Revista Mexicana de Astronomía y Astrofísica, **44**, 311–323 (2008)

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THE DELAYED CONTRIBUTION OF LOW AND INTERMEDIATE MASS STARS TO CHEMICAL GALACTIC ENRICHMENT: AN ANALYTICAL APPROACH

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Received 2008 May 19; accepted 2008 July 3

RESUMEN

Encontramos una nueva solución analítica para las ecuaciones de evolución química tomando en cuenta la contribución retrasada de todas las estrellas de $m < 8\ M_{\odot}$(LIMS) como una estrella representativa que enriquece al medio interestelar. Esta solución se construye para una tasa de formación estelar proporcional a la masa de gas en un modelo de caja cerrada. Obtenemos incrementos en C/O y N/O cuando O/H aumenta, comportamiento imposible de igualar con IRA. Nuestros resultados, obtenidos mediante dos ecuaciones analíticas, son muy similares a aquellos encontrados con modelos numéricos que consideran el tiempo de vida de cada estrella. Este modelo retrasado reproduce la evolución de C/O$-$O/H y $Y-O$ en la vecindad solar. Esta aproximación analítica es una herramienta útil para estudiar la evolución de elementos producidos por las LIMS cuando no se dispone de un código de evolución química.

ABSTRACT

We find a new analytical solution for the chemical evolution equations, taking into account the delayed contribution of all low and intermediate mass stars (LIMS) as one representative star that enriches the interstellar medium. This solution is built only for star formation rate proportional to the gas mass in a closed box model. We obtain increasing C/O and N/O ratios with increasing O/H, behavior impossible to match with the Instantaneous Recycling Approximation (IRA). Our results, obtained by two analytical equations, are very similar to those found by numerical models that consider the lifetimes of each star. This delayed model reproduces successfully the evolution of the C/O$-$O/H and $Y-O$ relations in the solar vicinity. This analytical approximation is a useful tool to study the chemical evolution of elements produced by LIMS when a galactic chemical evolutionary code is not available.

Key Words: galaxies: abundances — galaxies: evolution — ISM: abundances

1. INTRODUCTION

Chemical evolution models are used to describe the temporal variation of the gas mass and the abundances of the different chemical elements that are present in the gas. Their importance lies on the fact that the chemical history of the studied object (i.e. interstellar medium in galaxies and intergalactic medium) can be inferred. Moreover it is possible to get chemical information about stellar population properties and characteristics of the galaxies that we observe in the low redshift Universe.

The chemical evolution equations, derived by Tinsley (1974) and corrected later by Maeder (1992), are relatively complex and can be solved through numerical models. There are some analytical approximations

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that have the advantage of predicting the general behavior of chemical elements in a quick and easy way but some precision may be lost.

One of the best known analytical approximations is the Instantaneous Recycling Approximation (IRA) (Talbot & Arnett 1971) where the star lifetimes are negligible compared to the age of the galaxies. This approximation has been widely used because it simplifies the solution to the chemical evolution equations. However, massive star (MS) lifetimes are on the order of $10^6 - 10^7$ years, while the lifetimes for the low and intermediate mass stars (LIMS) are on the order of $10^8 - 10^{10}$ years, comparable to the lifetime of a galaxy. Hence, IRA provides only a very rough approximation for elements produced by LIMS.

Serrano & Peimbert (1983) proposed an analytic approximation related to the delays in chemical enrichment in N/O−O/H relation assuming N and O yields increase with Z. They present closed and open models (with and without gas flows, respectively) that takes into account the delay in nitrogen production due to LIMS concluding that nitrogen must be mainly a secondary element.

Later on, Pagel (1989) presented another approximation for open models introducing an arbitrary time delay in order to study the chemical evolution of elements produced by SNIa and LIMS, such as Fe and Ba (through the s-process). This time delay term makes the stars release the processed material to the interstellar medium (ISM) at a single time after star formation. After this single time delay, the contribution of all types of star is instantaneous, like in IRA.

The objective of this work is to present an alternative analytical solution to the chemical evolution equations that considers the LIMS lifetimes as only one group with delay times during the whole evolution. Those delay times are different for each chemical element and are computed based on the characteristics of the stellar yields and the stellar population. The delayed contribution to the chemical enrichment of elements produced by this type of star at different times should give results with a precision intermediate between the results obtained by using IRA and the ones obtained by numerical codes. As an application, this work aims to reproduce the C/O *vs* O/H and $Y(O)$ histories indicated by the stars and HII regions at the solar vicinity.

This approximation is a simple tool for theoretical and observational astronomers who need to include chemical aspects in their computations or to interpret observational data when they do not have access to a numerical code of chemical evolution of galaxies. Our equation for the mass abundance of element i, $X_i(t)$, would replace the popular equation of $Z(t)$ obtained assuming IRA in the closed box regimen, erroneously applied for elements produced by LIMS (e.g. He, C, N).

2. THE DELAYED CONTRIBUTION MODEL

This approximation finds an analytical solution to the chemical evolution equations. In the context of a closed box model, we take into account the stellar lifetimes (τ) of the LIMS as a group where each of its members die at the same time. In this approximation, after a star forming burst, all massive stars (MS) with initial mass m higher than 8 $M_\odot$ enrich the interstellar medium (ISM) instantaneously, while every LIMS with $m < 8\ M_\odot$ enriches the ISM after one single time τ_L, for the total mass ejected. We divide R, the material returned by the stars to the ISM, in two terms: R_L and R_M, the returned mass by the LIMS and the MS, respectively:

$$R_L = \int_{M_1}^{8M_\odot} (m - m_R)\phi(m)dm \ ,$$

$$R_M = \int_{8M_\odot}^{M_{\rm sup}} (m - m_R)\phi(m)dm \ ,$$

where m_R is the remnant mass, M_1 and $M_{\rm sup}$ are the lowest and highest masses that contribute to the ISM, and ϕ is the initial mass function.

We take the galactic chemical evolution equations for the gas mass in a closed box model

$$\frac{dM_{\rm gas}}{dt} = -\Psi(t) + E(t) \ ,$$

where $\Psi(t)$ is the star formation rate and $E(t)$ is the gas rate that is returned to the ISM from stars that die. In our approximation, we have divided $E(t)$ in two parts, the rate at which all MS return material to the ISM,

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considering that, $t-\tau(m)\approx t$:

$$E_M(t)=\int_{8M_\odot}^{M_{\rm sup}}(m-m_R)\Psi(t-\tau(m))\phi(m)dm=R_M\Psi(t)\ ;$$

and the rate at which the LIMS return material to the ISM, $t-\tau(m)\approx t-\tau_L$, so that:

$$E_L(t)=\int_{M_1}^{8M_\odot}(m-m_R)\Psi(t-\tau(m))\phi(m)dm=R_L\Psi(t-\tau_L)\ .$$

We have considered that the star formation rate is proportional to the gas mass with a constant efficiency ν, $\Psi(t)=\nu M_{\rm gas}(t)$; therefore,

$$\frac{dM_{\rm gas}}{dt}=-\nu(1-R_M)M_{\rm gas}(t)+\nu R_L M_{\rm gas}(t-\tau_L)\ .$$

The first term of this equation represents the contribution of the MS while the second one represents the delayed contribution of the LIMS.

In order to get the abundance evolution equation, we divide the stellar population yields P_i in two parts as we did with R, $P_i=P_{L_i}+P_{M_i}$, with P_{L_i} and P_{M_i} defined by:

$$P_{L_i}=\int_{M_1}^{8M_\odot}mp_i(m)\phi(m)dm\ ,$$

$$P_{M_i}=\int_{8M_\odot}^{M_{\rm sup}}mp_i(m)\phi(m)dm\ ,$$

where $p_i(m)$ represents the stellar yield of element i, that is the fraction of initial stellar mass processed and ejected by a star of initial mass m.

We take the galactic chemical evolution equation for the mass of gas in element i, $F_i(t)=X_i(t)M_{\rm gas}(t)$, in a closed box model,

$$\frac{dF_i}{dt}=-X_i(t)\Psi(t)+E_i(t)\ .$$

Dividing the E_i in two parts as we did before with $E(t)$ we get:

$$E_{M_i}(t)=\int_{8M_\odot}^{Msup}[(m-m_R)X_i(t-\tau(m))+mp_i(m)]\Psi(t-\tau(m))\phi(m)dm\ ,$$

$$E_{M_i}(t)=[R_M X_i(t)+P_{M_i}]\Psi(t)\ ,$$

$$E_{M_i}(t)=\nu R_M F_i(t)+\nu P_{Mi}M_{\rm gas}(t)\ .$$

And for the LIMS:

$$E_{L_i}(t)=\int_{M_1}^{8M_\odot}[(m-m_R)X_i(t-\tau(m))+mp_i(m)]\Psi(t-\tau(m))\phi(m)dm\ ,$$

$$E_{L_i}(t)=[R_L X_i(t-\tau_i)+P_{L_i}]\Psi(t-\tau_i)\ ,$$

$$E_{L_i}(t)=\nu R_L F_i(t-\tau_i)+\nu P_{L_i}M_{\rm gas}(t-\tau_i)\ .$$

We finally obtain:

$$\frac{dF_i}{dt}=-\nu(1-R_M)F_i(t)+\nu P_{M_i}M_{\rm gas}(t)+\nu R_L F_i(t-\tau_i)+\nu P_{L_i}M_{\rm gas}(t-\tau_i)\ .$$

For each element i, the group of LIMS enrich the ISM at a time τ_i after it was formed; τ_i has to be chosen as the representative ejecting time of the population. Equivalently, a representative mass is also required: this is defined as the mass up to which the accumulated chemical yield of stars of lower mass is equal to one half of the entire yield of the LIMS population. As the stellar yields are different for each chemical element and strongly depend on the metallicity, we calculate the value $M_{\rm repr}$ for each chemical element and for each metallicity:

$$\int_{M_1}^{M_{\rm repr}}mp_i\phi(m)dm=\frac{1}{2}\int_{M_1}^{8M_\odot}mp_i\phi(m)dm\ .$$

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3. SOLUTION TO THE CHEMICAL EVOLUTION EQUATIONS

We solve the differential equations for $M_{\rm gas}$ and F_i assuming for the moment that the metallicity of the stars that enrich the ISM is constant in the evolution, therefore the stellar properties, τ, R_M, P_M, R_L, P_L are constant.

3.1. *Gas Mass Evolution Equation*

The equation to be solved is:

$$\frac{dM_{\rm gas}}{dt} = -\nu(1-R_M)M_{\rm gas}(t) + \nu R_L M_{\rm gas}(t-\tau_L) \ . \tag{1}$$

For $t < \tau_L$, the equation can be solved by IRA, which gives:

$$M_{\rm gas}(t) = M_{\rm gas}(0)e^{-\xi t} \qquad 0 < t < \tau_L \ ,$$

where $\xi = \nu(1-R_M)$.

Having solved for the first interval, $0 < t < \tau_L$, we now substitute this solution in equation (1) when $\tau_L < t < 2\tau_L$ obtaining as solution:

$$M_{\rm gas}(t) = M_{\rm gas}(0)[e^{-\xi t} + e^{-\xi(t-\tau_L)}(t-\tau_L)(\nu R_L)] \ .$$

Solving for the other intervals:

$$M_{\rm gas}(t) = M_{\rm gas}(0)e^{-\xi t}\sum_{k=0}^{n-1}\frac{[e^{\xi\tau_L}(t-k\tau_L)(\nu R_L)]^k}{k!} \ , \tag{2}$$

for $(n-1)\tau_L < t < n\tau_L$.

Taking into account the delayed contribution due to the LIMS, the gas mass equation is the product of the term that comes from the contribution of the MS (IRA) times a summation that represents the contribution of the LIMS that were born at $t - k\tau_L$ and enriched the ISM at the time t.

3.2. *Gas Mass in Element i*

The equation to solve is:

$$\frac{dF_i}{dt} = -\nu(1-R_M)F_i(t) + \nu P_{M_i}M_{\rm gas}(t) + \nu R_L F_i(t-\tau_i) + \nu P_{L_i}M_{\rm gas}(t-\tau_i) \ . \tag{3}$$

Following a similar procedure to that of previous section:

$$F_i(t) = M_{\rm gas}(0)e^{-\xi t}[X_i(0) + \nu P_{M_i}t] \qquad 0 < t < \tau \ .$$

Solving for the second interval:

$$F_i(t) = M_{\rm gas}(0)[e^{-\xi t}[X_i(0)+\nu P_{M_i}t] + e^{-\xi(t-\tau_i)}(t-\tau_i)(\nu R_L)[X_i(0)+\nu P_{M_i}(t-\tau_i) + P_{L_i}/R_L]] \ ,$$

for $\tau_i < t < 2\tau_i$.

Solving for the other intervals:

$$F_i(t) = M_{\rm gas}(0)e^{-\xi t}\sum_{k=0}^{n-1}\frac{[e^{\xi\tau_i}(t-k\tau_i)(\nu R_L)]^k}{k!} \times ([X_i(0)+\nu P_{M_i}(t-k\tau_i)] + kP_{L_i}/R_L) \ , \tag{4}$$

for $(n-1)\tau_i < t < n\tau_i$.

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3.3. *Chemical Abundances by Mass $X_i(t)$*

The evolution equation for $X_i(t)$ is obtained from the ratio of the mass in the form of element i and the gas mass:

$$X_i(t) = \frac{F_i(t)}{M_{\rm gas}(t)} \ .$$

For the total gas mass we assume that the entire LIMS population ejects the material in form of element i at the same time as the material with all the elements, i.e., $\tau_i = \tau_L$. Then we finally get $X_i(t)$ dividing equation (4) by equation (2) where the abundance ratios are independent of the initial value of gas mass, $M_{\rm gas}(0)$. Therefore

$$X_i(t) = \frac{\sum_{k=0}^{n-1} G(k,t)Q(k,t)}{\sum_{k=0}^{n-1} G(k,t)} \ , \tag{5}$$

where $G(k,t) = [(t-k\tau_i)(e^{\xi\tau_i}\nu R_L)]^k/k!$ and $Q(k,t) = \nu P_{M_i}(t-k\tau_i) + X_i(0) + kP_{L_i}/R_L$, valid for $t - k\tau_i > 0$.

To solve the approximation of $\tau_i = \tau_L$ in $M_{\rm gas}(t)$, we take the gas mass as the average of the obtained masses of each chemical element:

$$M_{\rm gas}(t) = \frac{\sum_i M_{\rm gas}(t,\tau_i)}{5} \ ,$$

as the gas mass is independent of the element i, and 5 elements are considered here (He, C, N, O, and Z).

We have found that the values of $M_{\rm gas}(t)$ and each $M_{\rm gas}(t,\tau_i)$ have quite similar values, except for high ν and long τ_i. In such cases, $X_i(t)$ show an artificial secondary raise (see § 4 for details).

3.4. *Stellar Properties*

We have chosen an age, an initial mass function, and a yield set in order to study the applications and limits of our approximation. We consider 13 Gyr as the age of the models, the time elapsed since the beginning of the formation. The Initial Mass Function adopted is the one proposed by Kroupa, Tout, & Gilmore (1993) in the mass interval given by $0.1 < m/M_\odot < 80$, hence, $M_{\rm sup} = 80\ M_\odot$ and $M_1 = 1\ M_\odot$,the canonical lowest mass star that enriches the ISM in the IRA approximation.

After trying several stellar yields published in the literature, we selected those that provide us with the most complete information in terms of mass, metallicities and physical parameters such as stellar rotation, mass loss due to stellar winds and elements produced during the HBB stage. Once they were selected, we calculated the yields of the population based on its initial metallicity.

For LIMS we use the stellar yields and remnants provided by van den Hoek & Groenewegen (1997) that take in account the mass loss due to stellar winds. These yields are characterized by the parameter η that represents the mass loss; we chose a range for η for $Z_{\rm pop} = 0.001$, 0.004 and 0.02. In order to find the delay times of the LIMS group we have adopted the stellar lifetime by Schaller et al. (1992). We have assumed that no LIMS formed with $Z_{\rm pop} \sim 0$.

For MS we use the stellar yields and remnants provided by Hirschi (2007) for $Z_{\rm pop} = 10^{-8} \sim 0$, Meynet & Maeder (2002) for $Z_{\rm pop} = 10^{-5}$ and 0.004, and Maeder (1992) for $Z_{\rm pop} = 0.02$. We interpolate linearly by mass the stellar yields and remnant mass of LIMS and MS, we also extrapolate linearly in mass in order to reach $m_{\rm up} = 80\ M_\odot$.

We do not interpolate the stellar properties by metallicity and we combine MS and LIMS properties according to the initial stellar metallicity. For LIMS of $Z_{\rm pop} = 10^{-5}$ we assumed the $Z_{\rm pop} = 0.001$ yields and remnants.

Below we show the Tables 1 and 2 with physical properties derived here for our approximation, R_M, R_L, P_{M_i}, P_{L_i}, and τ_i. As you can notice, all values of τ_i are < 1 Gyr, with the exception of τ_O for $Z_{\rm pop} = 0.02$, which is 5.89 Gyr. This difference is important because it causes an artificial behavior in oxygen (only for the case with very high ν) and therefore in all abundance ratios related to oxygen.

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TABLE 1

RETURNED MASSES, STELLAR POPULATION YIELDS, REPRESENTATIVE MASSES (IN $M_\odot$) AND DELAY TIMES (IN GYR) FOR LOW AND INTERMEDIATE MASS STARS

$Z_{\rm pop}$	~ 0	0.001	0.004	0.02
R_L	—	0.217	0.225	0.238
$P_{He} \times 10^{-2}$	—	1.261	1.035	0.801
$M^{He}_{\rm repr}$	—	2.270	2.414	2.646
τ_{He}	—	0.719	0.635	0.623
$P_C \times 10^{-3}$	—	2.680	1.717	0.585
$M^{C}_{\rm repr}$	—	2.022	2.223	2.761
τ_C	—	0.968	0.788	0.554
$P_N \times 10^{-4}$	—	11.846	9.300	8.162
$M^{N}_{\rm repr}$	—	4.769	5.094	4.769
τ_N	—	0.110	0.099	0.121
$P_O \times 10^{-4}$	—	2.937	1.571	0.790
$M^{O}_{\rm repr}$	—	2.298	2.362	1.185
τ_O	—	0.697	0.672	5.890
$P_Z \times 10^{-3}$	—	4.206	2.832	1.492
$M^{Z}_{\rm repr}$	—	2.546	2.779	3.126
τ_Z	—	0.532	0.432	0.390

TABLE 2

RETURNED MASSES AND STELLAR POPULATION YIELDS FOR MASSIVE STARS

$Z_{\rm pop}$	~ 0	1.0×10^{-5}	0.004	Z=0.02
$R_M \times 10^{-2}$	7.345	7.339	7.370	7.438
$P_{He} \times 10^{-2}$	1.208	1.212	1.092	0.929
$P_C \times 10^{-3}$	2.694	0.854	0.820	2.802
$P_N \times 10^{-4}$	2.363	0.429	1.333	4.689
$P_O \times 10^{-3}$	6.261	6.104	7.285	2.953
$P_Z \times 10^{-2}$	0.988	1.012	1.154	0.893

4. RESULTS

We study the evolution of mass abundances H, He, C, N, O and Z, using the delayed contribution model taking into account primordial abundances, $X(0) = 0.75$, $Y(0) = 0.25$ for all metallicities, and we follow the evolution from $t = 0$ until $t = 13.0$ Gyr with a time step of $\Delta t = 0.01$ Gyr. In this section we do not show the evolution for $Z_{\rm pop} \sim 0$, since we assume that all Pop III stars are MS, so that C/O(t) and N/O(t) are constant according to IRA.

Next we discuss the result obtained when using our model for different values of the gas consumptions $\mu = M_{\rm gas}/M_{\rm tot}$ as well as for different metallicities.

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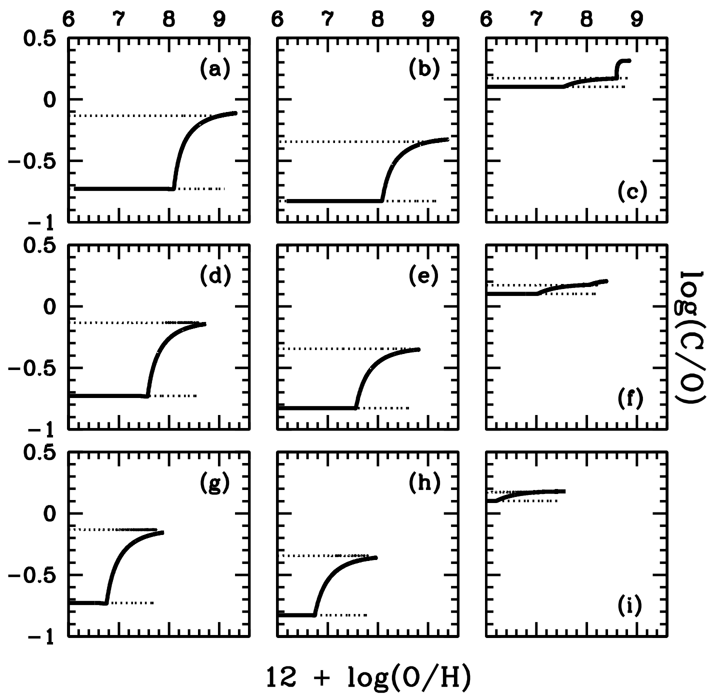

Fig. 1. Evolution of C/O abundance ratio versus O/H abundance ratio for metallicities of the stellar population $Z_{\rm pop} = 10^{-5}$ (Panels a, d, g), $Z_{\rm pop} = 0.004$ (Panels b, e, h), $Z_{\rm pop} = 0.02$ (Panels c, f, i) and three different values of gas consumption $\mu = 0.1$ (Panels a, b, c), $\mu = 0.5$ (Panels d, e, f), $\mu = 0.9$ (Panels g, h, i). The dotted lines represent IRA results when only MS (lower lines) or both MS and LIMS (upper lines) are considered.

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4.1. *Evolution of C/O vs O/H*

In Figure 1, we show the evolution of C/O vs O/H for three different metallicities and three μ values. In each model the value of μ is reached at $t_g = 13.0$ Gyr. The horizontal lines represent the results assuming IRA, the lower lines when only MS are considered and the higher lines when both MS and LIMS are considered. Assuming IRA, the abundance ratios by number can be written as, C/O(t)= $\frac{P_C/12.0}{P_O/16.0}$ and O/H(t)= $\frac{-P_O ln\mu/16.0}{0.75+(P_{He}+P_Z)ln\mu}$.

Initially, O/H depends on the star formation rate (SFR), the smaller the gas consumption μ the higher the O/H abundance value.

Massive stars are the first that die and so the C/O values obtained with the delayed model are identical to the ones obtained using IRA considering only massive stars. The effect of delay can be seen when the curve begins to increase its slope making the abundance ratio C/O grow. Afterwards, the curve flattens coinciding with the IRA case considering MS and LIMS together.

4.1.1. *Case* $\mu = 0.1$

In this case 90% of the initial gas mass has become stars at the end of the evolution and only 10% remains as ISM (See Figure 1, Panels a, b, c).

When the metallicity has values of $Z_{\rm pop} = 10^{-5}$ and 0.004, C/O increases for the first time when 12 + log(O/H) $\sim$ 8.0 dex respectively (Figure 1, Panels a, b) that is, at the times equal to 0.97 Gyr and 0.79 Gyr that are the delay times for carbon ejection from LIMS for these metallicities (see Table 1).

When $Z_{\rm pop} = 0.02$, C/O is higher than the lower $Z_{\rm pop}$ cases because P_C/P_O are higher for MS at solar metallicity. Moreover when $Z_{\rm pop} = 0.02$ the values for O/H are smaller than those for other metallicities at the same times. Both effects are due to MS of high Z; the metal rich massive star ejects more carbon in the stellar wind stage, leaving less carbon behind to be processed in order to become oxygen.

On the other hand, C yields of LIMS are lower at high Z, again because of the mass loss rate; stellar winds are intense in LIMS and stars have less mass to produce heavy elements. This makes a smaller difference between the C/O ratios with IRA for MS and MS + LIMS.

Moreover, for $Z_{\rm pop} = 0.02$ the first increment of C/O comes when 12+log(O/H) $\sim$ 7.5 dex (Figure 1 Panel c) corresponding to a time delay for carbon of 0.55 Gyr. The curve keeps growing due to the increase of carbon relative to oxygen. Afterwards, the second increase is artificial (at 12+log(O/H)$\sim$ 8.6 dex) and it is produced by the oxygen dilution due to LIMS. In the $\mu = 0.1$ case the SFR is a decreasing exponential function, therefore for $t > \tau_O$ the number of LIMS that are diluting is higher than the number of MS that are enriching with O the ISM.

4.1.2. *Case* $\mu = 0.5$

In this case 50% of the initial gas mass has become stars at the end of the evolution and 50% remains as ISM; therefore the SFR is lower and flatter than in the case $\mu = 0.1$. As a consequence the values for O/H are smaller in this case than what was previously found at the same times (see Figure 1, Panels d, e, f). On the other hand, the C/O ratios obtained in this case are almost the same as for $\mu = 0.1$ because $C/O \propto P_C/P_O$; therefore C/O depends on the stellar yields, not on μ.

The oxygen dilution effect is present again for $Z_{\rm pop} = 0.02$ but to a smaller degree than for $\mu = 0.1$ due to the SFR behaviour. In the $\mu = 0.5$ for $t > \tau_O$ the SFR is high so there are many MS enriching the gas, making difficult the oxygen dilution by LIMS.

4.1.3. *Case* $\mu = 0.9$

In this case only 10% of the initial gas mass has become stars at the end of the evolution and 90% remains as ISM; therefore the SFR is even lower and practically flat.

In Figure 1 Panels g, h, i, the results for this case are shown. This is the lowest SFR that we have studied, which implies that O/H ratio is very low too. In this case there is no second significant increase in C/O vs O/H, because the SFH is the flattest one and therefore the amounts of massive stars and LIMS are almost constant during the whole evolution.

4.2. *Evolution of N/O vs O/H*

In Figure 2 we show the N/O vs O/H evolution for the same μ and $Z_{\rm pop}$ values as in the previous section.

The general behavior of N/O ratio is similar to the C/O ratio because N and C are produced by MS and LIMS, and O is produced mainly by MS. Therefore when O/H ratio increases N/O ratio tends to increase when the LIMS eject material at the delay time. Since MS produce much less N than C, N/O is lower than C/O for early times. Also N/O evolution presents an artificial raise due to O dilution, as in the C/O history.

4.3. *Evolution of Helium with Oxygen*

In Figure 3 we show the helium by mass (Y) vs oxygen also by mass (O) for the same μ and $Z_{\rm pop}$ values as in §§ 4.1 and 4.2. Here we have assumed for all $Z_{\rm pop}$ that the initial Y and O values are 0.25 and 0.00, respectively.

Since the delay time for LIMS is lower than 0.7 Gyr, the ΔY values are very similar to those obtained assuming IRA for MS and LIMS. The slopes of $Y(O)$ changes with $Z_{\rm pop}$ due mainly to the Z dependence of oxygen yields through the stellar winds. Also $Y(O)$ presents the second significant increase caused by the artificial oxygen dilution due the LIMS.

4.4. *Limitation of the Approximation*

Based on C/O vs O/H and N/O vs O/H and Y vs O evolution the artificial second increase is present only for $Z_{\rm pop} = 0.02$ and $\mu = 0.1$. This second increase is due to a huge delay time and a decreasing SFR. According to our approximation when we study O(t), LIMS eject oxygen and the rest of the elements at the same time, $\tau_L = \tau_O$. Since P_{L_O} is very low, the rest of elements produced by LIMS dilute oxygen and there are not enough MS to counteract that dilution.

This behavior indicates that $\tau_L = \tau_i$ approximation is not valid for an element i mainly produced by MS, when the LIMS delayed contribution of the rest of elements is huge compared to the MS contribution of the element i at a fixed time.

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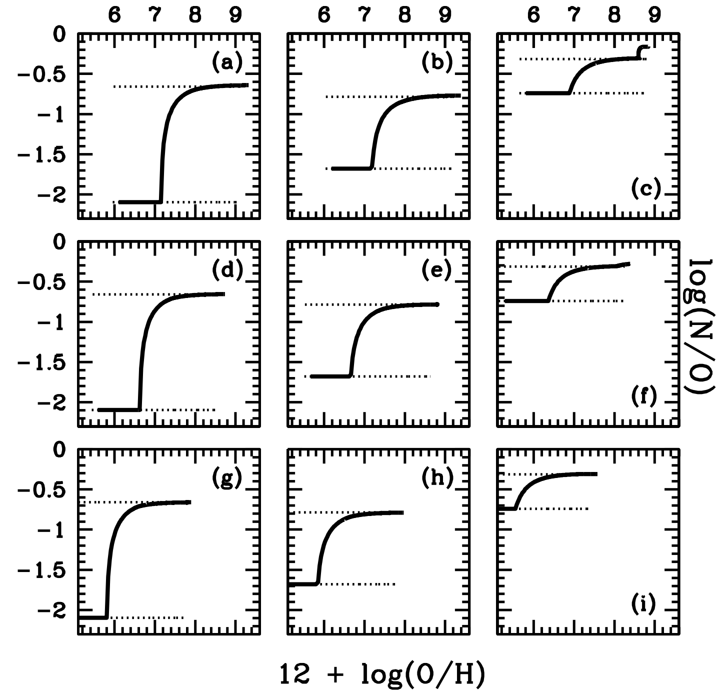

Fig. 2. Evolution of N/O abundance ratio versus O/H abundance ratio for metallicities of the stellar population $Z_{\rm pop} = 10^{-5}$, 0.004, and 0.02; and three different values of gas consumption $\mu = 0.1$, 0.5, and 0.9, as in Figure 1.

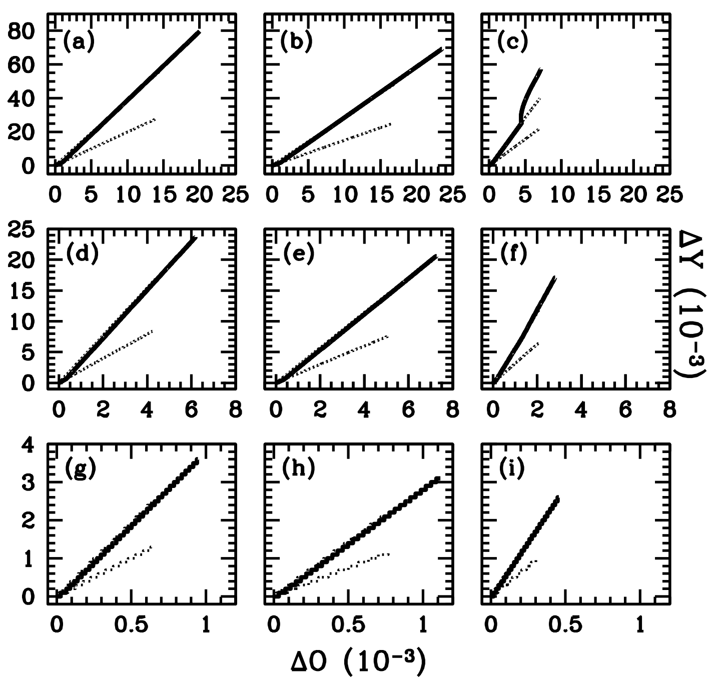

Fig. 3. Evolution of Helium versus Oxygen, both by mass, for the metallicities of stellar population $Z_{\rm pop} = 10^{-5}$, 0.004 and 0.02 and three different values of gas consumption $\mu = 0.1$, 0.5 and 0.9, as in Figure 1. The dotted lines represent IRA results, when only MS (flatter lines) or both MS and LIMS (steeper lines) are considered.

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5. THE SOLAR VICINITY

The solar vicinity is the place in the Universe with the largest number of observations; therefore, we use it to test our approximation. However, this approximation was developed for a constant metallicity of the stars, therefore P_i, τ_i and R_i are constant during the gas evolution of a galaxy. This assumption is not valid for a real galaxy, because the stars in a galaxy form with gas metallicity that changes in time. Therefore, the Figures 1, 2, and 3 are good illustrative examples, but not real ones.

We compute a model where the stars are formed with metallicities similar to the gas metallicity when the SFR is ongoing. We have made $M_{\rm gas}$ continuous and F_i when the gas reach the metallicities ($Z_{\rm gas}$) of the stellar population ($Z_{\rm pop}$) assumed in this work.

Formally, when $Z^j_{\rm pop} < Z_{\rm gas} < Z^{j+1}_{\rm pop}$ we assume that $P_i = (P^j_i + P^{j+1}_i)/2$ and $\tau_i = (\tau^j_i + \tau^{j+1}_i)/2$. Specifically for $0 \leq Z_{\rm gas} < 10^{-8}$ and $Z_{\rm gas} > 0.02$ we have assumed the stellar properties for $Z_{\rm pop} = 10^{-8}$ and $Z_{\rm pop} = 0.02$, respectively. However, for $10^{-5} \leq Z_{\rm gas} < 0.02$ we have considered average stellar properties between two consecutive $Z_{\rm pop}$'s.

With this combination we obtain more realistic results in the sense that we are taking into account the evolution of the stellar populations depending on its initial metallicity. Another advantage of this combination is that different star formation rate efficiencies can be used as a function of time, representing in a more realistic way the formation of different components of a galaxy.

5.1. *Observational Restrictions*

Now, we study the applicability and limitations of the delayed contribution model with different $Z_{\rm pop}$'s comparing our theoretical results with existing observations of the solar vicinity. We define the solar vicinity as the volume contained in a cylinder centered on the Sun with a radius of $\sim$ 1 kpc and a height enough to reach objects located in the Galactic halo.

Since C is better known than N, we will test our delayed approximation with the C/O vs O/H relation in the solar vicinity and our observational constraints for C and O are:

- H II regions to test the model results at the present time (Esteban et al. 2005; García-Rojas et al. 2004).
- Halo and disk main sequence stars at different times as past restrictions (Akerman et al. 2004).
- The Sun as a restriction at 4.5 Gyrs ago (Asplund, Grevesse, & Sauval 2004).

Since the Sun is located at 8 kpc from the Galactic center, we used the two HII regions studied by Esteban et al. (2005) and García-Rojas et al. (2004) nearest the Sun: the Orion Nebula and NGC 3576 at r = 7.46 kpc and r = 8.40 kpc, respectively. The C/H and O/H values have been increased by 0.10 and 0.08 dex, respectively, owing to the fraction of C and O embedded in dust grains (Esteban et al. 1998). Akerman et al. (2004) show values for 34 F and G dwarf stars from the galactic halo combining them with similar data of 19 stars of the disk.

5.2. *Chemical Evolution Models*

We have built four chemical evolution models to reproduce the O/H value in NGC 3576. Since our approximation was obtained for a closed box model with a SFR proportional to $M_{\rm gas}$, all models presented in this section follow those assumptions and, moreover, consider the same IMF, mass range and stellar yields specified in § 3.4.

Each model is characterized by the approximation used in the lifetime. Model 1 is our delay approximation, which assumes that the whole group of LIMS is represented by a specific star and its lifetime is considered. Models 2 and 3 are models in IRA, for MS only and MS and LIMS, respectively. In those models no lifetime is considered. Model 4 is obtained using the CHEVO code (Carigi 1994) that considers the lifetime of each star until it leaves the main sequence.

A closed box model with a $SFR = \nu M_{\rm gas}$ that reproduces a final O value has only one free parameter that is the efficiency ν. Therefore, the models 1, 2, 3 and 4 need ν =0.13, 0.13, 0.17, and 0.23 in order to get 12 + log(O/H) $\sim$ 8.82, resulting in values of μ =0.29, 0.21, 0.21, and 0.51, respectively.

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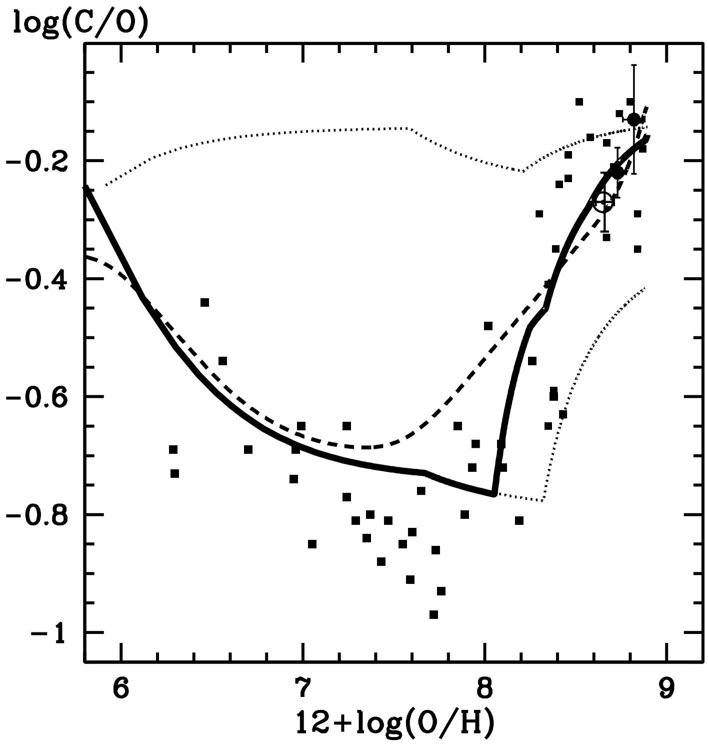

Fig. 4. Evolution of C/O versus O/H using the different models: *continuous line:* the delayed approximation, *dashed line:* taking into account the lifetime of each star, *lower dotted line:* IRA with MS, *upper dotted line:* IRA with MS+LIMS. Observations: *circles:* HII regions by Esteban et al. (2005) and García-Rojas et al. (2004), *squares:* dwarf stars by Akerman et al. (2004), $\odot$: solar values by Asplund et al. (2005).

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Since O is produced mainly by MS and models 1 and 2 assume IRA for MS only, the SFR is identical for both models, but the value of $M_{\rm gas}$ obtained by our delayed approximation is higher than in models with IRA, due to the contribution of the representative mass of the LIMS as a group. The SFR for model 3 is slightly higher due to the dilution of O caused by LIMS at each time and the $M_{\rm gas}$ are nearly identical because the high SFR in model 3 counteracts the material returned to the ISM by LIMS. In model 4, ν and μ values are higher than those obtained by the rest of the models because the CHEVO code needs a delay for each star that forms until its death enriching the ISM. Therefore, the SFR have to be higher than in the rest of models to reach the same O value. Moreover, the $M_{\rm gas}$ values obtained by model 4 in the last 9 Gyr are higher due to the delayed contribution of LIMS. For comparisons between closed box models with and without IRA see Figure 12 in Prantzos (2007).

Despite the fact that it is known that the [Fe/H] distribution shown in dwarf stars of the solar vicinity is impossible to reproduce with a closed box model, we use models 1–4 to check our approximation in C/O-O/H and $Y(O)$ relations.

5.3. *Evolution of C/O with O/H*

In Figure 4, we show the evolution C/O vs O/H of the models obtained with IRA, CHEVO and our delayed approximation.

Comparing models with the observational data, we conclude: (i) Model 3, assuming the canonical IRA for MS and LIMS, produces C/O values that are almost constant and much higher than the observed ones for most of the evolution. (ii) Model 2, assuming IRA only by MS, reproduces very well the C/O evolution until $12 + \log(\mathrm{O/H}) \sim 8$ because the ISM in the halo formation is determined mainly by MS. Also, this model shows the C/O increase due to the high C yield of MS at high Z. However, this increase is not to enough to reach

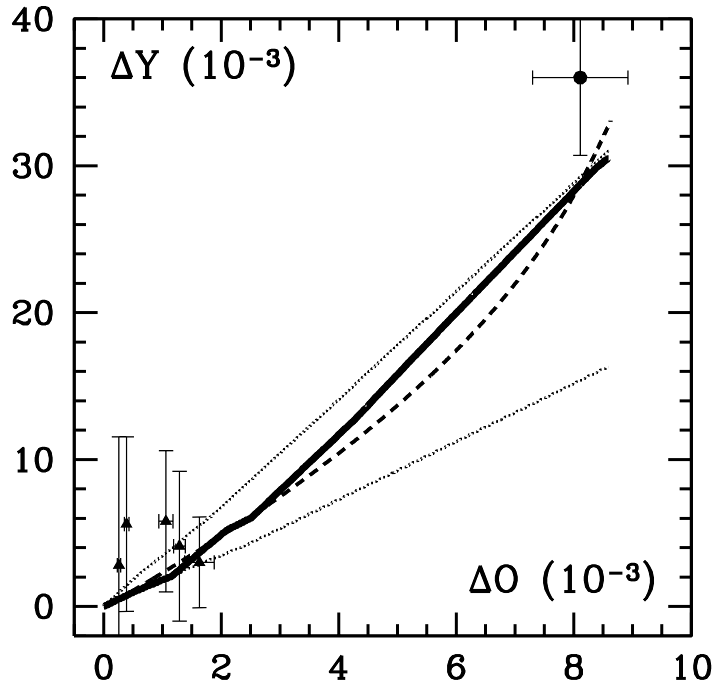

Fig. 5. Evolution of Helium versus Oxygen by mass. Models as Figure 4. *Observational data:* M17, HII region at $r = 6$ kpc by Carigi & Peimbert (2008) (*filled circle*), extragalactic HII regions by Peimbert et al. (2007) (*filled triangles*).

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the high C/O values present in disk stars and in HII regions. (iii) Model 1, assuming the delayed contribution of LIMS, reproduces quite well the whole C/O-O/H evolution and specifically the increase of C/O abundance ratio at high O/H.

It is notable that these results are very similar to results obtained with the numerical code that considers the lifetime of each star (model 4).

5.4. *Evolution of Helium with Oxygen*

In Figure 5, we show the evolution of He by mass (Y) and O by mass (O) obtained again using the models with IRA, CHEVO, and our delayed approximation. In order to present the power of our approximation independent of initial abundances we have plotted ΔY and ΔO.

Since there are no good observational data for He in dwarf stars in the solar vicinity and in the nearest HII regions, we have plotted the Y and O values from HII regions in dwarf galaxies and M17, an inner Galactic HII region, as representative of the past and future of the solar vicinity, respectively.

The $Y - O$ relation obtained from models 1 and 4 keeps an almost linear trend when $O < 2.5 \times 10^{-3}$ and $O < 4.0 \times 10^{-3}$, respectively, with a similar $\Delta Y/\Delta O$. Then the linearity is lost in both models and the slopes are more pronounced due mainly to the Z dependence of O yields. The difference in the changing O is caused by the interpolation assumed between consecutive $Z_{\rm pop}$. In the CHEVO code a linear interpolation in stellar yields is used when $Z_{\rm gas}$ increases with time, while in our delayed approximation we assumed constant yields: the average yields between consecutive $Z_{\rm pop}$.

Again the delayed approximation is better than IRA and is more similar to that obtained by the numerical code.

6. CONCLUSIONS

- We have found analytical equations for chemical evolution in the case of a closed box model and SFR proportional to the gas mass, where the delayed enrichment by LIMS is represented by a single type of star.

- The delay of LIMS with respect to the galactic enrichment for He, C and N produces an increase on C/O and N/O with O/H and in Y with O in good agreement with the results obtained by numerical models. With IRA, the C/O and N/O values are constant with O/H in disagreement with observed data and with model results that take into account all star lifetimes.
- For $\mu = 0.1$ and $Z_{\rm pop} = 0.02$, $Y(O)$, C/O, and N/O values show artificial secondary increases due to the O dilution produced by LIMS and not because of the increase of He, C and N produced by the LIMS.
- The delayed approximation was tested successfully in the solar vicinity reproducing the main trends of C/O-O/H relation shown by dwarf stars in agreement with results obtained with a numerical model that considers all star lifetimes. That relation cannot be reproduced at all by the instantaneous recycling approximation.
- The analytical equation (equation 5) obtained by our approximation is a useful tool to know the chemical evolution of those elements produced by LIMS when no galactic chemical evolutionary code is available.

Part of this work was submitted in the Physics Undergraduate Program at the Universidad Nacional Autónoma de México.

L. C. is grateful to a careful reading of the manuscript from A. E. Sansom. I. F. thanks A. Martínez-Sansigre for helpful comments. We acknowledge the anonymous referee for excellent suggestions. This work was partly supported by Conacyt grants 46904 and 60354.

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