\gamma = 1.5$ and $a = 0.0625$. The integration was performed with the code of L. A. Aguilar who uses an octupolar expansion of the potential [see Aguilar & Merritt (1990)]. Therefore, we generated the distribution with $250,000$ particles of equal mass and softening parameter plus one particle 25 times more massive than the others and with a softening parameter 5 times larger than that of the others, which is taken as the center of the expansion. The total mass was taken as unity and the distribution was truncated at a maximum radius of 9.30, which is the radius that includes 99% of the mass of the system. With $G = 1$, the crossing time of the system is $T_{\rm cr} = 1.275$.

Fig. 1 presents the results for a case where the softening parameter of the less massive particles was taken as $\epsilon = 0.01$. Both the density and the mass inside a certain radius are shown: they were computed sorting the particles in bins of $2,500$ particles each (i.e., about 1% of the total mass); the first bin includes also the more massive particle, so that it comprises $2,501$ particles, and its mass is slightly larger than that of the other bins. The radii used for the plot are those of the $1250th$ particle of the bin for the density, and of the last particle of the bin for the mass. Poissonian statistical errors amount to 2% (standard deviation) so that errors in the abcisae are 0.02 only, i.e., smaller than the size of the symbols used for the figure. Due to the small errors, the initial distribution coincides almost exactly with the theoretical one; it is shown as a full line, and its left tip corresponds to the radius that encloses the innermost $2,501$ particles at the beginning. The cusp is clearly smoothed in the case of the uncorrected distribution, but the profiles are very well conserved in the corrected cases. We notice that, after evolving the system for 10 $T_{\rm cr}$, the radius that encloses the innermost 1% of the mass increased 220% in the case of the original distribution, but only 14% in the case of the corrected one.

Figure 2 presents a blow-up of the innermost region of Fig. 1 where we have also added the results for a second model with $\epsilon = 0.005$. As could be expected, after 10 $T_{\rm cr}$, for both the corrected and uncorrected distributions the departures from the initial distribution are smaller for the less-softened model. In the new model, the radius that contains

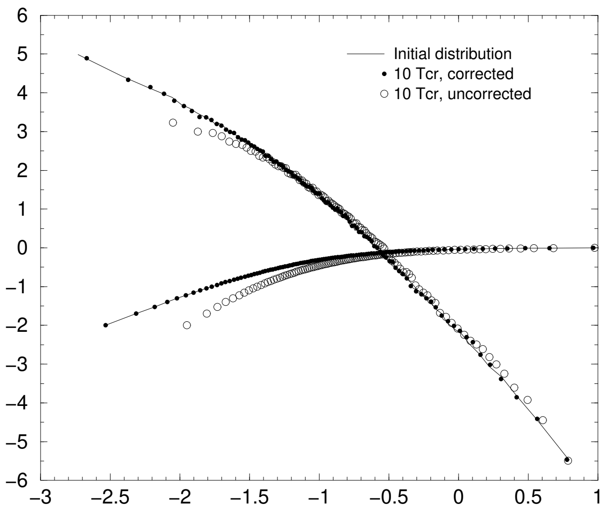

Fig. 3. Same as Fig. 1 but for a model of 25,000 particles with $\epsilon = 0.025$ evolved with the NBODY2 code.

the innermost 1% of the mass increased by 7% only in the corrected distribution, but by 165% in the uncorrected one.

In order to check whether our monopolar approximations (3) and (4) had worked well just because of the multipolar nature of Aguilar's code, we made an additional experiment using the NBODY2 code of Aarseth (1985) which uses the direct summation method. We used the same Dehnen distribution of our previous experiments, but in this case only 25,000 particles were included and the softening parameter was taken as $\epsilon = 0.025$. The results are shown in Figure 3, where 250 particle bins were used for the density and mass computations; the results are somewhat noisier due to the smaller number of particles included, but the compensation of the softening effects is clearly as good as in the previous examples.

4. CONCLUSION

We have presented a simple method to correct N-body distributions for the effects of softening and we showed, in particular, that the method is useful to preserve the central cusps of highly concentrated systems. Moreover, the method is model-independent and, once implemented in an N-body distribution generating code, it can be used for different distributions without modification.

Although we have used this method in systems with isotropic velocity distributions only, its extension to systems with anisotropic velocity distributions seems feasible. If $\sigma_{\rm r}$, σ_θ and σ_ϕ are the velocity dispersions in an spherical system of coordinates, assuming for a non-rotating spherical system that $\sigma_\theta = \sigma_\phi$, we can measure the anisotropy with the parameter:

$$\beta(r) = 1 - \frac{\sigma_\theta^2}{\sigma_{\rm r}^2}. \tag{6}$$

Then, from the theoretical phase-space density, one has to compute the function:

$$\zeta(r) = 2\int \frac{\beta(r)dr}{r}, \tag{7}$$

and, instead of equation (2), use the following one:

$$\rho(r)\sigma^2(r) = e^{-\zeta(\rm r)} \int_{\rm r}^{\infty} \rho(r')e^{\zeta(\rm r')}\frac{d\Phi}{dr'}dr', \tag{8}$$

except for that change, the other steps to follow would be the same.

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