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CCD PHOTOMETRY OF M92

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RESUMEN

Se presenta fotometría de Johnson en los filtros B y V para el cúmulo globular galáctico M92 (NGC 6341). Se obtuvieron resultados fotométricos para un total de $\sim$ 30,000 estrellas las cuales se grafican en un diagrama V versus $(B-V)$. Se ajustaron isócronas teóricas a este diagrama para obtener una estimación de la edad de M92. La edad que encontramos es $\sim 16 \times 10^9$ años con los siguientes valores para la metalicidad y la abundancia de Helio: [Fe/H] $= -2.03$, $Y = 0.235$. El módulo de distancia al cúmulo resulta ser de $m - M = 14.6$ de acuerdo con el encontrado por Stetson & Harris (1988). También llevamos a cabo conteos estelares para producir una función de luminosidad, la cual se ajusta en forma exitosa con los modelos teóricos usados para el diagrama color-magnitud.

ABSTRACT

We present Johnson B and V photometry for the galactic globular cluster M92 (NGC 6341). Photometric results for a total of $\sim$ 30,000 stars are obtained and are plotted on a V versus $(B-V)$ diagram. We fit theoretical isochrones to this diagram in order to get an estimate for the age of M92. The age which we find is $\sim 16 \times 10^9$ years with the following values for the metallicity and He-abundance: [Fe/H] $= -2.03$, $Y = 0.235$. The distance modulus to this cluster turns out to be $m - M = 14.6$ in accordance with that obtained by Stetson & Harris (1988). We also perform stellar counts in order to produce a luminosity function which is successfully fitted by the same theoretical models fitted to the colour-magnitude diagram.

Key Words: **GALAXY: HALO — GLOBULAR CLUSTERS: INDIVIDUAL (NGC 6341) — TECHNIQUES: PHOTOMETRIC**

1. INTRODUCTION

The study of globular clusters is very important due to a variety of reasons, such as the fact that they play an outstanding role in the establishment of galactic structure; they are also of vital importance in evolutionary studies of low mass, low-metallicity stars and in the dynamics of stellar systems in general. They may also be used in order to investigate the chemical evolution of galactic systems. For further information see Harris & Racine (1979), Freeman & Norris (1981), and VandenBerg, Stetson, & Bolte (1996).

It is believed that as remnants of an epoch of primordial stellar formation, globular clusters are present in all reasonably large galaxies and that they conform themselves as a galactic subsystem, which in general is part of the subsystem known as Galactic Halo.

In this paper we present a detailed study of the globular cluster M92, also known as NGC 6341 or C1715+432. This cluster is probably one of the oldest and most metal-poor globulars known so far. It therefore, provides us with the opportunity of studying very old and very metal-poor stars. In Stetson & Harris (1988) it is shown that M92 is older and closer to us than M15 which is considered to be the globular cluster most similar to M92 in our Galaxy. Because of its galactic position at $l = 68.4$ and $b = +34.9$, its light suffers little reddening; this, combined with its low metallicity results in an almost perfect fit of the theoretical models for old metal-poor stars (see Bolte & Hogan 1995) to the observations.

There have been several other photometric studies of M92. In particular, Johnson & Bolte (1998) give V and I photometry for this cluster. Grundahl et al. (2000) present Strömgren CCD photometry for M92 which they use in order to obtain a distance-independent determination of the age of M92 (≥ 16 Gyr). Andreuzzi et al. (2000) use *HST* (*Hubble Space Telescope*) observations down to $V \sim 27$ to obtain a luminosity function for this cluster. They find that the luminosity function becomes flatter towards the centre of the cluster, as it would be expected if M92 is a dynamically relaxed system. Furthermore, they show that segregation effects are at work in M92. We should also mention Peter Stetson's spectacular photometric standards project which provides a large number of standard stars, in several filters, for many different regions of the sky (see Stetson 2005). In particular for M92 we identify 3115 stars with values for B and V (for further details see below).

Lee et al. (2001) present *HST* IR (Infrared) observations of M92 through the $F110W$ and $F160W$ filters, which they use to show that the metal-poor inner halo cluster NGC 6287 appears to have essentially the same age (± 2 Gyr) as M92, which is one of the oldest clusters in our Galaxy.

Lee et al. (2003) present wide-field CCD photometry of M92 in the V and I bands. They obtain the luminosity function, the mass function, surface density profiles and surface number density maps for the cluster. They also find a change in the slope of the mass function between the inner ($5' \leq r \leq 9'$) and outer ($9' \leq r \leq 15'$) regions of the cluster, clearly indicating a mass segregation for the cluster. There is some evidence for a tidal tail of M92 oriented perpendicularly to the direction to the Galactic centre.

In § 2 we present our observations. Section 3 deals with our derived HR-diagram, § 4 talks about the luminosity function and § 5 presents our conclusions.

2. THE OBSERVATIONS

We have obtained photometric CCD observations of the globular cluster M92 (NGC 6341) in the B and V Johnson filters at the National Astronomical Observatory (OAN) at San Pedro Mártir, Baja California.

The data were obtained during one observing run in 1999 with a 1024×1024 CCD camera attached to the 1.5 m telescope with a plate scale of 0.3 arcsec per pixel. We tried to get the observations in exactly the same conditions as a set of photometric data for M92 taken in April 1995 by M. Bolte and E. Sandquist at the 0.9 m telescope at the Kitt Peak National Observatory. We intend to see whether we could reproduce the same results with our own telescope and CCD equipment.

The images were taken so that we could cover the entire region of the cluster out to a distance of $\sim 15'$ from the centre, both in RA and Dec.

We obtained 49 images whose centres are given in Table 1. All the observations were taken with the cluster near the meridian (± 2 hrs) so that the value for air mass was always pretty close to 1.0. So corrections for atmospheric extinction were almost unimportant. Table 1 gives the name of the image (Columns 1 and 4), and the RA (Columns 2 and 5) and Dec (Columns 3 and 6) of its centre. The size of each image is $307'' \times 307''$ and we cover a total field of $\sim 31' \times 31'$.

For each position, we took one 10 s exposure, three 60 s exposures, and one 300 s exposure for each filter, in order to attempt to see the centre of the cluster more clearly and also to get most of the stars towards the outer edge of the cluster. We also took twilight flats at the beginning and end of each observing night.

The reductions were carried out by means of IRAF and DAOPHOT II in a standard manner.

We compare our photometry with the standard photometry for M92 by Stetson & Harris (1988). In Figure 1 we present graphs for $\delta V = V_{\rm SH} - V_{\rm Ours}$ and $\delta B = B_{\rm SH} - B_{\rm Ours}$ versus $(B - V)$ and show a robust regression fit to the data.

In Figures 2 to 7 we present the comparison of our brighter photometric results with three old photographic photometry studies of M92; Sandage & Walker (1966) (SW), Sandage (1970) (SG), and Cathey (1974) (CY). As seen from the figures the colours and magnitude differences are never larger than 0.2 magnitudes. We also compare our photometry with that of Stetson (2005). Figure 10 and Figure 11 give the colour-magnitude diagrams for our data and for those of Stetson (2005). It is clear that both sets of data occupy the same locus over the HR-diagram, which implies that his photometry and ours are consistent with each other. Moreover, we have plotted a fiducial line obtained from his data over our fiducial line and that of Stetson & Harris (1988) (see Figure 9). They agree remarkably well, ensuring, once more, the consistency of our data with other data sets for M92. From all these results we conclude that our photometry is consistent with other studies over a very large time baseline.

TABLE 1

IMAGE POSITIONS

Image	RA (2000) (h, m, s)	Dec. (2000) (° ′ ″)	Image	RA (2000) (h m s)	Dec. (2000) (° ′ ″)
Im(−3,−3)	17 17 59.9	43 21 29.8	Im(1,−3)	17 17 59.9	43 04 25.8
Im(−3,−2)	17 17 42.8	43 21 29.7	Im(1,−2)	17 17 42.9	43 04 25.7
Im(−3,−1)	17 17 25.7	43 21 29.6	Im(1,−1)	17 17 25.8	43 04 25.6
Im(−3, 0)	17 17 08.6	43 21 29.5	Im(1, 0)	17 17 08.7	43 04 25.5
Im(−3, 1)	17 16 51.5	43 21 29.3	Im(1, 1)	17 16 51.6	43 04 25.3
Im(−3, 2)	17 16 34.4	43 21 29.2	Im(1, 2)	17 16 34.5	43 04 25.2
Im(−3, 3)	17 16 17.3	43 21 29.1	Im(1, 3)	17 16 17.4	43 04 25.1
Im(−2,−3)	17 17 59.9	43 17 13.8	Im(2,−3)	17 17 59.9	43 00 09.8
Im(−2,−2)	17 17 42.8	43 17 13.7	Im(2,−2)	17 17 42.9	43 00 09.7
Im(−2,−1)	17 17 25.7	43 17 13.6	Im(2,−1)	17 17 25.8	43 00 09.6
Im(−2, 0)	17 17 08.6	43 17 13.5	Im(2, 0)	17 17 08.7	43 00 09.5
Im(−2, 1)	17 16 51.5	43 17 13.3	Im(2, 1)	17 16 51.6	43 00 09.3
Im(−2, 2)	17 16 34.4	43 17 13.2	Im(2, 2)	17 16 34.5	43 00 09.2
Im(−2, 3)	17 16 17.3	43 17 13.1	Im(2, 3)	17 16 17.4	43 00 09.1
Im(−1,−3)	17 17 59.9	43 12 57.8	Im(3,−3)	17 18 00.0	42 55 53.8
Im(−1,−2)	17 17 42.9	43 12 57.7	Im(3,−2)	17 17 42.9	42 55 53.7
Im(−1,−1)	17 17 25.8	43 12 57.6	Im(3,−1)	17 17 25.8	42 55 53.6
Im(−1, 0)	17 17 08.7	43 12 57.5	Im(3, 0)	17 17 08.7	42 55 53.5
Im(−1, 1)	17 16 51.6	43 12 57.3	Im(−3, 1)	17 16 51.6	42 55 53.3
Im(−1, 2)	17 16 34.5	43 12 57.2	Im(−3, 2)	17 16 34.5	42 55 53.2
Im(−1, 3)	17 16 17.4	43 12 57.1	Im(−3, 3)	17 16 17.4	42 55 53.1
Im(0,−3)	17 17 59.9	43 08 41.8	...	...	...
Im(0,−2)	17 17 42.9	43 08 41.7	...	...	...
Im(0,−1)	17 17 25.8	43 08 41.6	...	...	...
Im(0, 0)	17 17 08.7	43 08 41.5	...	...	...
Im(0, 1)	17 16 51.6	43 08 41.3	...	...	...
Im(0, 2)	17 16 34.5	43 08 41.2	...	...	...
Im(0, 3)	17 16 17.4	43 08 41.1	...	...	...

3. THE HR DIAGRAM

From our reduced data we construct a colour-magnitude diagram for M92. One such diagram has the advantage of allowing a relatively easy direct comparison between the observations and the theory. We know that the colour-magnitude diagram for a globular cluster reveals the evolution that all the stars in the cluster have undergone since their birth, and this may be fitted to different theoretical models (see Hanes & Madore 1980; Bodenheimer 1996).

In Figure 8 we present the V versus $(B - V)$ diagram for more than 30,000 stars. This is a raw diagram because it contains cluster stars as well as foreground and background objects. It is interesting to point out that, in spite of the presence of alien objects, this diagram presents clearly all the features expected for a globular cluster: i.e., Main Sequence (MS), Horizontal Branch (HB), Giant Branch (GB), etc.

To eliminate alien objects we follow a statistical method which consists of three parts:

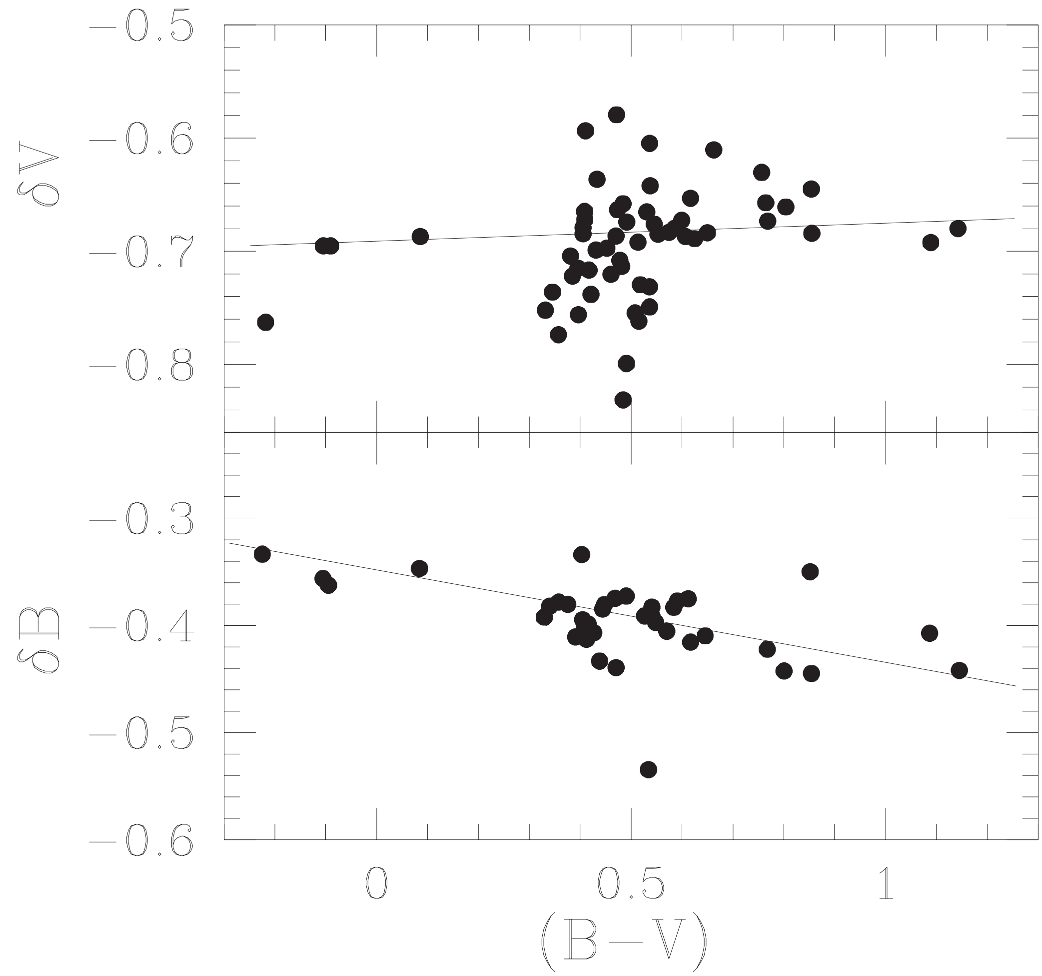

Fig. 1. Comparison of our photometric results with those of Stetson & Harris (1988)(SH). $\delta V = V_{\rm SH} - V_{\rm Ours}$, $\delta B = B_{\rm SH} - B_{\rm Ours}$.

(i) We assume a Gaussian distribution of colours at a fixed magnitude. We eliminate all the stars at more than 3σ from the colour index at which we have a maximum in the star count. We take a cut every 0.25 mag in V and find the $(B-V)$ colour at which there is a maximum number of stars. All those stars that lie at more than 3σ from this maximum are eliminated.

We took ~ 40 cuts which produced the maxima plotted on Fig. 9. The average value for σ for all these cuts turns out to be $\sigma = 0.04$; therefore, any star within a particular cut at more than 0.12 from the maximum is eliminated. Fig. 9 shows the points at which there is a maximum number of stars for each cut and also gives the Bolte (1996) and the Stetson (2005) fiducial lines. We see that all three loci coincide reasonably well.

(ii) We also use as a discrimination criterion the goodness of fit of the PSF (Point Spread Function) to a star. This goodness of fit is calculated by DAOPHOT and is expressed as the χ parameter. Numerical experiments indicate that any star with a χ value greater or equal to 1.5 should be eliminated. Following this criterion, we eliminate of the order of 600 stars.

(iii) We also eliminate objects with large photometric errors. We choose a maximum error in the $(B-V)$ colour of 0.03 magnitudes. We have, therefore, eliminated all objects with a $\sigma_{(B-V)} \geq 0.03$.

In Fig. 10 we present our clean colour-magnitude diagram. Out of the 30,000 plus stars in the raw diagram, there are 4000 plus stars in this clean diagram. The majority of the eliminated stars belong to the lower part of the diagram where most of the field and badly fitted objects congregate. The clean diagram, however, leaves intact most of the more evolved stars which delineate perfectly the GB and the HB of the cluster. We admit that our elimination

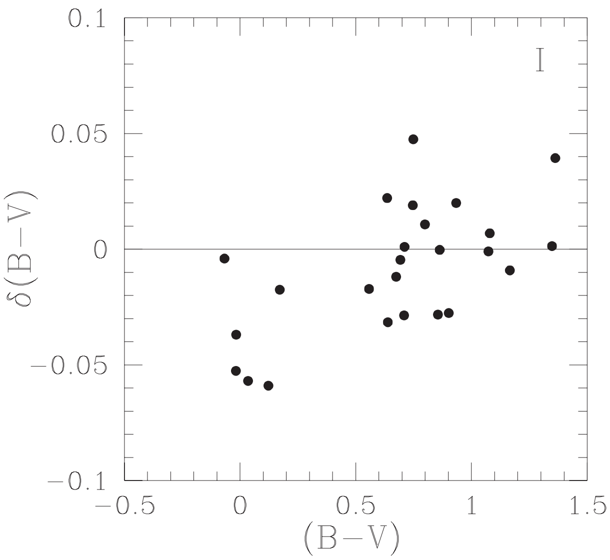

Fig. 2. Comparison of our photometric colour $(B-V)$ with the results of Sandage & Walker (1966) (I).

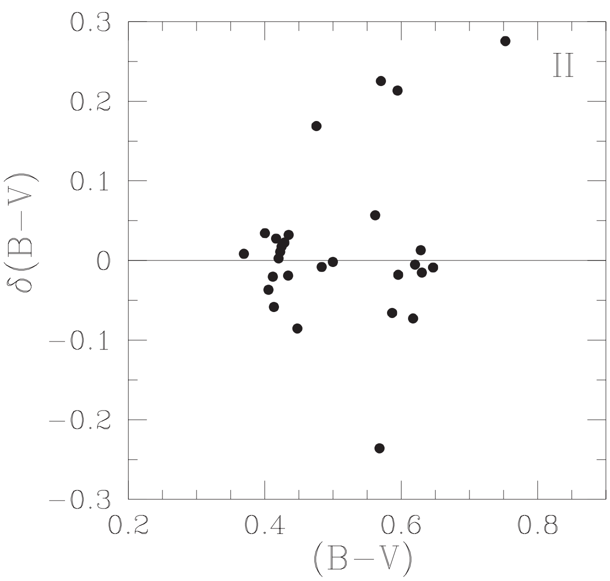

Fig. 3. Comparison of our photometric colour $(B-V)$ with the results Sandage (1970) (II).

criteria may be a little too strict, and that a good many stars that do belong to the cluster are eliminated. However, this permits us to be absolutely certain that the stars which we use, are really cluster members. Moreover, the results that are pursued by this paper are statistically well established by the number of stars left, and receive no ill effects by the eliminations.

Once we are certain that the majority of the stars in the HR diagram really belong to the cluster, we proceed to the fitting of a theoretical isochrone to

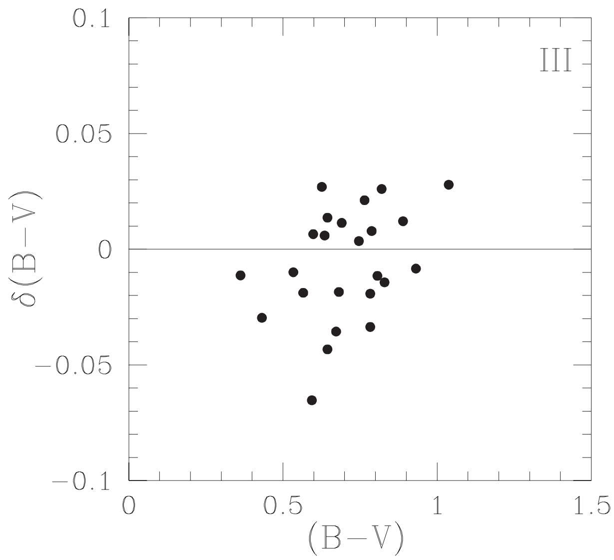

Fig. 4. Comparison of our photometric colour $(B-V)$ with the results of Cathey (1974) (III).

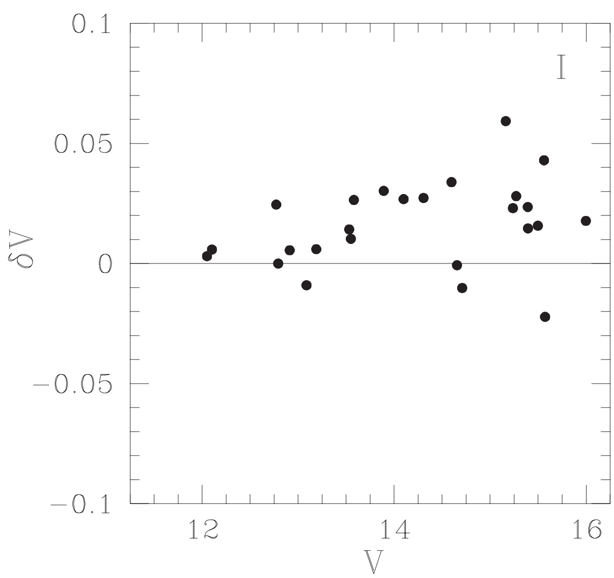

Fig. 5. Comparison of our photometric magnitude V with the results of Sandage & Walker (1966) (I).

the observational points.

A complete fit may be based on the determination of four key parameters which are: (i) reddening $E(B-V)$, (ii) metallicity [Fe/H], (iii) distance modulus $(m-M)$, and (iv) helium abundance Y. In what follows we shall find estimates for the value of these parameter for M92.

(1) Reddening $E(B-V)$: There have been many determinations of $E(B-V)$ for M92 in the literature (Alcaino (1977) (0.02), Harris & Racine (1979) (0.01), Sandage (1983) $(0.04-0.05)$, Stetson & Har-

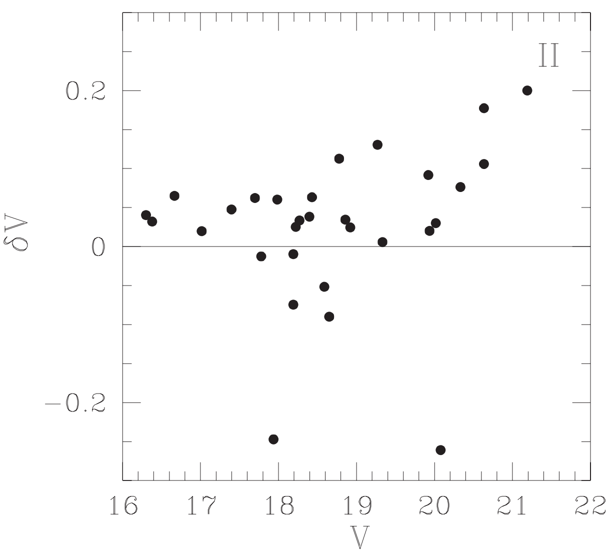

Fig. 6. Comparison of our photometric magnitude V with the results of Sandage (1970) (II).

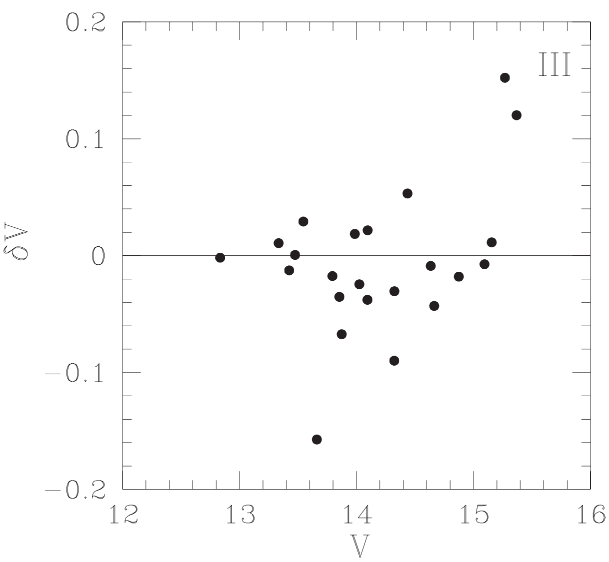

Fig. 7. Comparison of our photometric magnitude V with the results of Cathey (1974) (III).

ris (1988) (0.02), Bergbusch & VandenBerg (1992) (0.02), Bolte & Hogan (1995) (0.02)). They vary from 0.01 (Harris & Racine 1979) to 0.05 Sandage (1983). Based on the Burstein & Heiles (1982) H I maps, we determine that M92 lies between the 0.00 and 0.03 reddening contours which gives a mean value of $E(B-V) \sim 0.015$. Theoretical estimates from the Ruelas-Mayorga (1991) galactic model produce a value $E(B-V) \sim 0.025$. We therefore take the value $E(B-V) = 0.02$ as the reddening to M92.

(2) Metallicity [Fe/H]: M92 is one of the oldest and more metal-poor clusters in the Galaxy. Bergbusch & VandenBerg (1992) give a metallicity value for M92 of [Fe/H] ~ -2.2 "and perhaps smaller", although there are theoretical fits for colour-magnitude diagrams and luminosity functions that fit the data better with metallicity values in the interval $-2.03 \leq$ [Fe/H] ≤ -1.78.

We know that low metallicity values put the turn-off point of the MS at very blue $(B-V)$ values. But the fitting of this point depends on the assumed distance to the cluster. So, as an example we have that an estimate of the M92 metallicity of the order [Fe/H] ~ -2.03 (Bergbusch 1990) is correct if the cluster's age is approximately equal to 16 Gyr, but there are also excellent fits to the data for a metallicity of [Fe/H] ~ -2.27 with an age of 18 Gyr. Bolte & Hogan (1995) find that [Fe/H] ~ -2.26 is the value that better fits the lower part of the M92 MS.

Using a similar technique to the one developed by Sarajedini (1994) for the $(V,\ B-V)$ plane (see Sarajedini & Layden 1997), and applying it to the Bolte fiducial line and to our fiducial line (see Fig. 9) we obtain the following values for the metallicity:

$$[\mathrm{Fe/H}] = -2.34\ (\mathit{Bolte\ Fiducial\ line})\,,$$

$$[\mathrm{Fe/H}] = -2.31\ (\mathit{Our\ Fiducial\ line})\,.$$

In order to get a more representative value for the metallicity of M92, we shall calculate the average of all the values mentioned in this paper (−1.78, −2.03, −2.2, −2.26, −2.27, −2.31, −2.34) which produces a metallicity of:

$$[\mathrm{Fe/H}] = -2.17\,.$$

This agrees perfectly with the "canonical" value given by Bergbusch & VandenBerg (1992).

(3) Distance modulus: Adjusting the brightness of the HB stars in our observations ($V_{\mathrm{HB}} \sim 15.25$) to the absolute magnitude value for the RR-Lyrae variables in M92, given by Sandage (1983) ($< M_V(RR) >_{\mathrm{M92}} \sim 0.63$), we get a first approximation to the distance modulus of $15.25 - 0.63 = 14.62$. This value agrees with other values published in the literature: Alcaino (1977) (14.63), Harris & Racine (1979) (14.5 ± 0.3), Stetson & Harris (1988) (14.6) to cite just a few.

Tsujimoto, Miyamoto, & Yoshii (1998) have analysed data for 125 RR Lyraes ($-2.5 <$ [Fe/H] < 0.06) using the maximum likelihood technique proposed by Smith (1988), and derive:

$$\langle M_V \rangle_{RR} = (0.59 \pm 0.37) + (0.20 \pm 0.63)([\mathrm{Fe/H}] + 1.60)\,.$$

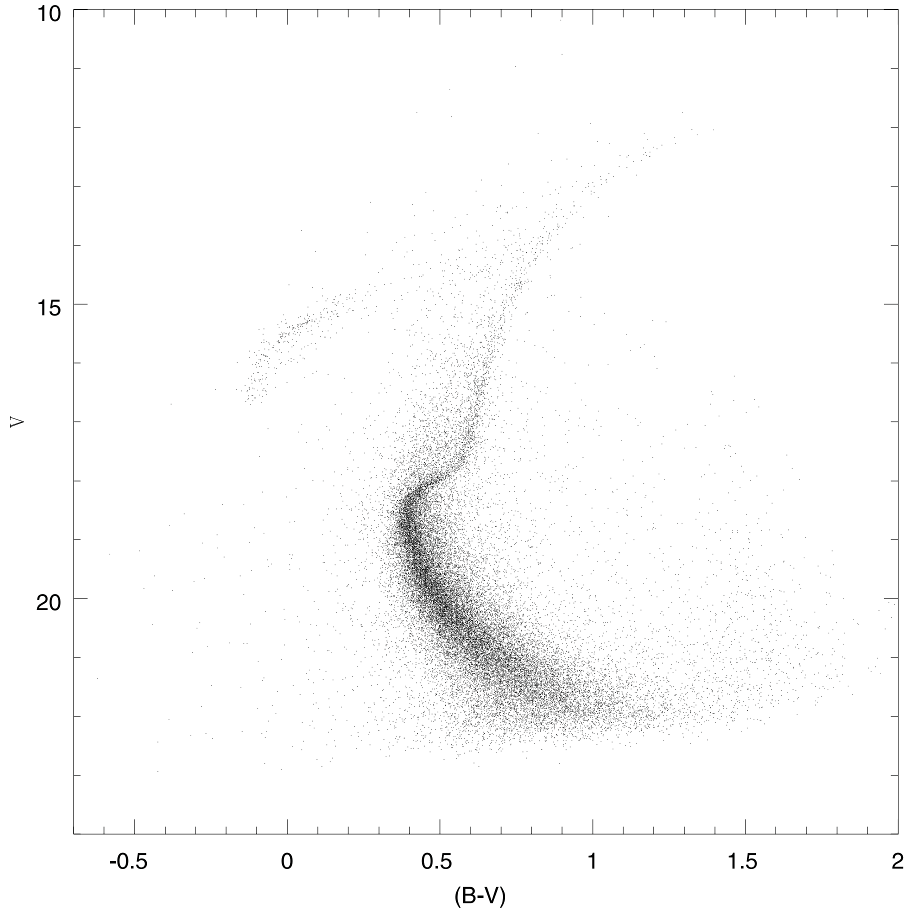

Fig. 8. Raw Colour-Magnitude diagram for M92.

This relation applied to M92 produces

$$\langle M_V \rangle_{\mathrm{M92}} = 0.50 \pm 0.65\,.$$

Statistical parallaxes of field RR-Lyrae stars drawn mostly from the Shanghai Observatory Catalogue (Wan, Mao, & Ji 1980) have been obtained, and are summarised in Table 3 of Reid (1999). There is a value for $\langle M_V \rangle_{\mathrm{RR}} \sim 0.83 \pm 0.23$ for $\langle[\mathrm{Fe/H}]\rangle$ values around -0.75 and another $\langle M_V \rangle_{\mathrm{RR}} \sim 0.85 \pm 0.15$ for $\langle[\mathrm{Fe/H}]\rangle$ values around -1.56. A linear extrapolation to the metallicity of M92 ([Fe/H] ~ -2.03) produces a value for

$$\langle M_V \rangle_{\mathrm{M92}} \sim 0.70 \pm 0.27\,.$$

Skillen et al. (1993) obtain a calibration based on combined infrared flux/Baade-Wesselink analysis of 29 stars which gives a mean relation of

$$\langle M_V \rangle_{\mathrm{RR}} = (0.21 \pm 0.05)[\mathrm{Fe/H}] + (1.04 \pm 0.10)\,,$$

which for M92 produces a result of

$$\langle M_V \rangle_{\mathrm{M92}} \sim 0.61 \pm 0.20\,.$$

McNamara (1997) has reanalysed these same 29 stars making use of more recent Kurucz model atmospheres and derives a steeper, more luminous calibration

$$\langle M_V \rangle_{\mathrm{RR}} = (0.29 \pm 0.05)[\mathrm{Fe/H}] + (0.98 \pm 0.04)\,,$$

which for M92 gives a value of

$$\langle M_V \rangle_{\mathrm{M92}} \sim 0.39 \pm 0.14\,.$$

Fernley (1993) uses his near-IR Sandage Period-shift Effect (SPSE) and a theoretical pulsation relation to derive

$$\langle M_V \rangle_{\mathrm{RR}} = 0.19[\mathrm{Fe/H}] + 0.84\,,$$

which applied to M92 gives

$$\langle M_V \rangle_{\mathrm{M92}} = 0.45 \pm 0.30\,.$$

Several techniques for the determination of $\langle M_V \rangle_{\mathrm{RR}}$ have been presented. They give different values for the absolute magnitude of the HB of M92: $0.50 \pm 0.65, 0.70 \pm 0.27, 0.61 \pm 0.20, 0.39 \pm 0.14$, and 0.45 ± 0.30.

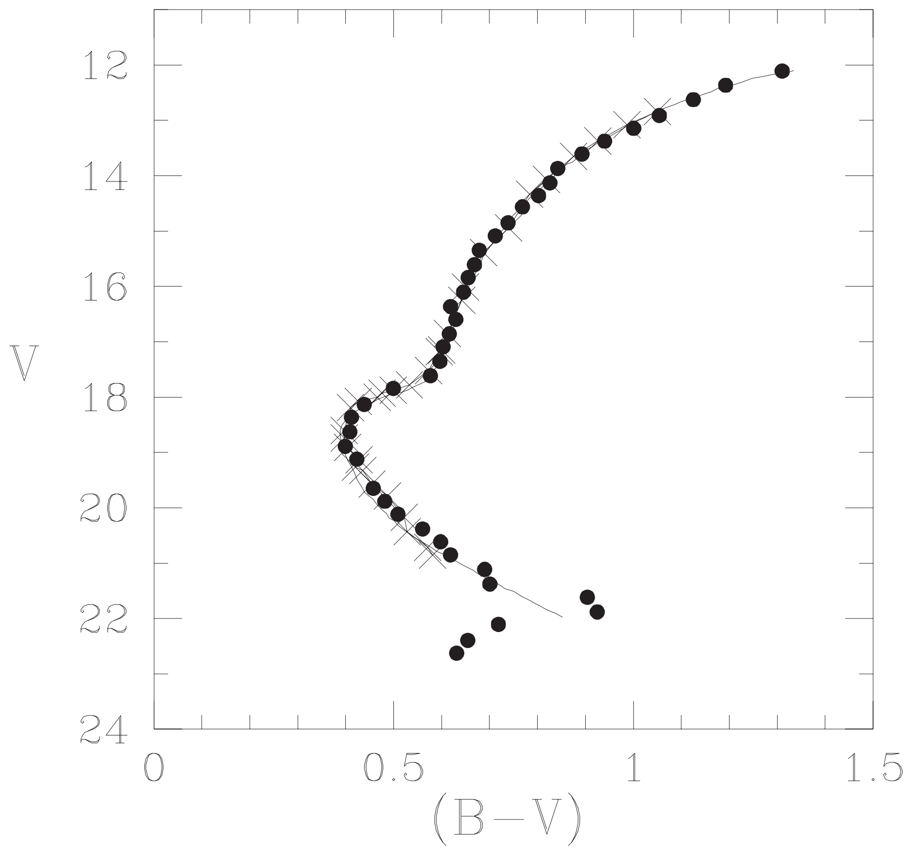

Fig. 9. Fiducial lines for M92. Solid dots (ours), Line (Bolte 1996), crosses (Stetson 2005).

Taking an average we obtain:

$$\langle M_V \rangle_{\rm M92} \sim 0.53 \pm 0.31\,,$$

which agrees reasonably well with the Sandage (1983) value.

Moreover, it is well known that the majority of analysis techniques favour the steep (M_V, [Fe/H]) relation derived originally by Sandage (1982).

We also use the subdwarf method in order to find the distance modulus to M92. To see further information on this method consult Carney (1979a,b), Alcaino & Liller (1980), Bolte (1987), and Stetson & Harris (1988).

To apply this method we use a group of subdwarf stars which is adequate for low-metallicity globular clusters (Bolte 1996); the data are given in Table 2. This table includes the values of absolute visual magnitude corrected by the Lutz-Kelker method and colours corrected for metallicity values smaller than −2.1. Column 1 gives the HD number, Column 2 the metallicity [Fe/H], Column 3 absolute magnitude (M_V), and Column 4 $(B-V)_{-2.1}$.

TABLE 2

LOCAL SUBDWARFS

HD	[Fe/H]	$M_V(LK)$	$(B-V)_{-2.1}$
25329	−1.34	7.159	0.800
64090	−1.73	6.328	0.591
103095	−1.36	6.701	0.693
134439	−1.40	6.851	0.714
134440	−1.52	7.230	0.804
194598	−1.34	4.755	0.424
201891	−1.02	4.891	0.442
+66268	−2.06	6.500	0.661
+114571	−3.60	8.536	1.080

Application of this method gives a value for $m - M = 14.71$ which is essentially the same as that mentioned previously and it also agrees with other values published in the literature: Bergbusch (1990) and Bolte (1996).

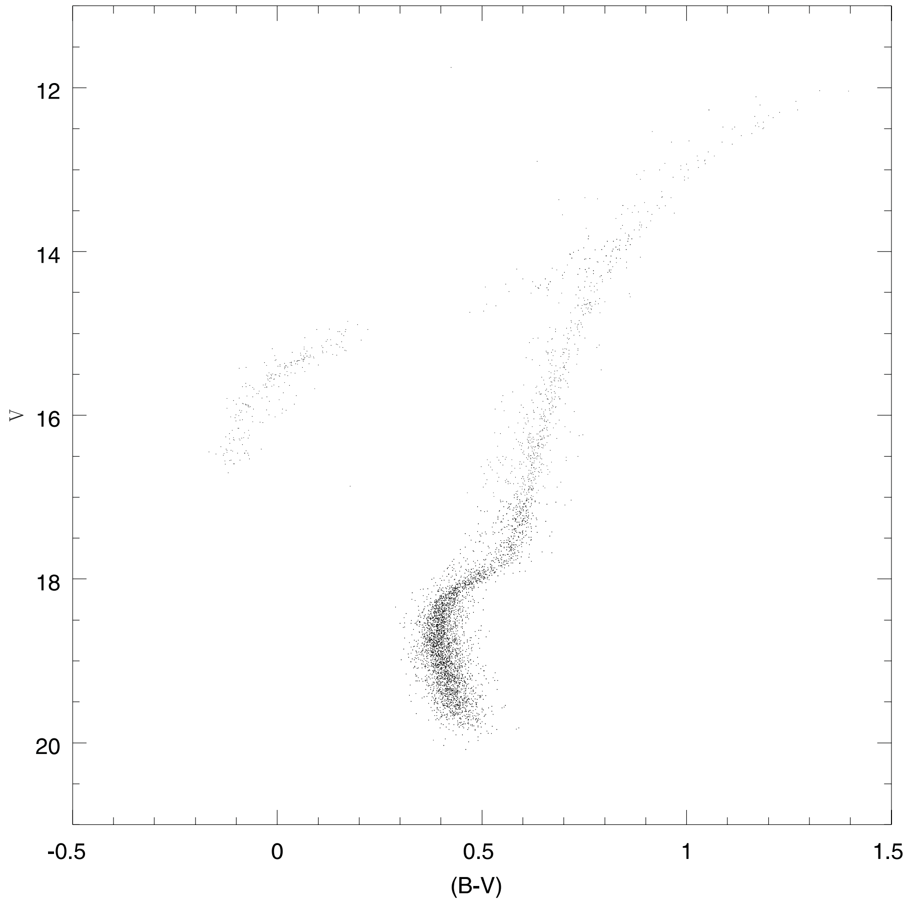

Fig. 10. Clean Colour-Magnitude diagram for M92.

(4) Helium Abundance Y: There are several methods in order to measure He-abundance; most of them produce a value for M92 in the interval $0.2 \pm 0.05 \leq$ [Fe/H] $\leq 0.3 \pm 0.05$ (see Buzzoni et al. 1983).

Yang et al. (1984) and Boesgaard & Steigman (1985), who study observations and predictions for the primordial He-abundance, conclude that the most adequate value for $Y_p \sim 0.23$. Recent determinations of the primordial He-abundance indicate that this value is closer to 0.24 (see Luridiana et al. (2003) ($Y_p = 0.2391 \pm 0.0020$) and Izotov & Thuan (2004) ($Y_p = 0.2421 \pm 0.0021$)). The value of the primordial He-abundance represents a lower bound for the He-abundance in the MS stars of globular clusters.

For M92, Sandage (1983) fits his data to a value $Y \sim 0.2$. This value is slightly smaller than that found by Stetson & Harris (1988) (0.24). Using the Bergbusch (1990) age-luminosity relations, the M92 data are well fit by the following two combinations of parameters

$$\begin{aligned} a)\ [\mathrm{Fe/H}] &= -2.03\,, \\ [\mathrm{O/Fe}] &= +0.7\,, \\ (m-M) &= 14.6\,, \\ Y &= 0.24\,; \\ b)\ [\mathrm{Fe/H}] &= -2.27\,, \\ [\mathrm{O/Fe}] &= +0.0\,, \\ (m-M) &= 14.74\,, \\ Y &= 0.0\,. \end{aligned}$$

Another way of determining the value of Y is by means of the R-method (see Iben & Rood 1969). This method consists basically in a comparison between the theoretical life-times of model stars in the Red Giant Branch (RGB) and the HB phases and the observed quotient of the numbers of stars in these phases

$$R = \frac{N_{\mathrm{HB}}}{N_{\mathrm{RGB}}} = \frac{t_{\mathrm{HB}}}{t_{\mathrm{RGB}}}.$$

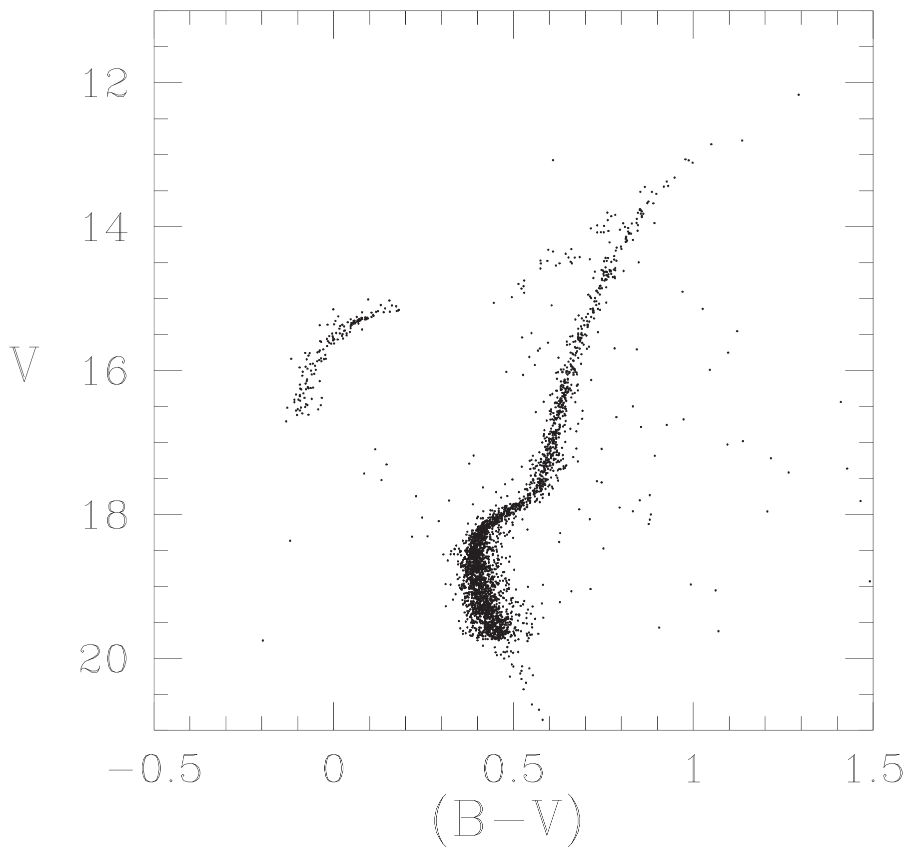

Fig. 11. Colour-Magnitude diagram for M92 from the data of Stetson's (2005).

Iben & Rood (1969) show that R depends almost exclusively on Y. Buzzoni et al. (1983) recalibrate this parameter for a wide selection of individual stars and clusters. They find the following relation:

$$Y = 0.380 \log R + 0.176 \,,$$

which applied to our data for M92 gives a $Y = 0.23 \pm 0.03$, since $R = 1.38 \pm 0.28$ (see below).

We shall, therefore, use theoretical isochrones contained within the interval of He-abundances $0.2 \leq Y \leq 0.3$.

In Buzzoni et al. (1983) there is also another method to obtain the He-abundance. They call it the R' method, where they define the parameter R' as:

$$R' = \frac{t_{\rm HB}}{(t_{\rm RGB} + t_{\rm AGB})} = \frac{N_{\rm HB}}{(N_{\rm RGB} + N_{\rm AGB})} \,.$$

The calibration they find for Y is as follows:

$$Y(R') = 0.457 \log R' + 0.204.$$

This formula is of great help in the determination of Y because it is generally not easy to distinguish the RGB-stars from the Asymptotic Giant Branch (AGB) stars.

The stars were counted on our catalogue, considering those stars with $V \leq 16.85 \pm 0.05$ and $B - V \leq 0.27 \pm 0.02$ as HB-stars and those with $V \leq 14.75 \pm 0.05$ and $B - V \geq 0.45 \pm 0.02$ as RGB+AGB stars. The results we obtain are as follows:

$$N_{\rm HB} = 225 \pm 15 \,,$$

$$N_{RGB+AGB} = 230 \pm 15 \,,$$

where the errors correspond to the $\sqrt{N}$. Therefore

$$R' = 0.98^{+0.12}_{-0.14}$$

and

$$Y(R') = 0.20 \pm 0.03.$$

From the counts reported above, we estimate that 163/230=71% of the total number of stars in RGB+AGB are in the RGB and only 29% of this

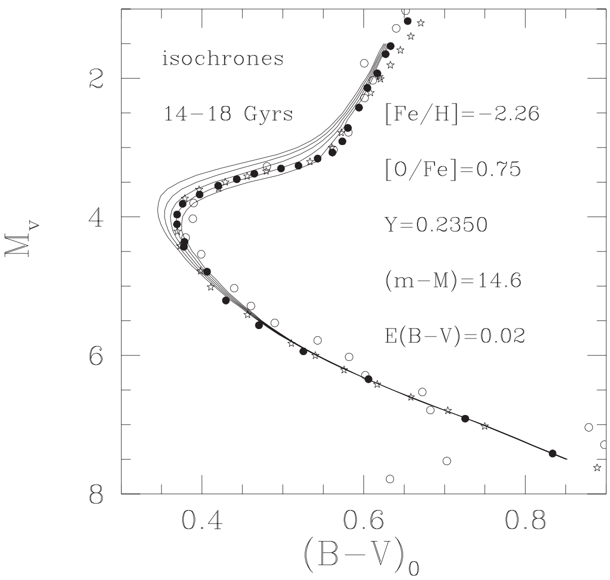

Fig. 12. Isochrone fit. Model 226 from Proffitt & VandenBerg (1991). Solid dots (Bolte 1996), Open dots (this work), Stars (Stetson & Harris 1988).

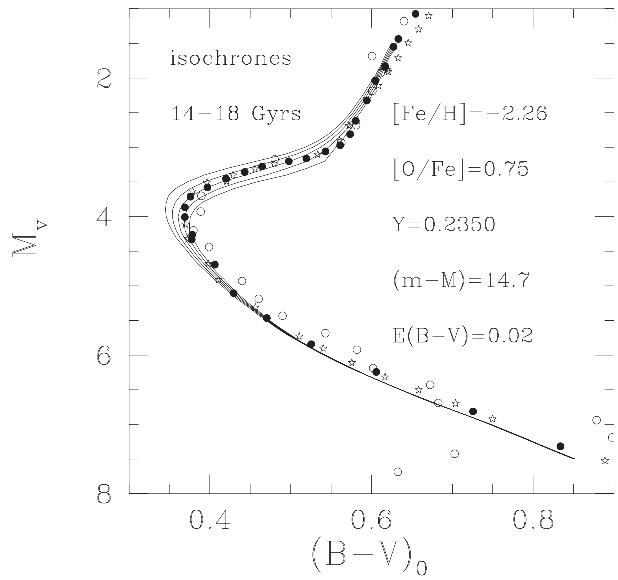

Fig. 13. Isochrone fit. Model 226 from Proffitt & VandenBerg (1991). Solid dots (Bolte 1996), Open dots (this work), Stars (Stetson & Harris 1988).

number of stars belongs to the AGB. Giving as a results:

$$N_{\rm RGB} = 163\,,$$

and

$$N_{\rm AGB} = 67\,.$$

The value $N_{\rm RGB} = 163$ is used above to produce a value for $R = 1.38$.

Now that we have established reasonable values for the key parameters; $E(B-V)$, [Fe/H], $(m-M)$, and Y of M92. We use the VandenBerg models (Proffitt & VandenBerg (1991) and VandenBerg (1992)) to fit our data.

Figures 12 to 17 show these fits, and in them we can see the following elements:

(a) Isochrones (solid lines) for the age interval 14 − 18 Gyr (in steps of 1 Gyr).

(b) Bolte's (1996) fiducial line (solid dots)= Mid points for the colour distributions for different magnitude cuts.

(c) Our fiducial line (open dots)= Maximum for the colour distribution for different magnitude cuts.

(d) The Stetson & Harris (1988) fiducial line (stars).

We do not plot on these figures the Stetson (2005) fiducial line because we already know (see Fig. 9) that it agrees very well with the fiducial line of Stetson & Harris (1988) and also with ours.

On each figure we indicate the values for the different parameters and the model used. We take particular care of the fits to the MS turn-off point and to the MS fit, since both these features are of great importance for the age determination of the cluster.

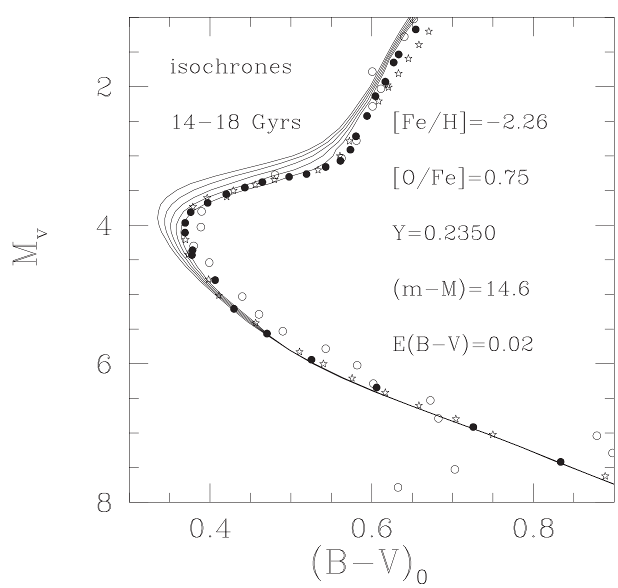

Fig. 14. Isochrone fit. Model 226 from Bergbusch & VandenBerg (1992). Solid dots (Bolte 1996), Open dots (this work), Stars (Stetson & Harris 1988).

The best fit to our points is that shown in Fig. 16, with the following parameters:

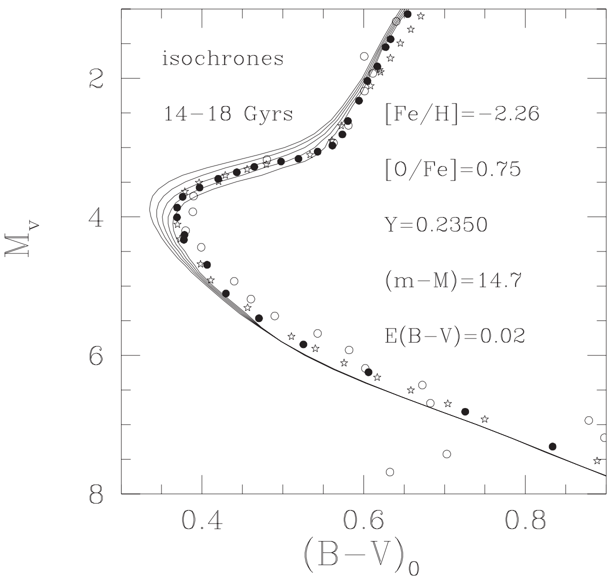

Fig. 15. Isochrone fit. Model 226 from Bergbusch & VandenBerg (1992). Solid dots (Bolte 1996), Open dots (this work), Stars (Stetson & Harris 1988).

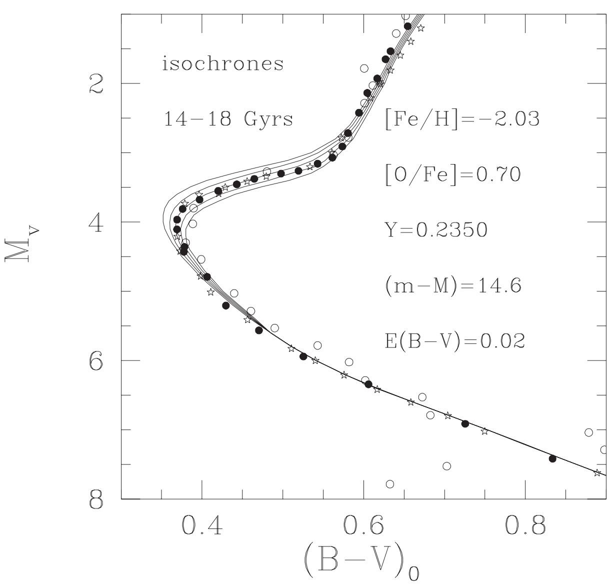

Fig. 16. Isochrone fit. Model 203 from Bergbusch & VandenBerg (1992). Solid dots (Bolte 1996), Open dots (this work), Stars (Stetson & Harris 1988).

$$[\mathrm{Fe/H}] = -2.03,$$
$$[\mathrm{O/Fe}] = 0.70,$$
$$Y = 0.2350,$$
$$(m - M) = 14.6$$
$$E(B - V) = 0.02$$

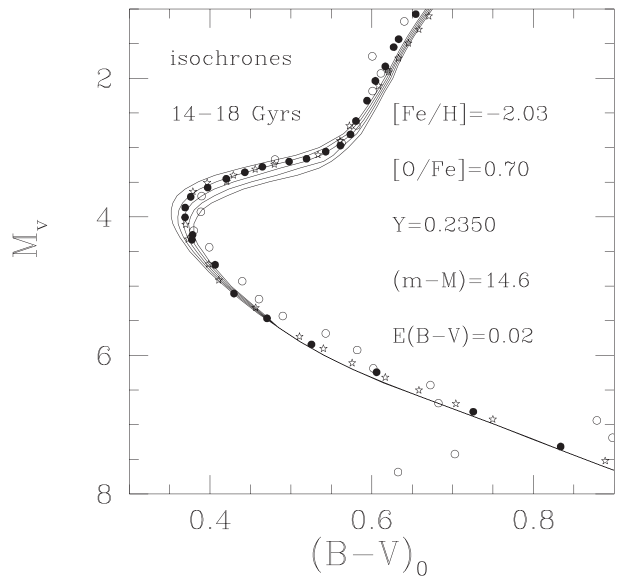

Fig. 17. Isochrone fit. Model 203 from Bergbusch & VandenBerg (1992). Solid dots (Bolte 1996), Open dots (this work), Stars (Stetson & Harris 1988).

and

$$\mathrm{Age} = 16\,\mathrm{Gyr}\,.$$

The fit shown in Fig. 12 is not bad either, but it is for a lower metallicity ([Fe/H] ~ -2.26) and a similar age. We therefore estimate an age of $16 - 17$ Gyr for M92 with a metallicity of $-2.26 \leq [\mathrm{Fe/H}] \leq -2.03$ and $(m{-}M) = 14.6$. These latter values are different from those which we mention above. However, within the observational and propagated uncertainties, the agreement is excellent.

According to Bolte & Hogan (1995), the errors in the fitting parameters carry a total uncertainty for the age of M92 of the order 13%; therefore we establish with certainty that:

$$\mathrm{Age(M92)} = 16 \pm 2 \ \mathrm{Gyr}\,.$$

There are other theoretical studies, such as Bergbusch & VandenBerg (2001) with which we could compare our data. Fig. 13 from this paper shows a fitting of the Stetson & Harris(1988) and Bolte's (1996) fiducial lines to a set of isochrones. Since our fiducial line is very similar to that of Stetson & Harris (1988) (see Fig. 9), this fit could be thought as appropriate for our data, requiring similar parameters to those obtained from our best fit to older isochrones (Fig. 16) except for the age which turns out to be larger for this fit by 2 Gyr.

Pietrinferni et al. (2004) have also published an interesting paper on theoretical isochrones. Their Fig. 12 presents a comparison between their predicted isochrones, and those of Bergbusch & VandenBerg (2001). This figure indicates that at ages of the order $\sim$ 2 Gyr their isochrones are hotter than those of Bergbusch & VandenBerg's by a maximum of $\sim$ 1200 K, whereas at higher ages ($\sim$ 10 Gyr and older) the agreement between these two sets of isochrones is excellent.

As we mention above, our data are well fitted by the isochrones of Bergbusch & VandenBerg's, and since these are well matched at older ages by those of Pietrinferni et al., it is clear that they also fit our data appropriately.

We are at present in the process of fitting in detail these two sets of isochrones to data for a number of globular clusters for which we have photometric data. The results of this investigation will be published in a forthcoming paper.

4. THE LUMINOSITY FUNCTION

The basic analysis of the Luminosity Function (LF) is done over two differentiated regions close to the MS turn-off point in the HR diagram (Fahlman 1993).

(i) In those phases that occur after the MS turn-off point, the evolution of the stars is quite rapid, so that:

$$n(m)dm = r(t-\tau)d\tau\ ,$$

where $\tau = t - t_0$ is the time that elapsed since a star of magnitude m left the MS until the time the observations are taken, i.e., it is the time the star has lived out of the MS, t is the age of the cluster, and t_0 the age at which the stars start to leave the MS. The function $r(t)$ represents the rate at which the stars change magnitude in units of number of stars of magnitude m_i per unit of time.

The time a particular star takes from its birth to the turn-off point is approximately the same for all the stars in a cluster, so for any cluster

$$r(t-\tau) = r(t_0) \sim constant = r_0\ ,$$

so

$$\Delta N_i = r_0 \delta\tau_i\ .$$

Therefore the number of stars with a magnitude m_i is directly proportional to the time they spend within that magnitude interval, so the number of stars in post-MS stages provides us with a powerful test to understand the physics of stars in stages beyond the MS.

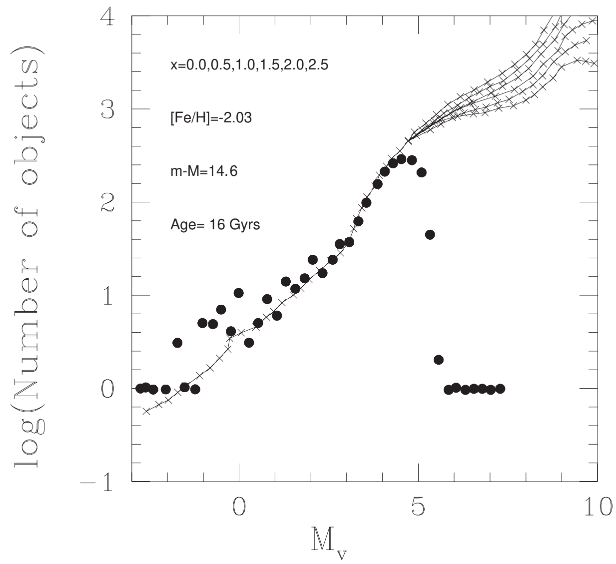

Fig. 18. Fit by model (crosses) to the observed Luminosity Function (solid dots)($r \leq 204''$) from the discriminated photometric catalogue. [Fe/H] $= -2.03$, $m - M = 14.6$, Age = 16 Gyr, Mass index $x = 0$ to $x = 2.5$.

(ii) Below the MS turn-off point the MS is nothing more than a sequence of mass, so

$$n(m)dm = n(M)dM\ ,$$

where M is the mass of the star and $n(M)$ is the initial mass function (IMF) of the cluster. It is common knowledge that the IMF is usually represented as:

$$n(M) \alpha M^{-(1+x)}\ ,$$

where x is the spectral-mass index.

In order to form a LF, we count the number of stars in each magnitude bin. We account for incompleteness effects by means of numerical experiments with artificial stars.

We divide our observational field in two sections: (a) An inner section with $r \leq 204''$, and (b) an outer section with $r \geq 204''$.

In Figures 18 to 21 we present the logarithm of the number of stars per magnitude bin for the discriminated photometric catalogue for the inner and outer sections of the cluster (Figs. 18 and 19) and in Figs. 20 and 21 we show the results for the total photometric catalogue. On all figures we give the predicted LF from the Bergbusch & VandenBerg (1992) models for an age of 16 Gyr, and they correspond to the model given in Fig. 16.

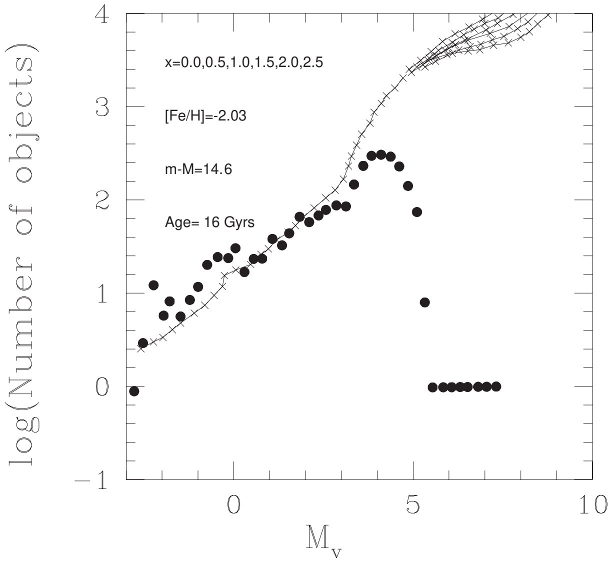

Fig. 19. Fit by model (crosses) to the observed Luminosity Function (solid dots)($r \geq 204''$) from the discriminated photometric catalogue. [Fe/H] $= -2.03$, $m - M = 14.6$, Age = 16 Gyr, Mass index $x = 0$ to $x = 2.5$.

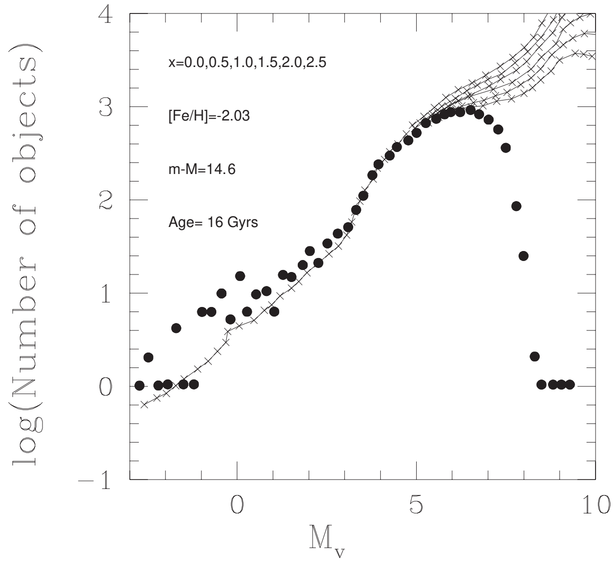

Fig. 21. Fit by model (crosses) to the observed Luminosity Function (solid dots) ($r \geq 204''$) from the complete photometric catalogue. [Fe/H] $= -2.03$, $m - M = 14.6$, Age = 16 Gyr, Mass index $x = 0$ to $x = 2.5$.

It is clear that incompleteness sets in at fainter magnitudes, and that it is more important for the case of the discriminated catalogue than for the case of the total catalogue, as it was expected, simply because there are fewer objects in the former catalogue than in the latter.

We compare our LF with the LF derived by Hartwick (1970). This study is quite valuable in the sense that Hartwick corrected the counts for his whole magnitude range, which permits comparisons of our data with his at very bright and very faint magnitudes. We present these comparisons in Fig-

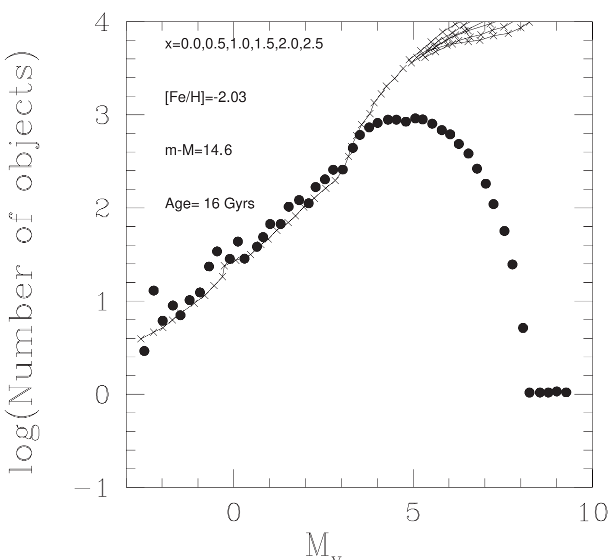

Fig. 20. Fit by model (crosses) to the observed Luminosity Function (solid dots)($r \leq 204''$) from the complete photometric catalogue. [Fe/H] $= -2.03$, $m - M = 14.6$, Age = 16 Gyr, Mass index $x = 0$ to $x = 2.5$.

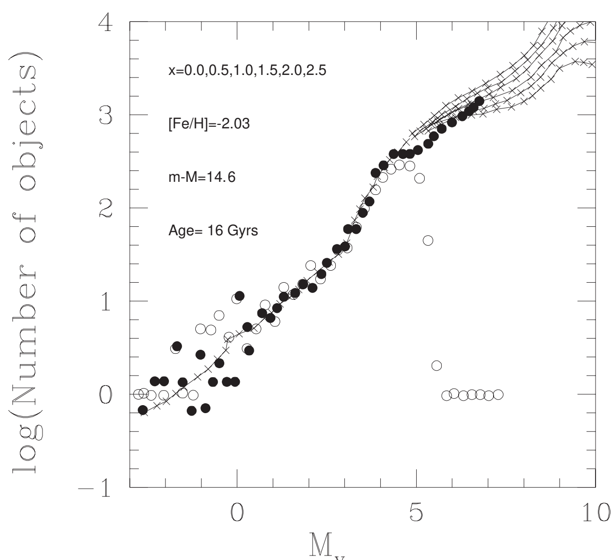

Fig. 22. Comparison of our Luminosity Function (open dots) ($r \geq 204''$) from the discriminated photometric catalogue with that of Hartwick (1970) (solid dots). The crosses represent theoretical models.

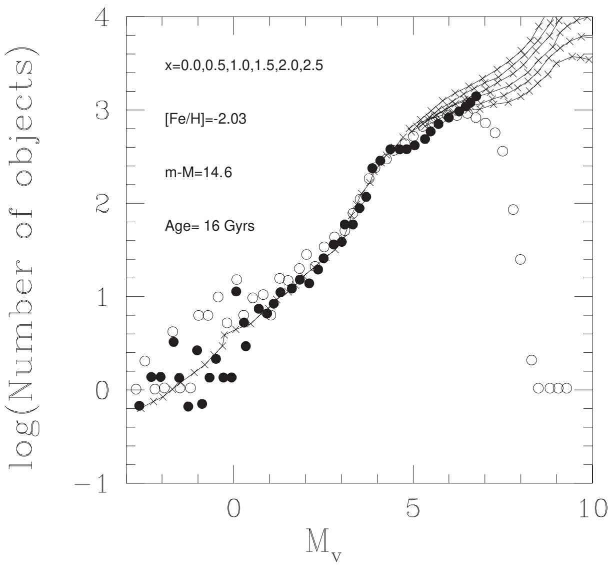

Fig. 23. Comparison of our Luminosity Function (open dots) ($r \geq 204''$) from the complete photometric catalogue with that of Hartwick (1970) (solid dots). The crosses represent theoretical models.

ures 22 and 23 for both the discriminated and the complete photometric catalogues for the outer section of the cluster.

As can be seen, the comparison is quite excellent for the whole magnitude range above the MS turn-off point ($M_V \leq 4.0$). We can now be certain that the level of incompleteness in our data, which is left over after the artificial star experiments, is acceptable and that our preliminary counts for bright magnitudes are correct. There is, however, an important numerical dispersion with respect to the theory at $M_V \leq 0$.

This dispersion may be accounted for from the point of view of the evolutionary models and not necessarily from any problems with the stellar counts.

5. CONCLUSIONS

We carried out successfully the reduction of a mosaic of 49 images in filters B and V which covered the globular cluster M92.

From the results of the reduction of these images, we formed a photometric catalogue containing more than 30,000 stars, which we used to construct a colour-magnitude diagram for this cluster.

By means of three different discrimination techniques ($\chi^2 \geq 1.5$, $\delta(B-V) \geq 0.03$ and $\mid (B-V) \mid \geq 3\sigma$) we considered, as proper cluster stars, approximately 4000 objects that are mostly post-MS stars ($V \leq 21$ mag).

We calculated fiducial lines for the HR-diagram based on the evolutionary models of Proffitt & VandenBerg (1991) and Bergbusch & VandenBerg (1992). The fitting of the fiducial lines depends importantly on the values of the following parameters: (a) Reddening ($E(B-V)$): the value we used was 0.02 although it could be as large as 0.04 (calculated from reddening models Ruelas-Mayorga 1991), (b) Distance modulus ($m-M$): we determined the distance modulus from fitting the cluster's MS to a sequence of low-metal abundance solar neighbourhood subdwarfs. We found $m-M = 14.71$; this figure is slightly higher than the most popular value given in the literature (14.6), (c) The metal abundance of the cluster was calculated by means of the Sarajedini method (Sarajedini & Layden 1997) and the value found is [Fe/H] ~ -2.3 which agrees, within the uncertainties, with values in the literature. It turns out that the metal abundance of M92 is one of the lowest for the group of galactic globular clusters. At times, it has been said that M92 is the most metal-poor globular in the Galaxy, (d) He-abundance (Y): We used the canonical value of $Y = 0.23$, although calculations with the R' method yield a He-abundance value of $Y(R') = 0.20 \pm 0.03$. Recent determinations of the primordial He-abundance indicate that its value is closer to 0.24 (see Luridiana et al. 2003; Izotov & Thuan 2004) posing an important consistency problem, since the primordial He-abundance should be lower or at least equal, to the He-abundance of the oldest and most metal-poor objects in the universe. At this point we do not offer any explanation for this consistency problem except that it is contained within the observational uncertainties.

Independently of the way we determine the values of the reddening, distance modulus, metallicity and He-abundance, we fit isochrones to the colour-magnitude diagram, and the best fits indicate values for the above mentioned parameters that sometimes differ from the former determinations. For example, isochrone fitting reveals that best fits are obtained for $E(B-V) = 0.02$ in agreement with what we determine before, [Fe/H] $= -2.03$ which is lower than the above value by almost ~ 0.3 dex, $m-M = 14.6$ which is slightly lower that the value mentioned above and $Y = 0.235$ which is higher than the value used above, and it also is more in tune with the recent higher determinations of the primordial He-abundance.

On the one hand, it is healthy that different methods produce values for the same physical quantities that are similar. However, the fact that the values do not turn up to be exactly the same reveals that our theories and methods are not perfect and that there is still room for improvement.

The colour-magnitude diagram turn-off point is best fitted with an isochrone of age equal to 16 ± 2 Gyr. This result is in agreement with that found by Stetson & Harris (1988).

We show that there are important incompleteness effects for stellar counts in magnitude ranges fainter than the MS. For brighter magnitudes we corrected the counts using artificial star experiments, and in doing so, we were able to find a LF which we compare with the model LF's of Bergbusch & VandenBerg (1992) for the magnitude interval $0 \leq M_V \leq 4$.

The best fits for the LF have the same parameter values as those for the best isochrone fits.

The fact that by using the R'-method we predict a lower than primordial value for the He-abundance, could be addressed by saying that peculiar evolutionary effects, which are not considered in the present models, produce a smaller than expected number of stars in the HB of the cluster or a larger than expected number in the RGB or the AGB. Both these effects would result in a smaller determination for the He-abundance. It has been mentioned that isothermal stellar nuclei might result in a larger quantity of H in a stellar nucleus at the time the star leaves the MS. This would slow down the evolution of the stars up the GB, causing an excess of stars in the sub-Giant Branch (SGB). Just as this effect alters the relative number of stars in the SGB, there might be other effects, as yet unknown, that could alter the relative star-number in other sections of the HR-diagram.

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