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KINEMATICS AND VELOCITY ELLIPSOID OF M GIANTS

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RESUMEN

Para estudiar la cinemática de las estrellas M gigante (clase de luminosidad III) se usaron 1,532 estrellas con movimientos propios incluidos en el catálogo *Hipparcos*, de las cuales 480 poseen velocidades radiales. Se excluyeron estrellas más lejanas que 700 pc porque inducen una notable inclinación en la distribución de las estrellas. Se calcularon varias soluciones de las cuales se tomó la solución dada por la robusta norma L_1 como la mejor. Se calcula simultáneamente una solución para los parámetros cinemáticos y para los coeficientes del elipsoide de velocidades. Los resultados obtenidos son razonables: velocidad solar de 24.20 ± 0.70km s^{-1}; constantes de Oort, en unidades de km s^{-1} kpc^{-1}, $A = 16.86 \pm 2.78$ y $B = -6.34 \pm 2.56$, las cuales indican una velocidad rotacional de 197.27 ± 26.80 km s^{-1}. El elipsoide de velocidades está inclinado marcadamente respecto al plano Galáctico en la dirección y.

ABSTRACT

To study the kinematics of M giant stars (luminosity class III) use is made of 1,532 stars with proper motions taken from the *Hipparcos* catalog, of which 480 have radial velocities. Stars farther off than 700 pc were excluded because they induce a noticeable tilt in the distribution of the stars. Various solutions were performed, and the one calculated by the robust L_1 norm was taken as the best. Kinematical parameters and the coefficients of the velocity ellipsoid are solved for simultaneously. The results obtained are reasonable: solar velocity of 24.20 ± 0.70 km s^{-1}; Oort's constant's, in units of km s^{-1} kpc^{-1}, $A = 16.86 \pm 2.78$ and $B = -6.34 \pm 2.56$, implying a rotational velocity of 197.27 ± 26.80 km s^{-1}. The velocity ellipsoid is tilted considerably with respect to the Galactic plane in the y direction.

Key Words: Galaxy: kinematics and dynamics — methods: numerical

1. INTRODUCTION

Various investigations of the kinematics of M giants have been published, Zhu (2000) and Mignard (2000), for example, although they include these stars as a subset of a larger group. I propose to study exclusively the kinematics of M giants, but to include radial velocities as well as proper motions, to also solve for the coefficients of the velocity ellipsoid, and to use a reduction method for the data, semi-definite programming (SDP), that offers the advantage that the solar motion calculated from the velocity ellipsoid must be the same as that calculated from the kinematical parameters. Nor is it necessary to use the same adjustment criterion for the two set of calculations: the kinematical parameters may be reduced by use of a least squares criterion whereas the velocity ellipsoid may be calculated with the robust L_1 criterion, or the same L_1 criterion may be used for both.

Why the M giants? Recently I published a study of the kinematics and velocity ellipsoid of the O-B5 giants (Branham 2006). The M giants are found at the other end of the spectrum, with different kinematical properties from the early stars, and to study them with techniques similar to those used with the O-B5 stars seems indicated. I have also published a method for deriving stellar space densities that includes a discussion of the space densities of M giants and supergiants (Branham 2003a). To study their kinematics affords a useful complement to their densities.

2. THE OBSERVATIONAL DATA AND REDUCTION MODELS

The proper motions and parallaxes used in this study were taken from the *Hipparcos* catalog (ESA 1997), the radial velocities from the Wilson (Nagy 1991) and Strasbourg Data Center (Barbier-Brossat, Petit, & Figon 1994) catalogs. Stars listed as spectral class M, luminosity class III, were extracted from the *Hipparcos* catalog and the star's HD number used to find if either of the two radial velocity catalogs contained an entry for that particular star. Not all of the data could be accepted. Negative parallaxes were excluded as were parallaxes smaller than 1 mas because the Ogorodnikov-Milne (OM) model was used for the equations of condition (Ogorodnikov 1965). This model, valid out to about 1 kpc, should be adequate because the minimum *Hipparcos* parallax used in this study, 1 mas, corresponds to a distance of 1 kpc. Parallaxes smaller than 1 mas have such large mean errors that their inclusion would seem unwarranted because of the uncertainty in their distances. Known multiple stars, flagged in the *Hipparcos* catalog, contaminate the proper motion by confusing orbital motion with genuine proper motion and should be excluded. And some of the Hipparcos solutions for the astrometric data in the catalog are substandard ($\chi^2 \succ 3$), also flagged in the catalog, and should likewise be excluded. I also decided to exclude the stars that have radial velocities but also exhibit large variability, *Hipparcos* variability index of 3. These stars may contaminate the radial velocity needed in the kinematical calculations with spurious variations.

In this way 1,573 M III stars were found, of which 483 have radial velocities. The distribution among the subdivisions of the M class is highly skewed: only 9.5% of the sample is M5 or later and no star is M9. A star's position and proper motion in right ascension (α) and declination (δ) were converted to position and proper motion in Galactic longitude (l) and latitude (b). One takes the required data from the Hipparcos catalog and converts to l and b by use of the relation

$$\begin{pmatrix} \cos l \cos b \\ \sin l \cos b \\ \sin b \end{pmatrix} = \mathbf{M} \cdot \begin{pmatrix} \cos \alpha \cos \delta \\ \sin \alpha \cos \delta \\ \sin \delta \end{pmatrix} \quad , \tag{1}$$

where $\mathbf{M}$ is an orthogonal matrix given by

$$\begin{pmatrix} -\sin \alpha_0 \cos l_0 + \cos \alpha_0 \sin \delta_0 \sin l_0 & -\sin \alpha_0 \sin l_0 - \cos \alpha_0 \sin \delta_0 \cos l_0 & \cos \alpha_0 \cos \delta_0 \\ \cos \alpha_0 \cos l_0 + \sin \alpha_0 \sin \delta_0 \sin l_0 & \cos \alpha_0 \sin l_0 - \sin \alpha_0 \sin \delta_0 \cos l_0 & \sin \alpha_0 \cos \delta_0 \\ -\cos \delta_0 \sin l_0 & \cos \delta_0 \cos l_0 & \sin \delta_0 \end{pmatrix} \quad ;$$

α_0 and δ_0 are the equatorial coordinates of the Galactic origin and l_0 the longitude of the ascending node of the Galactic plane on the equator. Their respective numerical values for J2000, calculated taking into account Land's (1979) remarks about the conversion of equatorial to Galactic coordinates, are: $\alpha_0 = 12^h 51^m 26.^s 2754$; $\delta_0 = 27°07' 41.'' 705$; $l_0 = 32°55' 54.'' 905$. Proper motion in Galactic longitude, μ_l, and in Galactic latitude, μ_b, follow from their counterparts in α and δ, μ_α and μ_δ, by use of the relations

$$\begin{aligned} \mu_l \cos b &= \mu_\alpha \cos \delta \cos \phi + \mu_\delta \sin \phi \quad ; \\ \mu_b &= -\mu_\alpha \cos \delta \sin \phi + \mu_\delta \cos \phi \quad , \end{aligned} \tag{2}$$

where ϕ is the Galactic parallactic angle.

Let x, y, z be rectangular coordinates with origin at the Sun: x points towards the Galactic center, y is perpendicular to x in the direction of increasing l, and z is positive for positive Galactic latitude. If π represents the star's parallax, measured in milli-arc-seconds (mas), then

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{\pi} \begin{pmatrix} \cos l \cos b \\ \sin l \cos b \\ \sin b \end{pmatrix} \quad . \tag{3}$$

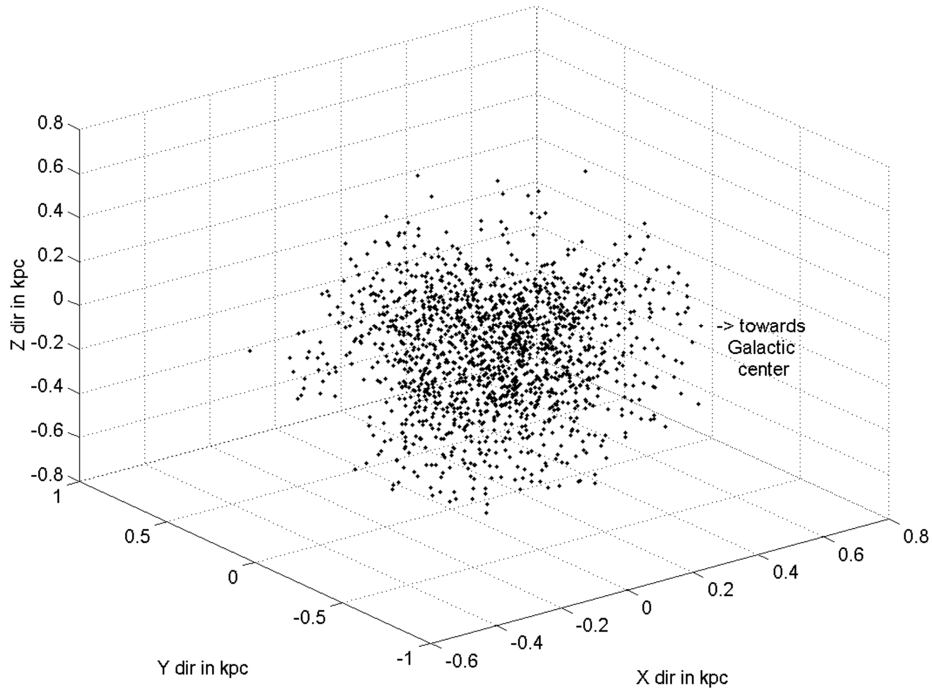

Fig. 1. Distribution of 1,532 M III stars.

Figure 1 shows the distribution of the stars in space and Figures 2–4 the distributions in the x-y, x-z, and y-z planes. There appears no concentration towards the Galactic plane, as is evident with the O-B5 stars, nor into spiral arms, also evident with the O-B5 stars.

Let the proper motion be measured in mas yr^{-1}, let $\dot{r}$ be the radial velocity in km s^{-1}, and X, Y, Z the components of the reflex solar motion in km s^{-1}. Define the auxiliary parameters (because we are now in the Galactic system rather than in the right ascension and declination system I will reuse the symbol α for another purpose)

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} \cos l \cos b \\ \sin l \cos b \\ \sin b \end{pmatrix} ; \quad \begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} -\sin l \\ \cos l \\ 0 \end{pmatrix} ; \quad \begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} -\cos l \sin b \\ -\sin l \sin b \\ \cos b \end{pmatrix} . \tag{4}$$

The OM model studies the motion of a group of stars whose centroid is located at distance R_0 from the Galactic center. r is the distance from the centroid (the Sun) to the star and V the velocity of the centroid at distance R from the Galactic center. From elementary calculus we have

$$V = V_0 + D \cdot r \quad , \tag{5}$$

where D is the displacement tensor of partial derivatives evaluated at R_0,

$$D = \begin{pmatrix} \partial V_x/\partial x & \partial V_x/\partial y & \partial V_x/\partial z \\ \partial V_y/\partial x & \partial V_y/\partial y & \partial V_y/\partial z \\ \partial V_z/\partial x & \partial V_z/\partial y & \partial V_z/\partial z \end{pmatrix}_{R=R_0} = \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{pmatrix} . \tag{6}$$

Equation (5) involves a total of twelve unknowns, the three components of the reflex solar motion and the nine components of the displacement tensor.

From equation (4) and equation (5) can be derived the equations of condition for Galactic kinematics:

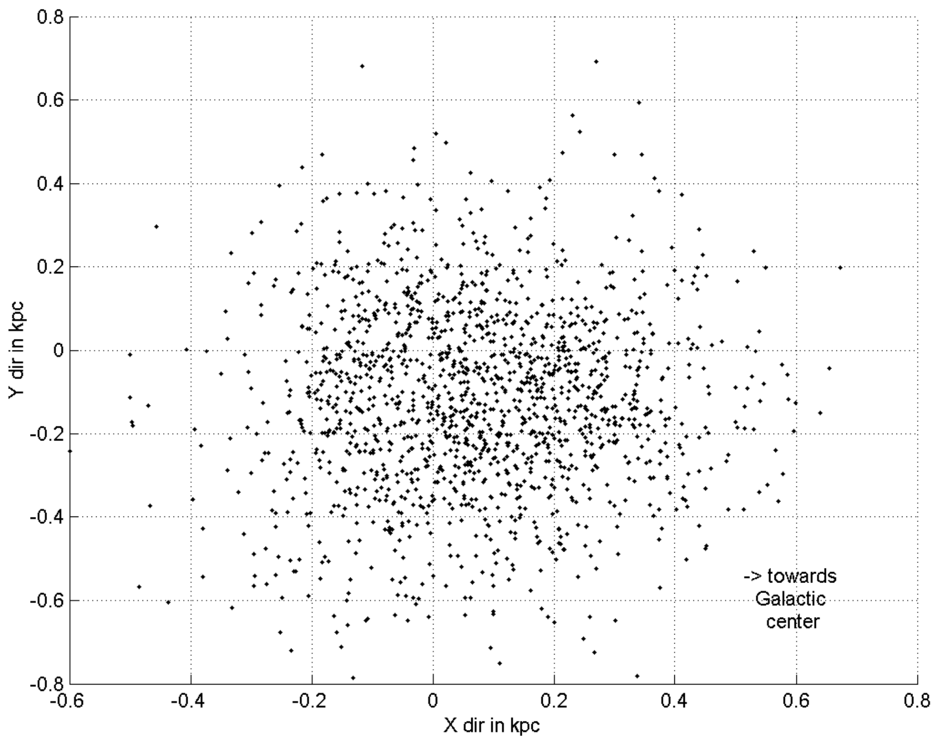

Fig. 2. Distribution in $x - y$ plane.

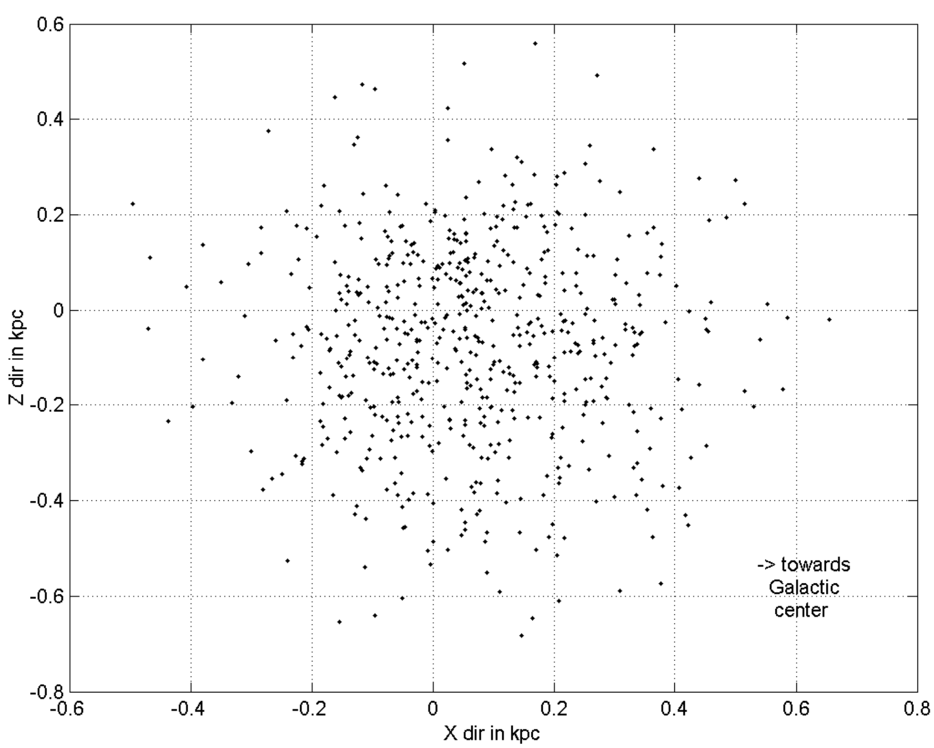

Fig. 3. Distribution in $x - z$ plane.

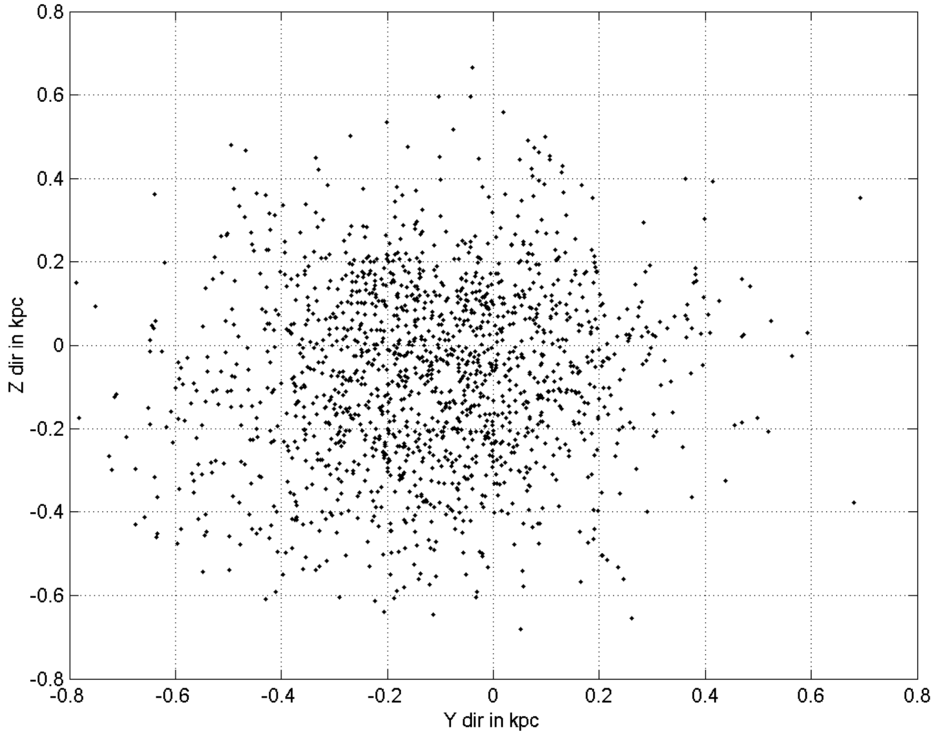

Fig. 4. Distribution in $y - z$ plane.

$$\alpha^2 u_x + \alpha\beta u_y + \alpha\gamma u_z + \alpha\beta v_x + \beta^2 v_y + \beta\gamma v_z + \alpha\gamma w_x + \beta\gamma w_y + \gamma^2 w_z - \pi\alpha X - \pi\beta Y - \pi\gamma Z = \pi\dot{r} \quad ; \tag{7}$$

$$\sec b(-\alpha\beta u_x - \beta^2 u_y - \beta\gamma u_z + \alpha^2 v_x + \alpha\beta v_y + \alpha\gamma v_z) + \pi\alpha_1 X + \pi\beta_1 Y + \pi\gamma_1 Z = \kappa\mu_l \quad ; \tag{8}$$

$$\begin{aligned} -\sec b[\alpha^2\gamma u_x + \alpha\beta\gamma u_y + \alpha\gamma^2 u_z + \alpha\beta\gamma v_x + \beta^2\gamma v_y + \beta\gamma^2 v_z + \alpha(\gamma^2 - 1)w_x + \beta(\gamma^2 1)w_y - \gamma(\alpha^2 + \beta^2)w_z \\ +\pi\alpha_2 X + \pi\beta_2 Y + \pi\gamma_2 Z = \kappa\mu_b \quad , \end{aligned} \tag{9}$$

where κ is a conversion constant with value 4.74047 (the velocity in km s^{-1} corresponding to 1 AU yr^{-1}). Because of substantial parallax error, the median parallax in the *Hipparcos* catalog is 4.790 mas with median error of 1.090 mas, the components of the equations associated with the solar velocity contain significant error. Smith and Eichhorn's procedure (1996) corrected the parallaxes.

To calculate the velocity ellipsoid let $\dot{x}, \dot{y}, \dot{z}$ be the space velocities of a star. These are found from differentiation of equation (3):

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -\sin l & -\cos l \sin b & \cos l \cos b \\ \cos l & -\sin l \sin b & \sin l \cos b \\ 0 & \cos b & \sin b \end{pmatrix} \cdot \begin{pmatrix} \kappa\mu_l \cos b/\pi \\ \kappa\mu_b/\pi \\ \dot{r} \end{pmatrix} \quad . \tag{10}$$

The quadric surface to fit to these velocities becomes

$$a\dot{x}^2 + b\dot{y}^2 + c\dot{z}^2 + d\dot{x}\dot{y} + e\dot{x}\dot{z} + f\dot{y}\dot{z} + g\dot{x} + h\dot{y} + k\dot{z} - q = 0 \quad , \tag{11}$$

where $a, ..., q$ are the ten coefficients to define the quadric that must be determined. The coefficients g, h, k determine the solar velocity. Equation (11) may be transformed into the more symmetric

$$\begin{pmatrix} \dot{x}-X & \dot{y}-Y & \dot{z}-Y \end{pmatrix} \cdot \begin{pmatrix} a & d/2 & e/2 \\ d/2 & b & f/2 \\ e/2 & f/2 & c \end{pmatrix} \cdot \begin{pmatrix} \dot{x}-X \\ \dot{y}-Y \\ \dot{z}-Z \end{pmatrix} = q \quad . \tag{12}$$

or, in more compact notation

$$(v-l)^T A(v-l) = q \quad ,$$

where A is the matrix and $v = \begin{pmatrix} \dot{x} & \dot{y} & \dot{z} \end{pmatrix}^T$ and $l = \begin{pmatrix} X & Y & Z \end{pmatrix}^T$. To assure that the equation indeed corresponds to an ellipsoid one must impose the condition that the matrix be positive-definite. And to avoid the trivial solution $a = b = \cdots = q = 0$ another condition must be imposed. The one I use is that the volume of the ellipsoid must be a maximum. Because the volume is proportional to the determinant of A, the condition becomes $\det(A) = \max$.

The solar velocity calculated in equation (12) must be the same as the velocity found from equations (7)–(9). This condition can be imposed as part of an SDP formulation of the reduction problem. See Branham (2006) for details. Suffice to say that SDP minimizes the norm, whether least squares or L_1, of the residuals from equations (7)–(9), calculates the coefficients of the velocity ellipsoid, and imposes the conditions that the quadric surface of equation (12) must indeed be an ellipsoid and that the solar velocity must be the same from both the kinematical and the velocity ellipsoid calculations.

3. SOME CORRECTIONS TO THE OBSERVATIONS AND COVARIANCE MATRICES

The total space motions needed in the velocity ellipsoid calculation should be corrected for the effects of Galactic rotation by modifying the proper motions and radial velocities used in the calculations to remove the rotation. This was done by the same procedure used in my prior publication (Branham 2006). Nor were incompleteness factors applied to the sample of the M III giants taken from the *Hipparcos* for the same reason as that given for the O-B5 giants: fault of overlap in magnitude between the *Hipparcos* catalog and possible proper motion catalogs such as Tycho-2.

The covariance matrix is given in equation (25) of my previous publication, and equation (26) shows how to calculate mean errors for quantities, such as the Oort constants, derived from the displacement tensor.

4. RESULTS

After the equations of condition for the kinematical parameters had been formed, I applied two checks for the adequacy of the reduction model. The first check simply calculates the singular values of the matrix of the equations of condition. An inadequate reduction model, for example one in which some unknowns are strongly correlated, often results in a high condition number for the matrix because of small singular values. The matrix's condition number of 11.87, however, is low. Table 1 shows the sorted singular values, none of them insignificant.

The second check calculates Eichhorn's efficiency (Eichhorn 1990). The efficiency of 0.95 strongly indicates that *all* of the variables in the model are necessary and with little correlation among themselves. (The efficiency varies from 0 to 1; an efficiency of 0 means that some unknowns are correlated and therefore redundant, an efficiency of 1 that the unknowns are independent.)

The first solution was calculated from the 3,629 equations of condition and 483 velocities for the velocity ellipsoid. This solution calculated residuals needed to find discordant data. To eliminate the discordant data and perform a second solution I used a filter justified by previous experience: exclude a residual that exceeded five times the mean absolute deviation (MAD) of the residuals. This eliminated 66 equations of condition, a modest 1.82% trim. This coincides well with what is given in Table 7.4 in Mihalas (1968), where 1.7% of M giants are high velocity. It is thus likely that most of the rejected stars are in fact high velocity rather than genuine outliers.

Before calculating a solution, however, one must address the question of whether we have a random sample of M giants. The answer, unfortunately, is negative. A simple calculation of the correlations among the

TABLE 1

SINGULAR VALUES OF THE EQUATIONS OF CONDITION

Singular value
12.53
15.71
15.98
16.16
16.36
19.53
19.91
20.97
23.17
138.63
140.97
157.67

rectangular coordinates of the stars' positions shows slight correlations of 1.3% for $x-y$, -2.3% for $x-z$, but a more disturbing 12.0% for $y-z$. A more refined calculation fits a plane to the data to see if there is a noticeable tilt with respect to the Galactic plane. To fit the plane I employed the same methodology as that of my study of the Sun's distance from the Galactic plane (Branham 2003b). The results showed that, although the Sun is well centered with respect to the M giants, only 1.6 pc from the centroid, the plane defined by the M giants is tilted 21.°9 with respect to the Galactic plane. Because the M stars are not associated with the Gould belt, this must be a selection effect. Further study showed that most of the tilt comes from stars farther away than 700 pc. After 41 of these stars were eliminated, leaving 1,532 stars with 480 radial velocities, the tilt decreased to 8.°8, although it was impossible to eliminate altogether without discarding an unreasonable number of stars. I, therefore, accepted the 8.°8 tilt as the best that can be done and based all further calculations on this selection of M giants: 1,532 stars with 480 radial velocities.

Table 2 shows the solution for the unknowns. For convenience the components for the displacement tensor are converted to the more familiar form of the solar motion, Oort constants, vertex deviation, and K term. Table 3 gives the coefficients of the velocity ellipsoid.

Is Table 2's solution good (according to some definition of "good")? The size of the A constant seems high compared with what others have obtained, B is on the low side albeit less discordant than A. The next section discusses this matter.

5. DISCUSSION

The A constant of Table 2 is higher than what both Zhu (2000) and Mignard (2000) have found, both of whom obtain values near 15–16 km s^{-1} kpc^{-1}. Their studies are not strictly comparable with mine because neither of them uses radial velocities. Zhu, moreover, considers not only M giants but also K giants and, rather than take the parallax from the *Hipparcos* catalog, prefers the spectroscopic parallax from the Skymap catalog (Sande 1999). Mignard uses the Hipparcos parallaxes, but applies no correction for parallax error. Rather, he calculates weights, based on various criteria, for the equations of condition. One should first see, by performing a solution using only the proper motions, if the addition of radial velocities seriously affects the calculation of the Oort constants. Equations (8) and (9) lead to a singular matrix if one solves for all twelve unknowns. One may, nevertheless, calculate a sub-rank solution for all of the unknowns by use of the singular value decomposition (SVD). To calculate this solution I use a program I wrote for the SVD that solves only for the kinematical parameters; no computation of the velocity ellipsoid is attempted. A glance at the relevant solution from Table 4 shows that the A constant still remains high even when radial velocities are suppressed.

One may question whether the Smith-Eichhorn corrections for parallax have removed all of the parallax error. Certain assumptions must be made to derive the corrections that may lack applicability to the actual data.

TABLE 2

SOLUTION FOR KINEMATIC PARAMETERS

Quantity	Value	Mean Error
$\sigma(1)$ (mean error of unit weight in mas km s^{-1})	86.48	...
u_x (in mas km s^{-1})	20.24	6.12
u_y (in mas km s^{-1})	30.84	4.49
u_z (in mas km s^{-1})	3.33	5.22
v_x (in mas km s^{-1})	13.18	5.08
v_y (in mas km s^{-1})	−5.60	5.58
v_z (in mas km s^{-1})	1.43	5.37
w_x (in mas km s^{-1})	4.94	4.60
w_y (in mas km s^{-1})	−6.28	4.14
w_z (in mas km s^{-1})	−1.30	5.35
S_0 (solar velocity in km s^{-1})	24.21	0.70
A (Oort constant in km s^{-1} kpc^{-1})	25.52	3.51
B (Oort constant in km s^{-1} kpc^{-1})	−8.83	3.23
l_1 (vertex deviation)	−15.°21	8.°29
K (K term in km s^{-1})	7.32	4.57

TABLE 3

VELOCITY DISPERSION AND VERTEX DEVIATION

Quantity	Value	Mean Error
Mean absolute deviation of residuals in radians	$2.08847 \cdot 10^{-5}$	...
S_0 (solar velocity in km s^{-1})	24.21	1.73
σ_x (velocity dispersion in x in km s^{-1})	57.40	1.67
σ_y (velocity dispersion in y in km s^{-1})	45.86	1.63
σ_z (velocity dispersion in z in km s^{-1})	33.84	1.02
l_x (longitude of σ_x)	−1.°72	4.°33
b_x (latitude of σ_x)	−12.°80	4.°43
l_y (longitude of σ_y)	78.°80	8.°13
b_y (latitude of σ_y)	35.°94	4.°33
l_z (longitude of σ_z)	−75.°34	5.°51
b_z (latitude of σ_z)	51.°15	4.°42

To check for this possibility I decided to perform a least squares-total least squares (LS-TLS) solution (Branham 2001) using the equations of condition with the parallaxes corrected for parallax error. To use LS-TLS assumes that residual error remains in the equations of condition and that this error is transmitted, by equations (7)-(9), into the calculation of the solar velocity; the other unknowns are considered error-free. If r represents the residuals from the equations of condition, rather than minimize $\mathsf{r}^T \cdot \mathsf{r}$, we minimize $\mathsf{r}^T \cdot \mathsf{r}/(1+S_0^2)^{1/2}$. Table 5 shows the kinematical parameters for the LS-TLS solution and Table 6 the velocity ellipsoid computation.

Given that outliers are present one may abandon the least squares approach for the kinematical parameters in favor of an L_1 solution for both these parameters and those for the velocity ellipsoid calculation. With the SDP approach it is easy to implement an L_1 solution for both classes of parameters. Table 7 shows the kinematical parameters for the L_1 solution, along with mean errors, and Table 8 the solution for the velocity ellipsoid. The solar velocity is the same as that given in Table 7, as it should be because we have enforced the condition that the two solutions give the same velocity. The mean errors, however, are different, as again they should be because different residuals, and hence a different dispersion for the residuals, go into the calculation

TABLE 4

SOLUTION FOR KINEMATIC PARAMETERS – SVD; ONLY PROPER MOTIONS

Quantity	Value	Mean Error
$\sigma(1)$ (mean error of unit weight in mas km s^{-1})	86.48	...
u_x (in mas km s^{-1})	12.06	5.07
u_y (in mas km s^{-1})	33.48	4.79
u_z (in mas km s^{-1})	2.62	4.91
v_x (in mas km s^{-1})	12.69	3.12
v_y (in mas km s^{-1})	−6.13	3.84
v_z (in mas km s^{-1})	−2.21	2.03
w_x (in mas km s^{-1})	3.00	2.41
w_y (in mas km s^{-1})	−8.93	3.78
w_z (in mas km s^{-1})	−5.92	1.70
S_0 (solar velocity in km s^{-1})	22.99	2.33
A (Oort constant in km s^{-1} kpc^{-1})	24.81	2.87
B (Oort constant in km s^{-1} kpc^{-1})	−10.39	2.85
l_1 (vertex deviation)	$-10.^\circ75$	$7.^\circ25$
K (K term in km s^{-1})	2.96	3.22

TABLE 5

SOLUTION FOR KINEMATIC PARAMETERS – LS-TLS

Quantity	Value	Mean Error
$\sigma(1)$ (mean error of unit weight in mas km s^{-1})	86.48	...
u_x (in mas km s^{-1})	9.68	6.12
u_y (in mas km s^{-1})	27.10	4.49
u_z (in mas km s^{-1})	6.55	5.22
v_x (in mas km s^{-1})	12.87	5.08
v_y (in mas km s^{-1})	−8.52	5.58
v_z (in mas km s^{-1})	7.14	5.37
w_x (in mas km s^{-1})	4.94	4.60
w_y (in mas km s^{-1})	−6.28	4.14
w_z (in mas km s^{-1})	−1.30	5.35
S_0 (solar velocity in km s^{-1})	24.18	0.70
A (Oort constant in km s^{-1} kpc^{-1})	21.96	3.50
B (Oort constant in km s^{-1} kpc^{-1})	−7.12	3.23
l_1 (vertex deviation)	$-24.^\circ49$	$9.^\circ64$
K (K term in km s^{-1})	0.58	3.54

of the mean error. To calculate the mean errors I decided to use the usual covariance matrix associated with a least squares solution, but taking the MAD as a measure of dispersion in lieu of the mean error of unit weight, rather than the mean errors calculated from an L_1 error distribution (Branham 1986). This decision is based on the residuals' kurtosis, 20.2, and Q factor, 0.40, more typical of a leptokurtic distribution than the L_1's platykurtic distribution. Thus, although the L_1 solution admirably handles the problem of discordant observations, the calculated mean errors would be grossly overestimated by use of an L_1 error distribution.

By examining Tables 2–8 one sees that, although the coefficients of the velocity ellipsoid do not vary by much, the kinematical parameters, particularly the Oort constants, do vary. The solar velocity is remarkably constant, but with the exception of the L_1 solution the A constant is consistently high; the B constant is low for

TABLE 6

VELOCITY DISPERSION AND VERTEX DEVIATION, LS-TLS

Quantity	Value	Mean Error
Mean absolute deviation of residuals in radians	$2.08847 \cdot 10^{-5}$	...
S_0 (solar velocity in km s^{-1})	24.18	1.66
σ_x (velocity dispersion in x in km s^{-1})	58.83	1.82
σ_y (velocity dispersion in y in km s^{-1})	45.70	1.59
σ_z (velocity dispersion in z in km s^{-1})	32.89	0.91
l_x (longitude of σ_x)	$-2.°56$	$3.°95$
b_x (latitude of σ_x)	$-15.°43$	$3.°78$
l_y (longitude of σ_y)	$76.°74$	$6.°84$
b_y (latitude of σ_y)	$29.°17$	$3.°76$
l_z (longitude of σ_z)	$-72.°02$	$4.°88$
b_z (latitude of σ_z)	$51.°80$	$3.°76$

TABLE 7

SOLUTION FOR KINEMATIC PARAMETERS; L_1

Quantity	Value	Mean Error
MAD (mean absolute deviation in mas km s^{-1})	69.42	...
u_x (in mas km s^{-1})	16.77	4.82
u_y (in mas km s^{-1})	19.78	3.55
u_z (in mas km s^{-1})	11.77	4.13
v_x (in mas km s^{-1})	7.09	4.03
v_y (in mas km s^{-1})	−3.62	4.41
v_z (in mas km s^{-1})	15.06	4.26
w_x (in mas km s^{-1})	1.57	3.67
w_y (in mas km s^{-1})	3.27	3.30
w_z (in mas km s^{-1})	−9.29	4.22
S_0 (solar velocity in km s^{-1})	24.20	0.70
A (Oort constant in km s^{-1} kpc^{-1})	16.86	2.78
B (Oort constant in km s^{-1} kpc^{-1})	−6.34	2.56
l_1 (vertex deviation)	$-9.°22$	$4.°97$
K (K term in km s^{-1})	6.58	3.60

all of the solutions. The K term varies from insignificant (LS-TLS solution), to marginally significant (SVD), to significant (SDP and L_1). Numerous investigations have shown that the K term only seems significant for the early stars, presumably because they have not yet reached dynamical equilibrium. If one were to cherry pick the results one would take the A constant from the L_1 solution, the B constant from the SVD solution, and the K term from the LS-TLS solution. But we cannot cherry pick the results and have to select one of the solutions. Of these I feel most comfortable with the L_1 solution. The criterion, being robust, eliminates no discordant data. A least squares solution, whether ordinary or of the LS-TLS variety, requires selecting a criterion for outlier rejection, and the results can depend heavily on the criterion. The A constant of Table 2 decreases to 20 km s^{-1} kpc^{-1} if I adopt a more extreme 10% trim for the residuals rather than the parsimonious 1.7% actually used. To adopt the LS-TLS solution implies residual error in the parallaxes for which there is no conclusive evidence; it remains a pure assumption. The SVD solution is suspect because it is subrank nor does it include the radial velocities.

TABLE 8

VELOCITY DISPERSION AND VERTEX DEVIATION, L1

Quantity	Value	Mean Error
Mean absolute deviation of residuals in radians	$2.08847{\cdot}10^{-5}$	...
S_0 (solar velocity in km s^{-1})	24.20	1.73
σ_x (velocity dispersion in x in km s^{-1})	57.40	1.67
σ_y (velocity dispersion in y in km s^{-1})	45.86	1.63
σ_z (velocity dispersion in z in km s^{-1})	33.84	1.02
l_x (longitude of σ_x)	$-1.^\circ72$	$4.^\circ33$
b_x (latitude of σ_x)	$-12.^\circ80$	$4.^\circ43$
l_y (longitude of σ_y)	$78.^\circ80$	$8.^\circ13$
b_y (latitude of σ_y)	$35.^\circ93$	$4.^\circ33$
l_z (longitude of σ_z)	$-75.^\circ34$	$5.^\circ51$
b_z (latitude of σ_z)	$51.^\circ15$	$4.^\circ42$

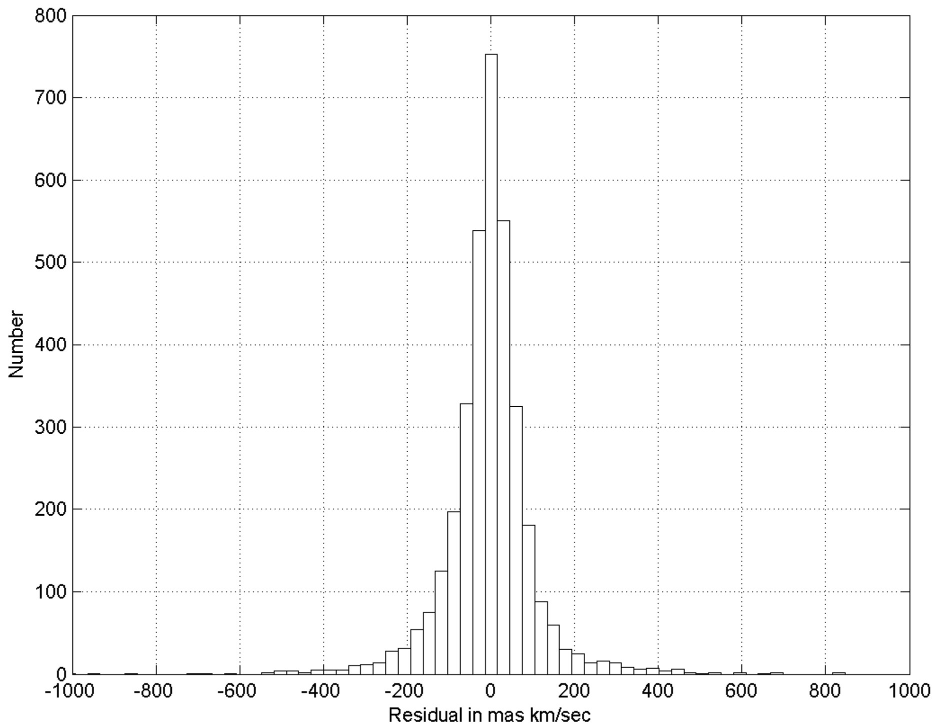

Fig. 5. Histogram of residuals for kinematical parameters.

Taking the L_1 solution as the best, Figure 5 shows a histogram of the residuals from this solution. At first glance the residuals, looking somewhat peaked and skewed, seem to differ from what the normal distribution gives. Statistics confirm this first impression. The coefficient of skewness is -0.94, 0 for the normal distribution; the kurtosis of 20.23 is far from the normal's 3, and the Q factor of 0.40 differs considerably from the normal's 2.58. A runs test for randomness, however, shows 1769 runs for the non-zero residuals out of an expected 1766. From the point of view of formal probability this means that there is a 92% chance that the residuals are random, and the solution may be considered satisfactory. Figure 6 shows the residuals from the velocity ellipsoid computation. The residuals are more skewed, coefficient of skewness 1.84, than their kinematical counterparts and exhibit fewer runs, 219 runs out of an expected 240. Formal probability says that there

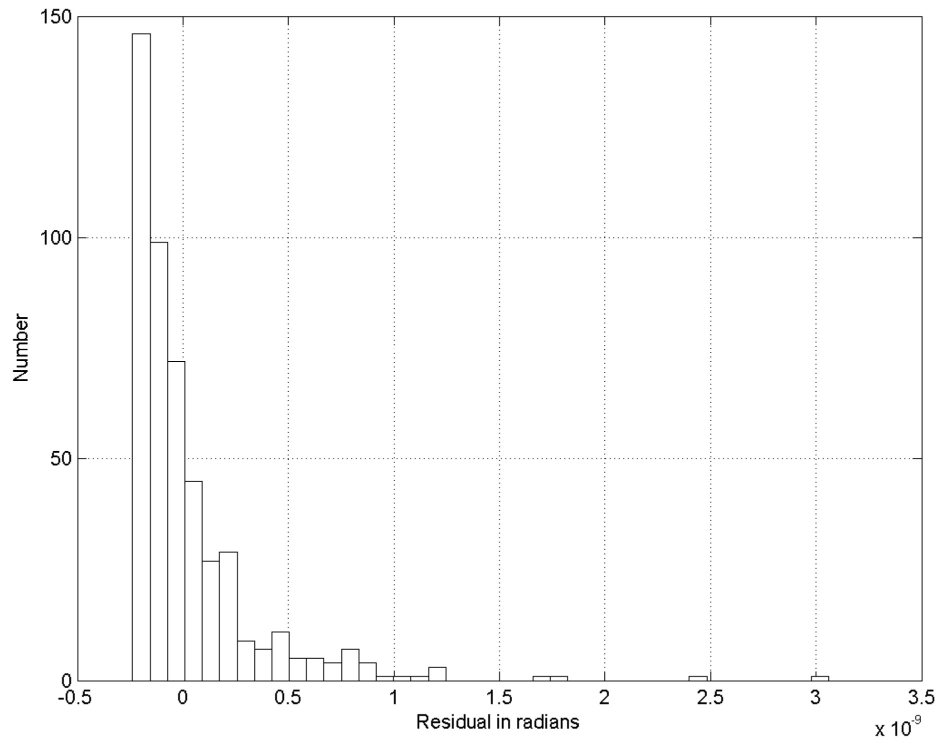

Fig. 6. Histogram of residuals from velocity ellipsoid calculation.

is only a 5.5% chance that these residuals are random, but given that the ellipsoidal hypothesis is only an approximation to the real, and complicated, distribution of stellar velocities, the behavior of the residuals can be considered satisfactory.

Looking at previous studies, such as those Delhaye (1965) summarizes, we find solar velocities in the general range of 17–30 km s^{-1}. My value lies in this range and is higher than what I found for the O-B5 III. This is in line with what numerous investigations have found, that the late stars have higher solar velocities than the early stars. The K term seems significant. The import of this remains to be seen. The A and B constants compare not badly with the IAU values of 14 km s^{-1} kpc^{-1} and -12 km s^{-1} kpc^{-1}, respectively, although B is definitely on the low side. But given its mean error it cannot be considered discordant. Most of the mean errors in Table 7 are higher than those I found for the O-B5 III stars (Branham 2006). This occurs because the dispersion in the residuals is higher: 69.42 mas km s^{-1} for the M III stars versus 24.03 mas km s^{-1} for the O-B5 III stars, although the former is a MAD and the latter a mean error of unit weight.

Figures 7–10 show the total space motions of the M giants and the fitted velocity ellipsoid. An elongated distribution in the velocities is apparent. The velocity ellipsoid is indicated by a dot where a straight line from the center of the ellipsoid to the star intersects the surface of the ellipsoid.

The rotational velocity V_0 at the Sun's distance from the Galactic center is

$$V_0 = (A - B)R_0 \quad , \tag{13}$$

where R_0 is the distance to the center of the Galaxy. Kerr and Lynden-Bell's (1986) estimate R_0 as 8.5±1.1 kpc. To calculate V_0 itself is easy enough given the values in Table 1 and R_0. Its mean error is a little more tricky because R_0 involves no dependence on the kinematical parameters of equations (7)–(9) but nevertheless incorporates a mean error and cannot be treated as a pure constant. Let $\mathbf{C}$ be the covariance matrix from the kinematical parameters. Augment $\mathbf{C}^{-1}$ to include R_0:

$$\begin{pmatrix} \mathbf{C}^{-1} & 0 \\ 0 & R_0 \end{pmatrix} \quad .$$

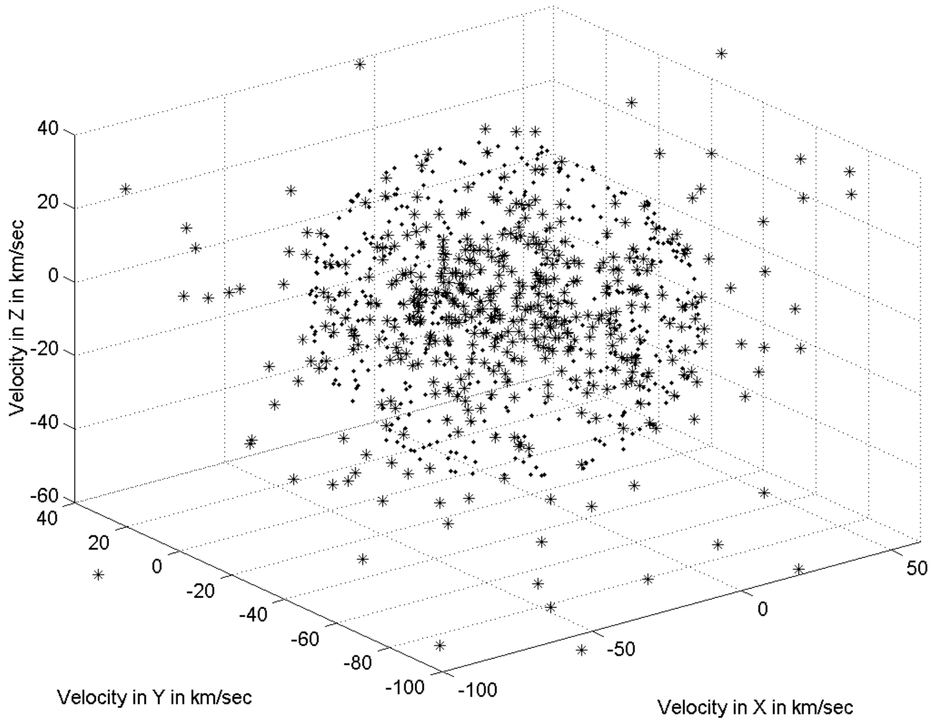

Fig. 7. Velocity ellipsoid in 3-dimensions; *=star, $\cdot$=projection onto ellipsoid.

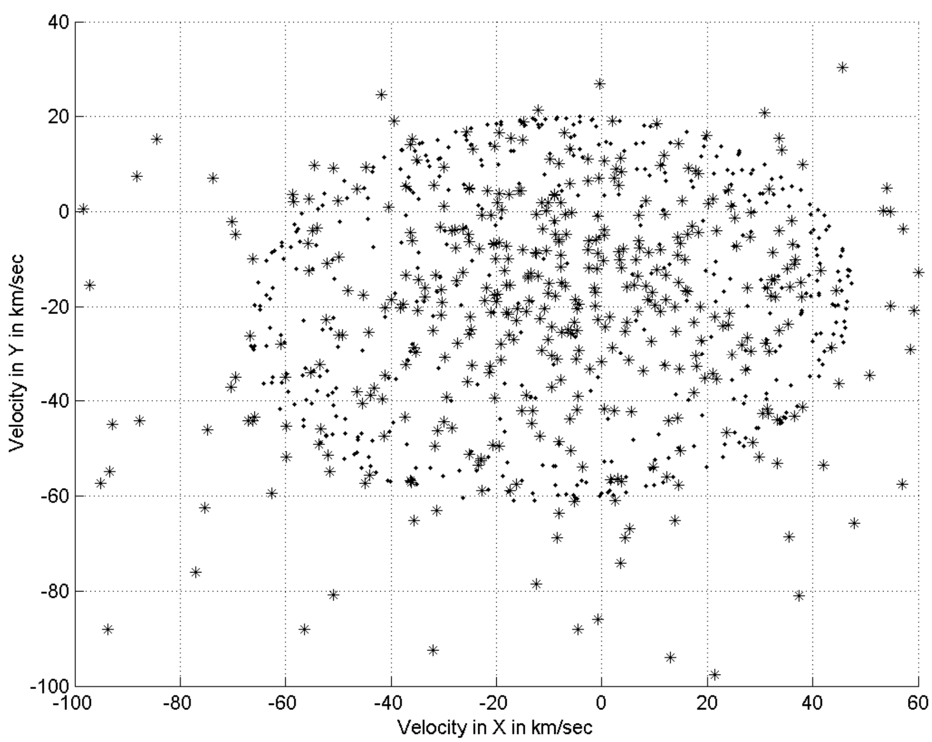

Fig. 8. Velocity ellipsoid in $x-y$ plane; *=star, $\cdot$=projection onto ellipsoid.

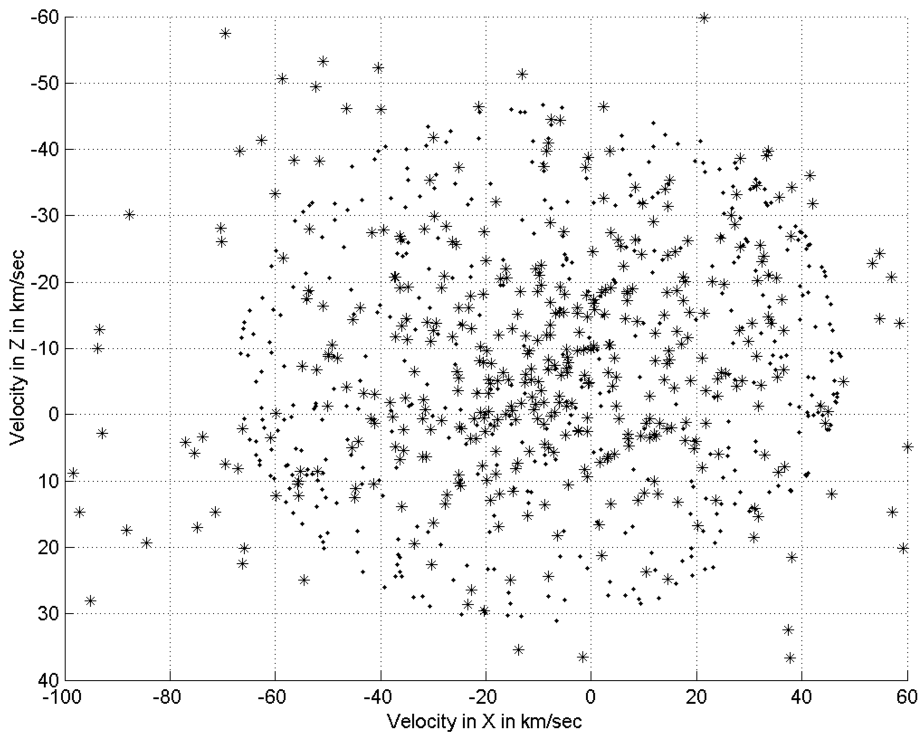

Fig. 9. Velocity ellipsoid in $x - z$ plane; *=star, $\cdot$ =projection onto ellipsoid.

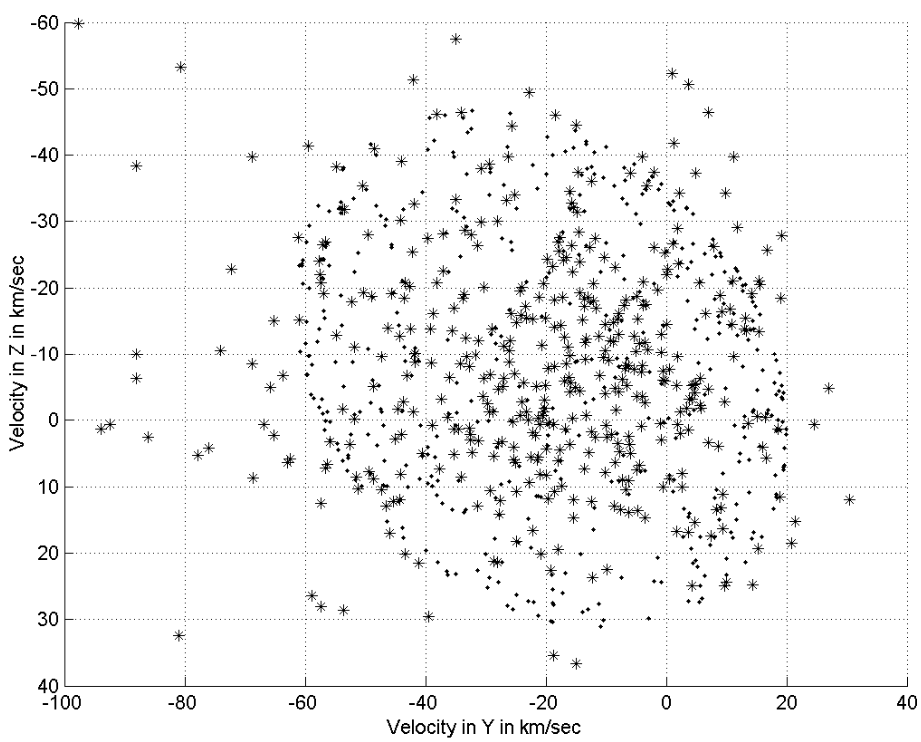

Fig. 10. Velocity ellipsoid in $y - z$ plane; *=star, $\cdot$ =projection onto ellipsoid.

The mean error for V_0 consists of two parts. The first part, $dV_{0,1}$, comes from

$$dV_{0,1} = \sigma(1) \left[\left(\begin{array}{ccc} \partial V_0/\partial u_x & \cdots & \partial V_0/\partial R_0 \end{array} \right) \cdot \left(\begin{array}{cc} \mathbf{C}^{-1} & 0 \\ 0 & R_0 \end{array} \right)^{-1} \cdot \left(\begin{array}{c} \partial V_0/\partial u_x \\ \vdots \\ \partial V_0/\partial R_0 \end{array} \right) \right]^{-1/2} .$$

and gives 7.97 km s^{-1}. The second part comes from $dV_{0,2} = (A - B)dR_0$ and equals 25.59 km s^{-1}. Because $dV_{0,1}$ and $dV_{0,2}$ are statistically independent, the total mean error becomes $dV_0 = (dV_{0,1}^2 + dV_{0,2}^2)^{1/2}$. The final result becomes $V_0 = 197.27 \pm 26.80$ km s^{-1}. This value compares well with the IAU's recommended value of 220$\pm$20 km s^{-1}.

Regarding the dispersions for the velocity ellipsoid, Table 8 shows values higher than those found in Delhaye. But this comes as no surprise because he summarizes individual determinations that use statistical moments whereas I use a different method, one that does not depend on the assumption of a normal distribution and one that calculates a unique ellipsoid, and concordance becomes unlikely. The dispersions are higher than those I found for the O-B5 III stars. It has been found that in general, see Delhaye (1965), the late stars have higher dispersions than the early stars. The mean errors of the orientation of the velocity ellipsoid are high, showing that it is easier to determine the dispersions of the ellipsoid than its orientation. The x-axis of the ellipsoid lies nearly in the Galactic plane, but the y-axis, perhaps surprisingly, is inclined significantly with respect to the plane.

6. CONCLUSIONS

Semi-definite programming proves itself once again a useful tool for problems of Galactic kinematics by allowing one to combine a solution for the kinematical parameters such as the Oort constants with one for the coefficients of the velocity ellipsoid. When applied to 1,532 M III stars the calculated solution is reasonable.

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