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ON THE MAGNETIC FIELD EVOLUTION IN SHELL-LIKE SUPERNOVA REMNANTS

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RESUMEN

En este artículo aplicamos y discutimos un método para la determinación de la evolución del campo magnético en los remanentes de supernova (SNRs) a partir de la relación entre la luminosidad en radio a la frecuencia ν y el diámetro ($L_\nu - D$). Asumimos que H evoluciona como $H \propto D^{-\delta}$, donde D es el diámetro del remanente. El valor $\delta \approx 1.2$ se obtiene con la hipótesis de equipartición a partir de las ecuaciones del cálculo revisado de equipartición (REC) y usando la muestra de datos de la galaxia de brote estelar M82. Intentamos investigar si los remanentes de la supernova en M82 están en estado de equipartición o no, comparando la δ empíricamente obtenida con el valor teórico esperado para equipartición. La diferencia entre el valor obtenido teóricamente para la equipartición con expansión adiabática ($\delta = 1.5$) y el valor empírico obtenido aquí se puede explicar principalmente como debido a efectos de selección en la sensibilidad, que tienden a aplanar la pendiente de la relación $L_\nu - D$ para las muestras extragalácticas.

ABSTRACT

In this paper we apply and discuss a method for the determination of the magnetic field (H) evolution in supernova remnants (SNRs) from radio luminosity at given frequency ν to diameter ($L_\nu - D$) correlation. We assumed that H evolves as $H \propto D^{-\delta}$, where D is the diameter of the remnant. A value $\delta \approx 1.2$ is obtained under the equipartition assumption from the equations for revised equipartition calculation (REC) and by using the data sample from the nearby starburst galaxy M82. We try to investigate whether or not SNRs in M82 are in the equipartition state. This is done by comparison of our empirically obtained δ with the theoretical value expected for equipartition conditions. The inconsistency between the value obtained for equipartition conditions and adiabatic expansion ($\delta = 1.5$) and the value empirically obtained herein can be explained mainly by the influence of sensitivity selection effects which tend to flatten the slope of the $L_\nu - D$ relations for extragalactic samples.

Key Words: **GALAXIES: INDIVIDUAL (M82) — ISM: MAGNETIC FIELDS — METHODS: STATISTICAL — RADIATION MECHANISMS: NON-THERMAL — RADIO CONTINUUM: ISM — SUPERNOVA REMNANTS**

1. INTRODUCTION

Supernova remnants (SNRs) are an important factor in the process of cosmic ray acceleration and matter circulation. Albeit very important, these processes are still not fully understood. Various theories were suggested during the last few decades with a view to understanding SNR properties. There is a general belief that the evolution of a SNR is strongly influenced by the properties of the local interstellar medium (ISM) in which SNR evolves. As SNRs are the luminous synchrotron emitters in the radio domain of the electromagnetic spectrum, the magnetic field inside them and the energy spectrum of relativistic particles can be determined. Here, we will mainly focus on magnetic field properties such as field strength and evolution. The most commonly used empirical relation in studies of SNR evolution

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is the radio surface brightness to diameter ($\Sigma - D$) relation. This is because the only statistically reliable data samples of SNRs are found in the radio domain. In order to study SNR evolution issues from a slightly different perspective, in this paper we apply a method that transforms $\Sigma - D$ into magnetic field to diameter ($H - D$) relation. In this way, we can discuss SNR evolutionary properties by comparing theories on H with an empirically extracted $H - D$ relation. A statistical, i.e., empirical study of H evolution nevertheless requires reliable data samples of SNRs in different types of interstellar media.

There is a number of ways to estimate H in SNRs. Unfortunately, few are reliable and even they are available only for a few well studied SNRs. The estimates are made by measuring rotation measures or spectral line splitting. The estimates can also be extracted from radiation fluxes from different parts of the electromagnetic spectrum such as radio, X-rays or γ-rays. However, there is another problem in performing a statistical study of H based on these estimates. The data samples of SNRs are biased by severe selection effects through the entire electromagnetic spectrum. SNRs are mainly identified in the radio domain. Unlike optical, X-rays or γ-rays, radio waves are less influenced by absorption and scattering in the interstellar medium. Also, radio interferometers have the best resolution among all the other observational devices, which also helps in the detection of remnants. Large and reliable data samples are of crucial importance for a good and well-founded statistical study of empirical $H - D$ relation. Today, this condition is partially fulfilled only by data samples in the radio domain. The empirical studies of SNR properties are also severely influenced by selection effects. It remains to be hoped that the observational instruments and techniques in the future will help us overcome these problems.

The main purpose of this paper is to apply and discuss a method for the determination of the $H - D$ slope from the correlation of radio luminosity at given frequency ν to diameter ($L_\nu - D$) in SNR samples that show existence of such a correlation ($\Sigma_\nu = L_\nu/(D^2\pi^2)$). The method is based on the energy equipartition assumption between magnetic field and relativistic particles. It uses the equations for revised equipartition calculation (REC). The equipartition calculation is the most commonly used manner of obtaining H estimates in valid radio SNR samples. The obtained $H - D$ slope from the only reliable data sample of SNRs in M82 is then compared with the slope arising from the theoretical models of SNR evolution. We can then argue whether or not M82 SNRs are in the equipartition state, and thereby give a contribution to the general evolutionary studies of SNRs. We also try to give estimates of the magnetic field strengths, particularly for SNRs in M82. In addition, we discuss the accuracy of magnetic field strength obtained under REC. This is done by comparing the values for H, obtained herein, with reliable ones available in literature (found for a few SNRs in the Large Magellanic Cloud and our Galaxy). It is noteworthy that the SNR luminosity is mainly determined by the density of the environments in which the SNR evolves. This is an important issue for the discussion of the influence of equipartition arguments on H.

This paper is organized as follows: Section 2 presents explanations of the required topics which are too broad to be mentioned later in the text. In Section 3 we describe and analyze the method and REC with its assumptions. Section 4 features a discussion on the obtained results for $H - D$ slope and magnetic field strength. There, we consider whether M82 SNRs are in the equipartition state or not. Finally, the conclusions of this work are given in Section 5.

2. THE $H - D$ DEPENDENCE

2.1. *History of $H - D$ Relation*

We assume that $H - D$ relation can be written in the form:

$$H \propto D^{-\delta} \ . \tag{1}$$

Historically, this form of the magnetic field evolution is used in all theoretical models that explain synchrotron emission from SNRs.

Shklovsky (1960) was the first to theoretically describe synchrotron emission from a spherically expanding nebula. He assumed that the magnetic field structure remains unchanged during the expansion. Consequently, the magnetic field flux is constant and $H \propto D^{-2}$, where D is the diameter of the remnant. Lequeux (1962) applied Shklovsky's theory to model shell type remnants, which led to $H \propto (l \times D)^{-1}$, where $l \propto D$ represents the thickness of the shell. Poveda & Woltjer (1968) and Kesteven (1968) also gave their contribution to the general model of a shell type remnant. They assumed that H is increased with the compression of the interstellar medium magnetic field (leading to $H = const$) and that the shell thickness remains constant during the expansion (which leads to $H \propto D^{-1}$). A theoretical interpretation of SNR synchrotron emission by Duric & Seaquist (1986) used the magnetic field model with $H \propto D^{-\delta}$, based on the work of Gull (1973) and Fedorenko (1983). According to the results of Gull, the magnetic field is compressed and amplified in the convection zone, to finally gain enough strength to power the bright synchrotron emission. Fedorenko stated that $1.5 \leq \delta \leq 2.0$. Tagieva (2002) obtained $H \propto D^{-0.8}$, by using the $\Sigma \propto D^{-2.38}$ relation (Case & Bhattacharya 1998). However, this result should be taken with great reserve because the $\Sigma - D$ relation from the work of Case & Bhattacharya (1998) is plagued by severe selection effects (Urošević et al. 2005). Also, Tagieva did not take into account the influence of the density of the environment in which the SNRs evolve. An interesting discussion about the magnetic field and the equipartition arguments for five Galactic SNRs, based on the results empirically obtained from X-ray data, can be found in the work of Bamba et al. (2005). The predecessor of this paper is the work of Vukotić et al. (2006).

2.2. *Magnetic Field Calculation from Radio Synchrotron Luminosities*

The magnetic field is calculated from the following formula for synchrotron emission of relativistic electrons (Beck & Krause 2005, hereafter BK):

$$L_\nu = 4\pi f V c_2(\gamma) n_{\mathrm{e},0} \cdot \cdot E_0^\gamma (\nu/2c_1)^{(1-\gamma)/2} {H_\perp}^{(\gamma+1)/2} \ . \quad (2)$$

We adjusted the formula from BK to suit our needs. Here, f is the fraction of the radio source volume occupied by the radiative shell. We assumed that $f = 0.25$. This is consistent with SNRs having strong shocks where the compression ratio is 4. However, should this not be the case, a variation of f will still not have any significant effect on values for H, because of the small value of the exponent $1/(\alpha+3)$ in Eq. (12). Further, the total volume of SNR is designated by V. Instead of spectral intensity along the radiation ray path (I_ν) in BK, we used the spectral luminosity of the source, because the majority of sources in the data samples used are seen almost as point-like sources, having only the flux density data integrated over the whole source available. According to BK this may lead to an overestimation of values for H. This effect is discussed further in Section 4. The rest is the same as in BK, $c_2(\gamma)$ (in units erg^{-2} s^{-1} G^{-1}) is identical to $c_5(\gamma)$ in Pacholczyk (1970), $n_{\mathrm{e},0}$ is the number density of cosmic ray electrons per unit energy interval for the normalization energy E_0, $c_1 = 3e/(4\pi m_{\mathrm{e}}^3 c^5) = 6.26428 \cdot 10^{18}$erg^{-2} s^{-1} G^{-1}, $H_\perp$ is the magnetic field strength in the plane of the sky and finally γ represents the exponent in the cosmic ray power law energy spectrum (see Appendix A in BK). Closer inspection of Eq. (2) shows that in order to calculate H from L_ν, some assumption regarding the relationship between H and $n_{\mathrm{e},0}$ has to be made.

2.3. *Data Samples*

Currently, it seems that there is no better way to determine H by using only data on L_ν and spectral index α ($\gamma = 2\alpha + 1$) than the equipartition or the minimum-energy assumption. This method is useful for SNR samples where all other data are lacking. However, Galactic SNR data samples are strongly biased by selection effects. The farther the object, the greater its brightness detection limit. The extragalactic samples suffer from milder selection effects. Their brightness detection limits (sensitivity lines) do not differ from one SNR to another because all the SNRs in the sample are approximately at the same distance. In this study, we have relied on the only statistically trustworthy sample of SNRs from a nearby starburst galaxy, M82 (Huang et al. 1994). The equations that we used in calculating H are presented in Section 3. Inspection of those equations shows that any $H - D$ correlation requires the existence of a $L_\nu - D$ correlation. If a $L_\nu - D$ correlation does not exist, than it makes no sense to extract an $H - D$ relation from $L_\nu - D$ data. If SNR data samples show a non-existing or poor $L_\nu - D$ correlation there are two possibilities: either the SNR luminosity does not evolve with the diameter, which is unlikely, or the sample is made of SNRs that evolve in different environments and is influenced by selection effects. This is explained in the next paragraph.

In their work, Arbutina et al. (2004) showed that the best $L_\nu - D$ correlation exists for SNRs in M82. They also showed that some correlation exists for Galactic SNRs associated with large molecular clouds. Arbutina & Urošević (2005) imply that the evolution of SNR radio surface brightness depends on the properties of the interstellar medium, primarily on the density. They formed three SNR data samples from the existing ones (Galactic and extragalactic): Galactic SNRs associated with large molecular clouds (GMC), oxygen-rich and Balmer-dominated SNRs. The main intent of Arbutina & Urošević (2005) was to group SNRs by their properties, primarily the density of the interstellar medium in which they evolve (and also by SN type). They also argued that the M82 sample is the best possible sample that one can currently find. All SNRs from M82 are likely to evolve in a similar environment of dense molecular clouds. Consequently, they are very luminous and being extragalactic, they exhibit milder selection effects. The reliability of the M82

TABLE 1

RESULTS FOR δ

Direct	
Shklovsky (1960) ($n_{\mathrm{e},0} \propto D^{-(2\alpha+3)}$)	0.125
Berezhko & Volk (2004) ($n_{\mathrm{e},0} \propto D^{-3}$)	0.875
Classical	
equipartition	1.26
Revised	
equipartition	1.22

sample is also discussed in Urošević et al. (2005). By performing a Monte Carlo simulation, the authors showed that the M82 sample is not severely affected by sensitivity selection effects, as is the case of other extragalactic samples (LMC, SMC, M31, M33).

In this paper we have applied REC and calculated δ for the M82 sample, as the best sample for a statistical study, and additionally we have analysed three samples from Arbutina & Urošević (2005): GMC, oxygen-rich and Balmer dominated SNRs. We did not use the last three samples to calculate the slope δ because they are of poorer quality. Instead, we used them to check the consistency of the obtained H values with the global picture of SNR evolution in different environments. Also, through a literature search we found the magnetic field strengths for some SNRs from Table 2 and compared them with the values obtained in this paper. We searched the catalog of observational data on Galactic SNRs from Guseinov, Ankay, & Tagieva (2003, 2004a, 2004b) and papers available on the Web-based Astrophysical Data Service (*http://adswww.harvard.edu/*).[3]

In the calculation we used the radio flux density per unit frequency interval S_ν and radio spectral index α data (Table 2). These two properties are related by:

$$S_\nu = \beta \nu^{-\alpha} \ , \tag{3}$$

where β is the flux scale factor. The luminosity is calculated as $L_\nu = 4\pi d^2 S_\nu$, where d is the distance to a SNR. In the case of extragalactic SNRs we assume that d is the same for all SNRs, and equal to the distance to the host galaxy.

2.4. *Magnetic Field and Relativistic Particles*

Since our studies are based on the radio synchrotron luminosity of SNRs, we cannot treat magnetic field separately from relativistic particles. These two properties of an SNR are strongly coupled and it makes no sense to study them separately.

As mentioned before, calculation of H from Equation (2) requires an assumption about $n_{\mathrm{e},0}$. This quantity also evolves with D. In Table 1 and Section 4.2 we present and discuss various assumptions about the $n_{\mathrm{e},0}(D)$ evolution and its effect on $H(D)$ evolution (assuming an empirical $L-D$ relation). Some of the $n_{\mathrm{e},0}$ evolution patterns are only illustrative and are used for estimating the effect of different patterns on δ. The pattern we used in our method to calculate H arises from the equipartition of energies implying that energy densities stored in the magnetic field and relativistic particles are approximately equal. The equipartition assumption is widely used for H strength estimates, based purely on the radio data, in SNRs, galaxies, etc. It gives reasonable values for δ and H. Taking all of this into account we based our method on the equipartition of energies.

The revised equipartition calculation (REC) used to calculate H is presented in detail in the work of BK. According to BK, REC gives better results than the classical equipartition calculation (CEC) presented by Pacholczyk (1970).

2.5. *Evolution of the Magnetic Field in SNRs*

In this subsection we present the theoretical values for δ that characterize a particular SNR evolution phase. These values, together with the ones obtained by our empirical method, are used in Section 4 in the discussion of the most probable evolution scenarios for SNRs in M82.

If SNRs are young, in the early Sedov or free expansion phase, they expand practically adiabatically, since radiative energy losses are negligible. Under the adiabatic expansion assumption i.e. conservation of energy in cosmic rays and magnetic field ($d/dt(W) = 0$), and equipartition conditions ($w_{\mathrm{CR}} = w_{\mathrm{H}}$), where W is the total energy and the quantities w_{CR} and w_{H} are the energy densities of cosmic rays and magnetic field, respectively, it follows that $\delta = 1.5$. Indeed:

$$\frac{d}{dt}(W) = \frac{d}{dt}(wV) \propto \frac{d}{dt}(w_{\mathrm{H}}V) \propto \frac{d}{dt}(H^2D^3) \ , \tag{4}$$

$$\frac{d}{dt}(W) = 0 \Longrightarrow H \propto D^{-3/2} \ , \tag{5}$$

where w is the total energy density. In conclusion, SNRs in the free expansion or early Sedov phase will have $\delta = 1.5$ if they are in the equipartition state.

On the other hand, if SNRs are older, in the late Sedov or radiative phase, the value may be closer to

[3]ADS is NASA-funded project which maintains three bibliographic databases containing more than 4.7 million records.

$\delta = 1.25$. The radiative phase is characterized by significant energy losses and SNR would later expand with a velocity $v \propto D^{-5/2}$ (pressure-driven snowplow). If $n_{\mathrm{e},0} \propto n_{\mathrm{p},0} \propto n_{\mathrm{H}} v$ (Berezhko & Völk 2004, hereafter BV), assuming equipartition, $H^2 \propto n_{\mathrm{e},0}$, δ would be 5/4=1.25. The quantity $n_{\mathrm{p},0}$ is the number density of cosmic ray protons per unit energy interval for the normalization energy E_0, and n_{H} is the hydrogen number density.

It is a general belief that, during the expansion, SNRs strongly amplify the interstellar magnetic field. Two basic mechanisms of magnetic field amplification operate in SNRs. The first one is the Rayleigh-Taylor instability at the contact discontinuity between the supernova ejecta and the ISM swept by the SNR forward shock. This scenario leads to $1.5 \leq \delta \leq 2$ (Fedorenko 1983) and is preferred in young SNRs. The second mechanism operates right behind the shock, where the magnetic field is amplified by strongly excited magnetohydrodynamic waves. This is the probable mechanism for older remnants.

3. ANALYSIS AND RESULTS

There are two most commonly used assumptions regarding the magnetic field and cosmic ray energy content: (1) the minimum of total energy stored in the particles and magnetic field, and (2) the equipartition between these energies. The minimum energy assumption gives 4/3 for the ratio of the energies stored in the particles and magnetic field, which is ~ 1. These two assumptions are thereby often treated as synonymous and both procedures are referred to as the equipartition calculation. There are also two different methods for obtaining these two estimates: classical (Pacholczyk 1970) and revised (BK) equipartition, i.e. minimum-energy calculation. We will only present the formulas that we have used in calculating H and reader is referred to the mentioned papers for a detailed treatment of the subject.

3.1. *Classical Calculation*

The classical formulae are:

$$H^{\mathrm{min}} = 4.5^{2/7}(1+k)^{2/7} \cdot c_{12}{}^{2/7} f^{-2/7} \left(D/2\right)^{-6/7} L^{2/7} \ , \quad (6)$$

$$H^{\mathrm{eqp}} = 6^{2/7}(1+k)^{2/7} \cdot c_{12}{}^{2/7} f^{-2/7} \left(D/2\right)^{-6/7} L^{2/7} \ . \quad (7)$$

In these expressions we have introduced the following quantities: k is the ratio of the energies of the heavy relativistic particles and relativistic electrons, c_{12} and c_{13} are functions which are weakly dependent on α and are tabulated by Pacholczyk (1970). The radio luminosity L integrated between radio synchrotron spectrum cutoff frequencies ν_1 and ν_2 is calculated from:

$$L = 4\pi d^2 \int_{\nu_1=10^7\ \mathrm{Hz}}^{\nu_2=10^{11}\ \mathrm{Hz}} S_\nu \, d\nu \ . \quad (8)$$

Using Equation (3) we can eliminate β and obtain L. We used $k = 40$ which should be adequate for strong shocks in SNRs. Being luminous synchrotron emitters and having small linear diameters, SNRs in M82 are likely to be young and to have strong shocks, but their true nature is still a subject of debate. We obtained δ from Equations (6) and (7) by replacing L with the $L_\nu - D$ relation from Arbutina et al. (2004). Replacing L with L_ν does not have any noticeable effect on δ. We also assumed that H depends on D only trough L or L_ν. Therefore,

$$H \propto \left(D^{-3} L_\nu\right)^{2/7} \propto \left(D^{-4.4}\right)^{2/7} \ , \quad (9)$$

if $L(D) \propto D^{-1.4}$ (Arbutina et al. 2004). This gives $\delta = 1.26$. To prove the assumptions in Equation (9), we have calculated L from equations (3) and (8), and H from Equation (7). Then we fit a linear regression in the $\log H - \log D$ plane to obtain $\delta = 1.26 \pm 0.08$. This shows that $c_{12}(\alpha)$ does not change with D, so it does not affect δ, which is why we can calculate δ directly from the slope s of the $L_\nu \propto D^{-s}$ relation,

$$\delta = (3+s)\frac{2}{7} \ , \quad (10)$$

as in Eq. (9). Equations (6) and (7) differ by a constant, giving exactly the same δ. In the sequel we do not show results for minimum energy estimates of H.

3.2. *Revised Formulas*

The main revision of the classical formulas consist of using K instead of k. The quantities K and k stand for the ratios of proton to electron number densities and energy densities, respectively. In the CEC, integration of the radiation energy spectrum between fixed frequency limits is performed. As opposed to this, in REC the integration is performed over the energy spectrum of relativistic particles. This gives more accurate results (see BK).

The revised formulas are:

$$H_{\mathrm{rev}}^{\mathrm{min}} = \Big[4\pi K A(\gamma, L_\nu, \nu, V, f, i) \cdot C(\gamma, E_2)(\alpha+1)\Big]^{1/(\alpha+3)} \ , \quad (11)$$

$$H_{\rm rev}^{\rm eqp} = \Big[8\pi K A(\gamma, L_\nu, \nu, V, f, i) \cdot \cdot C(\gamma, E_2)\Big]^{1/(\alpha+3)} , \quad (12)$$

where

$$C(\gamma, E_2) = E_0^2 \cdot \left\{ \tfrac{1}{2}\left(\tfrac{E_0}{E_{\rm p}}\right)^{\gamma-2} + + \tfrac{1}{2-\gamma}\left[\left(\tfrac{E_0}{E_2}\right)^{\gamma-2} - \left(\tfrac{E_0}{E_{\rm p}}\right)^{\gamma-2}\right]\right\} \quad \text{for } \gamma \neq 2 , \quad (13)$$

$$C(\gamma, E_2) = E_0^2\left[\frac{1}{2} + \ln\frac{E_2}{E_{\rm p}}\right] \quad \text{for } \gamma = 2 , \quad (14)$$

and

$$A(\gamma, L_\nu, \nu, V, f, i) = \frac{L_\nu (\nu/2c_1)^{(\gamma-1)/2}}{4\pi c_2(\gamma) {E_0}^{\gamma} f V c_4(i)} . \quad (15)$$

In the above equations the following quantities appear: K is the ratio of proton-to-electron number densities per particle energy interval for the normalization energy E_0, E_2 represents the high-energy limit for the spectrum of cosmic ray particles. The spectral break at low energies for protons is designated as $E_{\rm p} = 938.28\,{\rm MeV} = 1.5033 \cdot 10^{-3}$ erg and finally $c_4(i)$ is used to replace the projected field component $H_\perp$ with the total field H (see Appendix A in BK), with i being the projection angle.

Equations (11) and (12) were originally taken from BK, with a few adjustments. To make the equations hold for $\gamma \leq 2$ we used $E_2 = 3 \times 10^{15}$ eV (Vink 2004). Instead of a $(K+1)$ factor we used only K which is justified for a proton-dominated plasma, and because the original formulae do not include the effect of possible synchrotron losses that affect the electron power law energy spectrum. Using K instead of $(K+1)$ may provide an even better approximation when taking into account synchrotron losses. To put it simply, it is as if there were almost no electrons in cosmic rays, and only protons remained. This can be justified by the fact that protons are far more energetic than electrons and show smaller synchrotron losses. Such an assumption does not have any significant effect on the values for H because of the $1/(\alpha+3)$ exponent in Equations (11) and (12). In this case, Equation (9) transforms into

$$H \propto \left(D^{-3} L_\nu\right)^{1/(\alpha+3)} \propto \left(D^{-4.4}\right)^{1/(\alpha+3)} , \quad (16)$$

and Equation (10) becomes

$$\delta = (3+s)\frac{1}{\overline{\alpha}+3} . \quad (17)$$

In Eq. (16) we applied the $L_\nu - D$ correlation, to obtain $\delta = 1.22$, while fitting gives $\delta = 1.19 \pm 0.08$. For α we used an average spectral index of the whole sample ($\overline{\alpha} = 0.6$). The value for δ from Equation (17), and the one obtained by fitting calculated values for H using Equation (12), are almost identical. The difference is negligible and we could have, as in CEC, calculated δ from the slope of $L_\nu - D$ relation.

In calculating H, we assumed that the magnetic field in the radiative shell of SNR is completely turbulent and has an isotropic angle distribution in three dimensions, giving $c_4 = (2/3)^{(\gamma+1)/4}$ (Appendix A in BK). This is the best assumption to be made when the majority of SNRs are point-like sources, i.e. without maps for H. We also used $K = (E_{\rm p}/E_{\rm e})^{(\gamma-1)/2}$ (Appendix A in BK), where $E_{\rm e} = 511$ keV $= 8.187\ 10^{-7}$ erg designates the spectral break at low energies for electrons. The data for 21 SNRs from M82 from the work of Urošević et al. (2005), and the obtained values for H, are shown in Table 2. As it can be seen, the magnetic field strengths are up to 10 mG. Using $L_\nu/(4\pi f V)$ in our formulae instead of I_ν/l (BK) could lead to an overestimation of the average field. Nevertheless, if the magnetic field is significantly overestimated it should not have a significant effect on the value for δ. There is also a possibility that M82 remnants are pulsar driven wind nebulae (PWNe). Unlike shell type SNRs, PWNe have different mechanisms that maintain magnetic fields. Magnetic field strengths in PWNe are comparable with the ones we obtained from REC for M82 SNRs. This possibility is investigated further in Section 3.4.

3.3. *Direct Derivation*

It is possible to derive δ directly from Eq. (2) if there is an additional assumption concerning the evolution of $n_{\rm e,0}$ with D. We consider models used by Shklovsky (1960), and the assumption of conservation of cosmic ray energy i.e. adiabatic expansion (e.g. BV). Respectively, these are

$$n_{\rm e,0} \propto D^{-(2\alpha+3)} \quad (18)$$

and

$$n_{\rm e,0} \propto D^{-3} . \quad (19)$$

Equation (2) together with the $L_\nu - D$ relation gives

$$H \propto \left(\frac{D^{-4.4}}{n_{\rm e,0}}\right)^{1/(\alpha+1)} . \quad (20)$$

For an average spectral index $\alpha = 0.6$ the results are presented in Table 1. Here, we found fitting unnecessary because we already saw in Sections 3.1 and 3.2 that the rest of the quantities from Equation 2

TABLE 2

SNRS DATA[a] AND RESULTS

Catalog name	Other name	Type[1]	D (pc)	S_1 flux density at 1 GHz (mJy)	α	Distance (kpc)	$H^{\rm eqp}$ class. equip. (G)	$H^{\rm eqp}_{\rm rev}$ rev. equip. (G)	H_1 literature (G)
M82 39.1+57.4	···	MC	0.9	8.28	0.50	3.9×10^3	6.03E-03	8.76E-03	···
M82 39.4+56.1	···	MC	3.23	4.25	0.58	3.9×10^3	1.68E-03	2.10E-03	···
M82 39.6+53.4	···	MC	2.65	2.68	0.45	3.9×10^3	1.74E-03	2.96E-03	···
M82 40.6+56.1	···	MC	3.02	4.97	0.72	3.9×10^3	1.94E-03	2.24E-03	···
M82 40.7+55.1	···	MC	1.93	15.56	0.58	3.9×10^3	3.78E-03	4.64E-03	···
M82 41.3+59.6	···	MC	1.02	6.19	0.52	3.9×10^3	4.99E-03	6.85E-03	···
M82 42.7+55.7	···	MC	4.30	6.10	0.71	3.9×10^3	1.51E-03	1.78E-03	···
M82 42.8+61.3	···	MC	1.97	3.58	0.63	3.9×10^3	2.47E-03	2.92E-03	···
M82 43.2+58.4	···	MC	1.05	12.61	0.66	3.9×10^3	6.11E-03	6.83E-03	···
M82 43.3+59.2	···	MC	0.60	29.54	0.68	3.9×10^3	1.27E-02	1.35E-02	···
M82 44.3+59.3	···	MC	1.96	5.46	0.64	3.9×10^3	2.80E-03	3.27E-03	···
M82 44.5+58.2	···	MC	2.25	3.55	0.50	3.9×10^3	2.16E-03	3.13E-03	···
M82 45.2+61.3	···	MC	1.12	19.54	0.67	3.9×10^3	6.58E-03	7.28E-03	···
M82 45.3+65.2	···	MC	2.05	5.80	0.82	3.9×10^3	2.96E-03	3.32E-03	···
M82 45.4+67.4	···	MC	2.23	5.01	0.67	3.9×10^3	2.47E-03	2.86E-03	···
M82 45.8+65.3	···	MC	2.13	3.74	0.46	3.9×10^3	2.30E-03	3.79E-03	···
M82 45.9+63.9	···	MC	2.22	4.25	0.41	3.9×10^3	2.32E-03	4.70E-03	···
M82 46.5+63.9	···	MC	1.39	6.93	0.74	3.9×10^3	4.18E-03	4.60E-03	···
M82 46.7+67.0	···	MC	2.95	4.39	0.76	3.9×10^3	1.94E-03	2.25E-03	···
M82 41.9+58.0	···	MC	0.52	154.96	0.75	3.9×10^3	2.38E-02	2.32E-02	···
M82 44.0+59.6	···	MC	0.79	54.89	0.48	3.9×10^3	1.16E-02	1.80E-02	···
G 111.7-2.1	Cas A	O	4.9	2720×10^3	0.77	3.4	1.02E-03	1.23E-03	5.5E-04[b]
G 260.4-3.4	Pup A	O	35.2	130×10^3	0.5	2.2	5.73E-05	8.32E-05	···
LMC 0525-69.6	N132 D	O	25	5800	0.7	55	2.07E-04	2.71E-04	$<$ 4E-05[c]
SMC 0103-72.6	···	O	55	250	0.5	65	4.53E-05	6.58E-05	···
NGC 4449	···	O	0.6	20	0.75	4200	1.22E-02	1.25E-02	···
G 42.8+0.6	···	MC	76.8	3×10^3	0.5	11	2.51E-05	3.64E-05	···
G 78.2+2.1	γ Cygni	MC	20.9	340×10^3	0.5	1.2	8.34E-05	1.21E-04	···
G 84.2-0.8	···	MC	23.6	11×10^3	0.5	4.5	6.00E-05	8.71E-05	···
G 89.0+4.7	HB 21	MC	24.2	220×10^3	0.4	0.8	5.20E-05	9.87E-05	···
G 132.7+1.3	HB 3	MC	51.2	45×10^3	0.6	2.2	3.10E-05	4.24E-05	···
G 166.2+2.5	OA 184	MC	183.8	11×10^3	0.57	8	1.43E-05	2.08E-05	···
G 309.8+0.0	···	MC	23	17×10^3	0.5	3.6	6.11E-05	8.88E-05	···
G 315.4-2.3	MSH 14-63	MC	28.1	49×10^3	0.6	2.3	5.45E-05	7.34E-05	···
G 349.7+0.2	···	MC	8.7	20×10^3	0.5	14.8	3.3E-04	4.80E-04	3.5E-04[d]
G 4.5+6.8	Kepler	B	2.4	19×10^3	0.64	2.9	3.95E-04	4.97E-04	2.15E-04[b]
G 120.1+1.4	Tycho	B	5	56×10^3	0.61	2.3	2.49E-04	3.21E-04	3E-04[b]
G 327.6+14.6	SN 1006	B	19	19×10^3	0.6	2.2	5.67E-05	7.62E-05	1.6E-04[b]
LMC 0505-67.9	DEM L71	B	19	9	0.5	55	3.96E-05	5.75E-05	···
LMC 0509-68.7	N103 B	B	7	1100	0.6	55	3.72E-04	4.75E-04	···
LMC 0509-67.5	···	B	7	70	0.5	55	1.68E-04	2.43E-04	···
LMC 0519-69.0	···	B	8	150	0.5	55	1.86E-04	2.70E-04	···
LMC 0548-70.4	···	B	25	100	0.6	55	6.29E-05	8.44E-05	···
SMC 0104-72.3	···	B	29	12	0.5	65	3.29E-05	4.78E-05	···

[a]M82 data are taken from Table A.1 in Urošević et al. (2005) with S_1 being scaled from 1.4 to 1 GHz. The rest of the used data are same as in the papers of Arbutina et al. (2004) and Arbutina & Urošević (2005), with updated distance data of Galactic MC SNRs from Green (2004).

[b]Völk, Berezhko, & Ksenofontov (2005).

[c]Dickel & Milne (1995).

[d]Brogan et al. (2000).

[1]MC – Associated with giant molecular clouds, O – Oxygen-rich, B – Balmer-dominated.

do not change with D, at least not in a way to affect δ. By using the direct method we can only get values for H scaled to a constant because of the proportionality of Equations (18) and (19).

3.4. *Calculated and Literature-found H Values for GMC, Oxygen-rich and Balmer-dominated SNRs*

With a view to checking values obtained for H we performed the same REC on the SNRs associated with large molecular clouds, on oxygen-rich and on Balmer dominated SNRs. According to Arbutina & Urošević (2005), these SNRs form parallel tracks in the radio surface brightness – diameter plane. If the environmental density is higher we expect the SNR to be brighter. The implication is that SNRs with the same D should have different luminosities if the environmental densities are different. According to Equation (12), SNRs that evolve in a more dense environment should also have stronger H than SNRs with the same diameter that evolve in a less dense environment. The data used and the obtained CEC and REC results for all groups of SNRs are presented in Table 2.

Figure 1 presents a plot of all REC values from Table 2. It shows that SNRs in a more dense environment (M82, GMC, oxygen-rich) appear to form a track in $H-D$ plane, while Balmer-dominated SNRs form another track that lies beneath the first one. Due to dispersion and incompleteness of the data samples, any statistical study of the tracks should be avoided, for now. We can, however, arrive at some qualitative conclusions. In Figure 1 we can see that REC does not change the $L_\nu - D$ evolution pattern. This is very convenient for the estimate of the reliability of H in M82 SNRs. From Figure 1 it is clear that the H values for SNRs in M82 seem consistent with the values for GMC and oxygen-rich remnants. They all evolve in a dense environment and accordingly may have a similar $H-D$ evolution pattern. Their H values, according to Arbutina & Urošević (2005), are different in comparison to the values for Balmer-dominated SNRs. This is because Balmer-dominated SNRs are likely to evolve in a low density environment. In the group that consists of Balmer-dominated, oxygen-rich and GMC SNRs, used in this work, we did not include PWNe, because REC is made for shell type SNRs. Accordingly, to avoid possible PWNe, we did not include SNRs with $\alpha \leq 0.4$, which is the characteristic of PWNe (Gaensler & Slane 2006). From Figure 1 we can see that most of SNRs in M82 are, probably, not PWNe because they fit the evolution pattern for SNRs in dense environments. In addition, the higher spectral indices

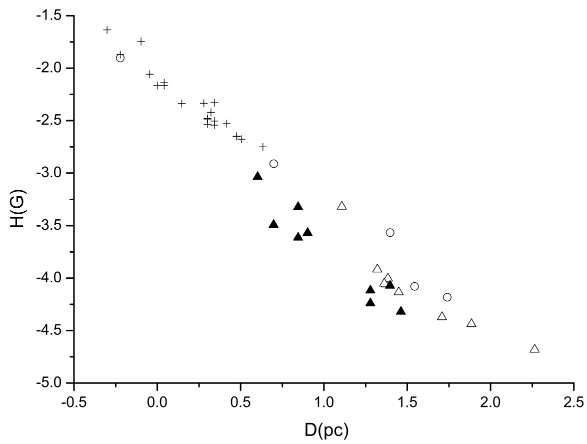

Fig. 1. The revised equipartition data in the $\log H - \log D$ plane. The SNRs are represented by: crosses (M82), open circles (oxygen-rich), open triangles (Galactic SNRs associated with large molecular clouds), filled triangles (Balmer-dominated SNRs).

of the M82 SNRs (average $\alpha \approx 0.6$; see Table 2) are not characteristic for PWNe. However, the possibility that at least some of these objects are PWNe should not be easily put aside. For now, we can only wait for the observational instruments to advance, and for a possible detection of pulsars in M82.

Table 2 also shows the best available literature-found values for H inferred from other methods, for Galactic and LMC SNRs. The agreement of these values with the values obtained from REC is another way to show the reliability of H estimates for SNRs in M82. This is one of the subjects discussed in Section 4.

4. DISCUSSION

4.1. *Values Obtained for H*

Both classical and revised equipartition calculation contain various uncertainties and assumptions and as such, are of limited applicability (BK). Nevertheless, by performing CEC and REC we arrived at the conclusion that all the imperfections do not have a noticeable effect on δ, but could have a significant impact on the values for H. Inspection of Table 2 shows that the obtained H values are higher than those found in literature. Such overestimates are probably due to replacement of I_ν with L_ν (BK). The assumptions regarding f and K in REC equations are not of great importance because of the small $1/(\alpha+3)$ exponent. Due to the $L_\nu \to I_\nu$ replacement, the magnitude of the overestimate is strongly affected by SNR morphology and consistently shows considerable variations from one SNR to another

(Table 2). The morphology variations should not depend on diameter, which means that overestimates of H are mainly arising from morphology-related factors, and they should only cause data scattering in the $H-D$ plane, without affecting δ. Table 2 also shows that an average overestimate by a factor of 2 can be adopted. Coupled with the explanation of Figure 1 (Section 3.4), this shows that the H values for SNRs in M82 are estimated reliably to an order of magnitude. This means that M82 does contain SNRs with magnetic fields of up to 10^{-2} G. However, this should be taken with reserve because of the possibility that some SNRs in M82 are perhaps PWNe.

RECs used in this paper thus give reliable estimates accurate to an order of magnitude. This is of small significance in studies of nearby, well resolved SNRs with data from all parts of the electromagnetic spectrum, but may be of great applicability in statistical and empirical studies of SNRs residing in other galaxies, that are unresolved and often have only radio data available. As already mentioned, Galactic SNR samples are strongly influenced by selection effects and cannot be used in statistical and empirical studies of SNRs evolution properties. For now, the only SNR samples that can be used for reliable statistical and empirical studies reside in other galaxies. With these samples, the obtained values for H will be probably overestimated by a factor of 2, but will be accurate to an order of magnitude, as in this paper. In the next section we discuss the results on the magnetic field evolution obtained when our method is applied to SNRs in M82. This should illustrate how the method can be used for getting closer insight into the SNRs evolution properties, i.e. SNRs evolution phases, and how it can be used to check the validity of the equipartition assumption.

4.2. *Magnetic Field Evolution of SNRs in M82*

If the sample is statistically reliable, the obtained H may be overestimated, but chosen REC parameter values should not have a significant effect on δ. The difference between δ obtained from classical and revised methods is mainly due to the exponents in equations (7) and (12). These exponents will be equal for an average spectral index $\alpha = 0.5$. For SNRs in M82 $\bar{\alpha} = 0.6$ is used, and therefore we obtain slightly different slopes in the $H-D$ plane. In the work of Berkhuijsen (1986), the author implies that α could depend on the density of the ISM in which the SNRs evolve as: $\alpha = (0.075 \pm 0.024) \log n_0 + (0.538 \pm 0.012)$, where n_0 is the density of the ISM. This means, according to Eq. (17), that the lower track in Figure 1, that consist of SNRs evolving in low-density environments, should have a somewhat shallower slope when compared with the track above (high density environments). However, considering Eq. (17) and the just mentioned relation, it is clear that for typical values of α and n_0, there will be no significant effect on δ. Consequently, the tracks from Fig. 1 should be considered as parallel. Taking all of above into account, we conclude that δ is strongly affected by the assumptions regarding $n_{\mathrm{e},0}$. "Directly" obtained values for δ of 0.125 and 0.875 (Table 1) are only illustrative. Shklovsky's model have a rather historical meaning, since no additional particle acceleration (by the shock) during evolution is assumed (besides the initial acceleration in the supernova explosion). This leaves us with equipartition as our best assumption.

Table 1 shows that the equipartition arguments combined with the possible $L_\nu - D$ dependence give $\delta \approx 1.2$. This value is slightly lower than the theoretical value $\delta = 1.5$ obtained under equipartition and adiabatic approximations (Section 2.5). If SNRs in M82 are young, in early Sedov or free expansion phase, this difference can be explained by the sensitivity selection effects related to the M82 sample. The Monte Carlo simulations in Urošević et al. (2005) show that the measured slopes of extragalactic surface brightness to diameter ($\Sigma_\nu - D$) relations are shallower due to the sensitivity selection effects. Therefore, the apparent $\Sigma_\nu - D$ (and $L_\nu - D$) slope for M82 is lower than the real slope. The lower $L_\nu - D$ slope gives lower δ. This means that equipartition arguments for the SNRs in M82 sample may still be applicable, whereas a small difference between the theoretical and empirical δ can be ascribed to selection effects.

On the other hand, $\delta = 1.2$ might indicate that not all SNRs from the M82 sample are in the equipartition state. If, for example, the larger ones are in the late Sedov phase where the magnetic field remains constant (BV), the empirical δ would be a compromise between values 0 and 1.5. The evolutionary status of SNRs remains greatly uncertain. The SNRs in M82 may be in free expansion, as well as in the Sedov, or even in the radiative phase. Chevalier & Fransson (2001) proposed that M82 SNRs may be in the radiative phase because they evolve in a very dense environment. In this case, δ may be 1.25, close to the empirical value. As the previous ones, this scenario too, remains uncertain.

4.3. *Interstellar Magnetic Field in M82*

Condon (1992) estimated the field strength in M82 to be $H \approx 100$ μG from classical minimum energy calculation, considering that the central emitting region of M82 is $30'' \times 10''$ and probably 0.5 kpc thick. Hargrave (1974) estimated the central emitting region in M82 to be $50'' \times 15''$. Using revised equipartition we estimated a value of ≈ 190 μG for the average interstellar magnetic field in the central emitting region of M82 using the data $S_{1.4\ \mathrm{GHz}} =$ 8.2 Jy and $\alpha = 0.68$ from Klein, Wielebinski, & Morsi (1988). We assumed that $f = 1$ and that M82 radiates mainly from its central region of ≈ 500 pc in diameter. This estimate is rough and should be taken with some reserve. Such ISM magnetic field strength is among the highest field strengths when compared to other galaxies. This, however, may imply that the M82 central region contains interstellar matter made of very dense molecular clouds. This is consistent with the high values of H in M82 SNRs, supporting the possibility that their luminous synchrotron emission is mainly due to very dense environments and not due to pulsar driven wind nebulae.

The values of up to 10 mG for H in M82 SNRs, however, imply that the magnetic field is strongly amplified from the average ISM values of $100 - 200$ μG.

5. CONCLUSIONS

In this paper we presented and discussed a method for the determination of the magnetic field evolution pattern in SNRs only from the radio luminosity data samples. Such samples are the only ones available for statistical and empirical studies of SNR evolution properties. The best sample, for now, consists of SNRs in M82, since these remnants seem to evolve in a similar environment and share similar properties, and are not severely influenced by selection effects.

In order to calculate H from REC we were forced to make some assumptions. The only significant effect on values for H, regarding the assumptions, comes from replacing I_ν in REC formulas from BK with L_ν, which is done in order to apply REC on practically point-like sources. The other assumptions are less important because of the small exponent in REC equations. Obtained under the equipartition assumption, δ is a direct consequence of the $L_\nu - D$ slope and has a reasonable theoretical explanation. The assumptions do not change the evolutionary picture from the $L_\nu - D$ plane. This means that our empirical estimate of δ is likely to be reliably determined. When compared with the more reliable values found in the literature, the obtained H values appear to be overestimated approximately by a factor of 2. We conclude that H values for all SNRs, even the ones from M82, are accurate to an order of magnitude.

To answer whether or not M82 SNRs are in an equipartition state we have compared the δ obtained by our method with the theoretical values. The empirically obtained δ from the $L_\nu - D$ correlation under the equipartition assumption is probably theoretically explainable by the following two scenarios:

(i) The slight difference between the theoretically derived $H - D$ slope ($\delta = 1.5$) under the adiabatic approximation and the equipartition assumption, and the slope obtained in this paper using the empirical $L_\nu - D$ correlation and REC ($\delta \approx 1.2$) can be explained by the sensitivity selection effects which affect the sample of SNRs in M82. In this way, the starting assumption concerning the approximate equipartition between the energy stored in the relativistic particles and in the magnetic field, could be justified. Therefore, we can conclude that SNRs in the M82 sample are probably close to the equipartition state.

(ii) Finally, equipartition conditions may not be fulfilled for all remnants. If, for instance, they are in different stages of evolution, δ may be between 0 and 1.5.

If SNRs are in the adiabatic phase, the most probable explanation for the lower empirically obtained value for δ is the sensitivity selection effect in the M82 sample, perhaps in combination with slight deviations from equipartition, but the problem is the unresolved evolutionary status of the M82 SNRs. Additional observations of SNRs in nearby starburst galaxies are needed for any firmer conclusions to be reached.

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