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BIASES IN THE KINEMATIC PARALLAXES OF GALACTIC PLANETARY NEBULAE

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RESUMEN

Se ha notado recientemente que las paralajes cinemáticas de las nebulosas planetarias (NP's) podrían tener errores significativos. El patrón de velocidades de las envolturas, el cual determina la expansión lateral de los flujos, parece diferir de las velocidades en la línea de visión determinadas a través de espectroscopía visual. Sin embargo, se asume usualmente que las distancias medias a un gran número de NP's deberían ser razonablemente seguras. Dado que las fuentes están orientadas aleatoriamente con respecto a la línea de visión, entonces los sesgos de las distancias individuales deberían desaparecer al promediarlas. Señalamos que esto puede estar equivocado cuando las fuentes no son esféricas. Si se miden las velocidades de expansión laterales $V_{\perp}$ a lo largo del eje menor de las fuentes, entonces las distancias tenderán (al promediarlas) a ser demasiado grandes. Alternativamente, si se determina consistentemente $V_{\perp}$ a lo largo del eje mayor de los flujos, entonces los valores de $<D>$ tenderán a ser bajos. Aunque la magnitud del error es difícil de evaluar, puede ser del orden de $\Delta D/D \sim 10\%$.

ABSTRACT

It has recently been noted that the kinematic parallaxes of planetary nebulae (PNe) may be significantly in error. The pattern velocities of the shells, which determine the lateral expansion of the outflows, are likely to differ from line-of-sight velocities determined through visual spectroscopy. It is usually assumed that the mean distances to large ensembles of PNe should be reasonably secure, however. Given that sources are randomly oriented with respect to the line-of-sight, then individual distance biases should wash-out in the mean. We point out that this is unlikely to be the case where sources possess non-spherical structures, however. If one measures lateral expansion velocities $V_{\perp}$ along the minimum axes of the sources, then distances will (in the mean) tend to be somewhat too large. Alternatively, if one consistently determines $V_{\perp}$ along the largest axes of the outflows, then values of $<D>$ will tend to be low. Although the size of error is difficult to assess, it may approach of order $\Delta D/D \sim 10\%$.

Key Words: **ISM: JETS AND OUTFLOWS — PLANETARY NEBULAE: GENERAL**

1. INTRODUCTION

The distances of planetary nebulae are very poorly known. More-or-less direct values are available based upon measures of trigonometric parallax (e.g., Harris et al. 1997; Acker et al. 1998; Gutiérrez-Moreno et al. 1999), kinematic parallax (e.g., Liller & Liller 1968; Reed et al. 1999; Hajian & Terzian 1996; Hajian, Terzian, & Bignell 1993), radial velocities (Acker 1978; Phillips 2001a), spectroscopic parallax (Ciardullo et al. 1999; Méndez & Niemela 1981), trends in nebular extinction (e.g., Gathier, Pottasch, & Pel 1986; Martin 1994; Kaler & Lutz 1983), Na D line absorption (Napiwotzki & Schönberner 1995), and determinations of central star gravities (e.g., Méndez et al. 1988b; Méndez, Kudritzki, & Herreri 1992). However, such direct estimates are available for only 100 sources in all; a total which constitutes little more than $\sim 7\%$ of known galactic PNe. Similarly, it appears that even these few, direct, estimates cannot always be regarded as reliable. Random uncertainties in distance appear to be of order $\sim 30\%$ (Phillips 2002a), whilst certain of the procedures are prone to systematic errors as well.

The gravitational distances of Méndez et al. (1988b), for instance, are $\sim 20\%$ greater than those of Méndez et al. (1992); a difference which arises because the analysis of Méndez et al. (1992) takes account of stellar winds, as well as the spherical extensions of the central stars. Similarly, it has been noted that although the impact of ion-dynamical effects on Balmer line profiles is probably quite small (Napiwotzki & Rauch 1994), the contribution of metals to atmospheric opacities may be of critical importance (Werner 1996).

Although this procedure is therefore open to uncertainty, it is encouraging to note that the most recent estimates of D_{GRAV} are consistent with trigonometric parallaxes (Napiwotzki 2001).

A further procedure for evaluating D is based upon measures of "kinematic parallax". This method, which is observationally difficult to undertake, makes use of the fact that secular expansion of the shells can be directly measured. Images are taken at radio or optical wavelengths, and displacements of sharp, well defined features are assessed as a function of time. This angular displacement is then combined with measures of line-of-sight velocity to evaluate values of D_{KIN} for individual shells (see e.g., Pottasch (1984) for a more detailed description of this method).

Early applications of the procedure usually assumed that the line-of-sight velocity $V_{//}$, determined using measurements of Doppler-shifted line splitting, were similar to velocities $V_{\perp}$ in the plane of the sky. Such estimates still dominate the corpus of published distances (e.g., Liller & Liller 1968; Hajian & Terzian 1996; Hajian et al. 1993), although certain more recent measures have employed sophisticated expansion templates (e.g., Li, Harrington, & Borkowski 2002; Reed et al. 1999).

Such a procedure is open to various biases, however. Mellema (2004) has used a simple two-wind model, and analysis of I- front expansion to determine that $V_{//}$ and $V_{\perp}$ may differ by as much as ~ 20 to 30%, for instance. This will cause a corresponding error in the evaluation of D_{KIN}. However, although there is little doubt that such a difference in velocities must indeed occur, the actual kinematics of the shells are likely to be somewhat more complex. It is now accepted that whilst the interior shocked stellar wind drives expansion of the primary outflow envelope, thermal expansion of this envelope leads to a further "thickening" of the shells —to an increase of $\Delta R/R$ with time (Marigo et al. 2001), and the "filling" of highly evolved outflow structures (Phillips, Cuesta, & Kemp 2005). The interior kinematics of the shells are likely to reflect these competing (and conflicting) mechanisms.

On the other hand, it is also possible to view this problem from a purely observational standpoint as well. It has been established that many PNe envelopes appear to be undergoing a Hubble-type expansion, such that expansion velocities V_{EXP} are proportional to the nebular radius R (e.g., Weedman 1968; Wilson 1958; Corradi et al. 2001; Corradi & Schwarz 1993a,b; Redman et al. 2000). It is not known if this applicable to all PNe shells, but it does appear to be the case in those for which such trends have been investigated.

Given that lateral expansion velocities $V_{\perp}$ are usually measured towards the peripheries of the shells, where velocities are higher, whilst line-of-sight velocities refer to components located at smaller radii, then one would again expect biases in distance and velocity similar to those postulated by Mellema (2004).

This is, however, by no means the only bias to which this procedure may be subject. Thus, Wade, Harlow, & Ciardullo (2000) have noted that the kinematic distances of novae are affected by shell geometry. If (as they suppose) such envelopes have prolate structures, then it appears that novae distances may be in error by tens of percent. To put it at its most simple, if the shells are thin and prolate, and follow a Hubble velocity relation, then it is likely that velocities along the line-of-sight will differ from those which are perpendicular to the line of sight.

Although many PNe shells appear to have large relative thicknesses $\Delta R/R$, it is nevertheless apparent that some such similar consideration may apply for these sources as well. Not only is it likely that $V_{EXP} \propto R$, but it is clear that many such sources possess non-spherical structures. In the case of circular and elliptical shell morphologies, for instance, it seems likely that the mean axial ratio is of order ~ 0.8 (Phillips 2000), although it remains unclear whether the intrinsic structures of such outflows should be regarded as oblate or prolate.

In either case, it is clear that one expects the envelopes to possess differing lateral and line-of-sight velocities. This will, in turn, affect the kinematic parallaxes of these sources. But what of the distances to PNe taken as a whole? Would one not expect such geometrical biases to even out, such that mean PNe distances are broadly correct? Whilst some PNe distances will be underestimated, others are likely to be overestimated, so that the mean of all such distances will be similar to those determined through other procedures.

This has, up to now, been an implicit (if unwritten) assumption behind many such analyses. It also has an important bearing upon the evaluation of statistical distances, since these are based upon measures of "standard" source distances, including those determined through kinematic parallax.

It is our purpose, in the following, to suggest that this assumption is likely to be false —that when one determines mean distance scales based upon kinematic parallax, using large ensembles of randomly oriented nebulae, then these values will likely be systematically in error because of non-spherical shell structures. Thus, not only are such distances biased because of differences between velocities, but the structures of the sources may also play a part in biasing these results. We describe, in the following section, the reasons for this conclusion.

2. THE MEAN DISTANCES OF ELLIPSOIDAL PNE

If one assumes that PNe shells expand according to a Hubble expansion law, and have structures which can be approximated by prolate or oblate spheroids, then it may be shown that the ratio between the maximum central velocity $V_{//}(max)$, and the maximum velocity perpendicular to the line-of-sight $V_{\perp}(max)$ is given through

$$\Gamma = \Gamma_{MAX} = \frac{V_{//}}{V_{\perp}} = \left(cos^2\phi + \left[\frac{a}{b}\right]^2 sin^2\phi\right)^{-0.5}, \tag{1a}$$

where

$$\left(sin^2\phi + \left[\frac{b}{a}\right]^2 cos^2\phi\right)^{0.5} \leq 1,$$

and

$$\Gamma_{MAX} = \frac{\left(cos^2\phi + \left[\frac{a}{b}\right]^2 sin^2\phi\right)^{-0.5}}{\left(sin^2\phi + \left[\frac{b}{a}\right]^2 cos^2\phi\right)^{0.5}}, \tag{1b}$$

where

$$\left(sin^2\phi + \left[\frac{b}{a}\right]^2 cos^2\phi\right)^{0.5} > 1,$$

with ϕ the angle between the plane of symmetry and the line-of-sight vector, measured with respect to the surface. Similarly, a is the radius of the plane of symmetry, and b is the length of the axis perpendicular to this plane. A slice through this structure which includes axis b will take the form of an ellipse with eccentricity $\epsilon = 1 - b^2/a^2$ (where $a \geq b$). It follows that where $a/b < 1$ then the structures are prolate, whilst a value $a/b > 1$ implies an oblate shell structure. We shall not in the following consider the case of bipolar envelopes, although it is likely that our overall conclusions will apply to these structures as well.

The average of expression (1) with respect to ϕ, integrated between the limits 0 and $\pi/2$, and weighted by $cos\,\phi$ (to take account of variations in the number of randomly oriented sight-lines), then gives the mean ratio $<\Gamma>=<\Gamma_{MAX}>$ for a random distribution of nebular orientations.

In determining distances using kinematic parallaxes, much depends upon where one measures the lateral expansion of the shells; whether it is along the projected major axis of the shell, along the minor axis, or for some value of radius intermediate between these two.

We have considered four cases in the present analysis. In the first of these, it is assumed that time-lapse images are used to measure expansion along the largest projected shell radius ($\theta = \theta_{MAX}$). For this case, the mean ratio $<\Gamma_{MAX}>$ is determined using the integral of expression (1). We have also determined a ratio $<\Gamma_{MIN}>$ corresponding to expansion along the smallest radius θ_{MIN}, and a ratio $<\Gamma_{MEAN}>$ corresponding to projected radii $\theta_{MEAN} = (\theta_{MAX} + \theta_{MIN})/2$. Finally, the ratio $<\Gamma_{HARM}>$ is appropriate for the harmonic mean radius $\theta_{HARM} = (\theta_{MAX}\theta_{MIN})^{0.5}$. These differing values of $<\Gamma>$ are represented in Fig. 1 as a function of the outflow axial ratio a/b.

It may be seen that except in the case of spherical shells, for which $b/a = 1$, then the mean ratio $<\Gamma>$ will be either larger or less than unity. Where expansion is measured along θ_{MIN}, for instance, then $<\Gamma>$ is always > 1. This will imply distance determinations which are systematically too large. For all of the other cases it appears likely that distances will be too small —although the bias is particularly extreme where one determines expansion along the largest projected radius (θ_{MAX}).

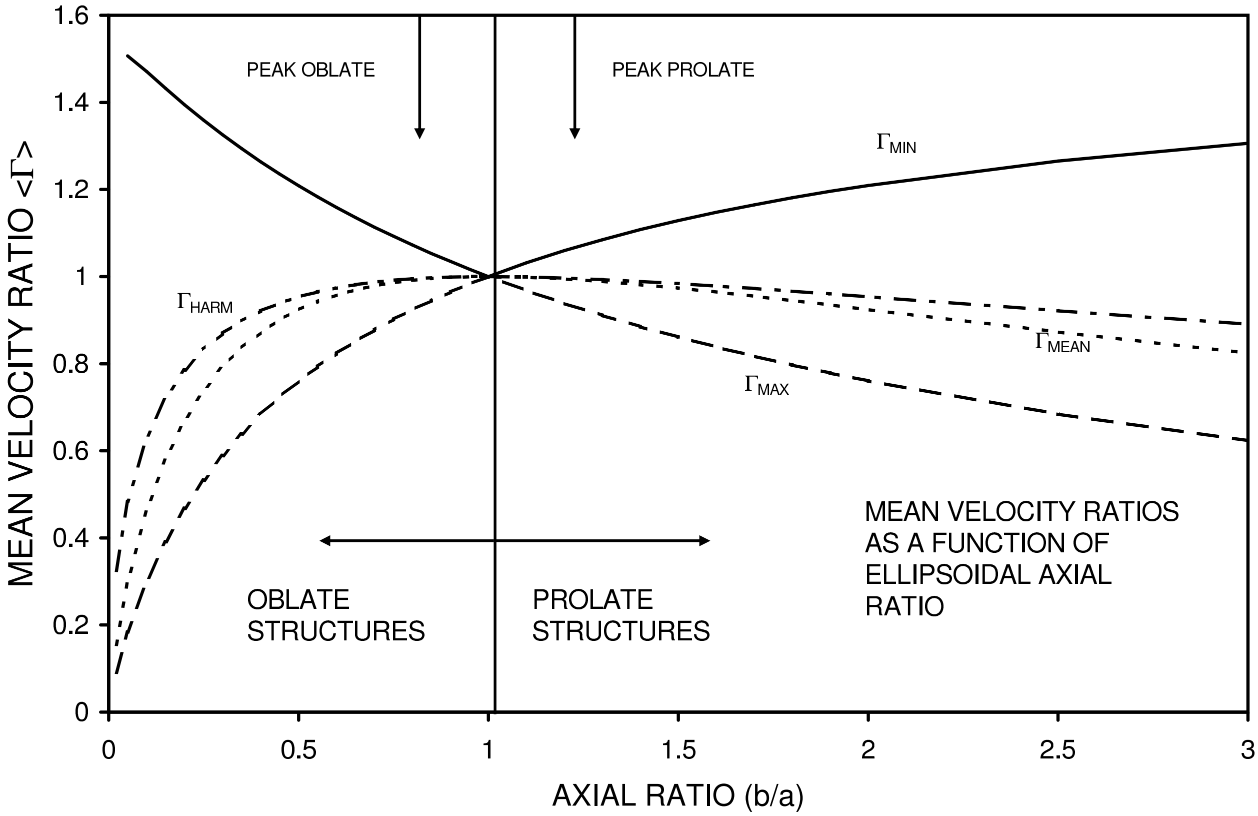

Fig. 1. The variation of the mean velocity ratio $<\Gamma>$ as a function of shell axial ratio b/a. Where values of $<\Gamma>$ are greater or less than unity, then kinematic distances will tend to be respectively too large or too small. The differing values of Γ depend upon whether lateral velocities refer to the largest projected shell radius (Γ_{MAX}), smallest projected radius (Γ_{MIN}), the mean projected radius (Γ_{MEAN}), or the harmonic mean radius (Γ_{HAR}).

It is important at this point to note what this analysis does not imply. It does not imply that all distances will be uniformly low when expansion is measured along the maximal outflow axis θ_{MAX}. On the contrary, it is likely that many of these distances will be rather too large. What it does imply, on the other hand, is that the average of many independent distance measurements, for differing PNe, is likely to be somewhat lower than is intrinsically the case.

The mean size of this bias is difficult to assess. It depends upon whether shells are prolate, oblate, or have some differing structure entirely. This is not known at present. However, given that intrinsic shell structures have axial ratios peaking around $b/a \sim 0.81$ for oblate shells, and $a/b \sim 0.83$ for prolate shells (Phillips 2000), then it would seem that potential biases may be as great as $\sim 10\%$ (see Figure 1).

We note, finally, that such an analysis is intended to be purely indicative, and represents little more than a warning concerning potential errors. It is strictly applicable in the case of thin-shell structures, such as are predicted to occur during intermediate phases of evolution (e.g., Marigo et al. 2001; Marten & Schönberner 1991). However, it is also likely to be indicative of trends for thicker shell structures as well, for which $\Delta R/R \gg 0.1$.

It is therefore clear that distances evaluated using kinematic parallaxes are open to two inter-related biases, one deriving from shell kinematics, and the other arising from non-spherically symmetric expansion. Whilst differences between "pattern" and material velocities would yield distances which are systematically too large (Mellema 2004), biases arising from non-spherical shell structures may go either way, depending upon which of the axes are employed. Since most measures of expansion seem to take place along the maximal axes of the outflows, then the overriding trend is likely to be for $<D>$ to be too small.

In either case, it is clear that such distance determinations require to be treated with caution, and may bias evaluations of mean statistical scales.

3. CONCLUSIONS

We possess relatively few direct measures of planetary nebula distances, and it is therefore important that those which we do possess show no systematic biases. This is, taken as a whole, likely to be the case. However, it seems that gravitational distances need to be treated with caution, since they depend upon the accuracy of the modeling used to determine g. Differing analyses appear to lead to differing values of D. Similarly, it seems likely that differences between line-of-sight expansion velocities, measured through optical spectroscopy, and the pattern velocities responsible for lateral expansion of the shells, may cause systematic biases in the assessment of kinematic distances.

We have pointed out that the procedure for determining kinematic parallaxes may also be prone to a further bias. If the expansion of the envelopes can be approximated by a Hubble type law, whereby V_{EXP} is proportional to nebular radius, then non-spherical expansion may lead to differences between lateral velocities $V_{\perp}$, and line-of sight velocities $V_{//}$. We point out that this difference between $V_{\perp}$ and $V_{//}$ remains important even when considering large ensembles of PNe, oriented randomly relative to the line of sight. Where lateral expansion is measured along the projected minor axes of the outflows, then mean distances will tend to be too large. Where measurements are along the major axes of the sources, then $<D>$ will tend to be be low.

Such biases are important. Not only do they compromise the limited corpus of distances which we at present possess, but they may also bias statistical distance scales as well.

Whether mean distances are too large or too small is difficult at present to assess, although the level of bias may be of order $\sim 10\%$.

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