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ENERGY AND ANGULAR MOMENTUM OF DILATON BLACK HOLES

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RESUMEN

Dando seguimiento a un artículo previo, revisamos los resultados para la energía y momento angular de un hoyo negro de Kerr-Newman, y extendemos el cálculo para el caso de un dilaton en rotación, obtenido a partir del modelo de Garfinkle et al. (1991, 1992). Mostramos que hay, en lo que se refiere solamente a la energía y momento angular, una interacción entre los campos, de forma que, el gravitacional y el electromagnético pueden ser ocultados por la intensidad del campo escalar.

ABSTRACT

Following a prior paper, we review the results for the energy and angular momentum of a Kerr-Newman black hole, and then calculate the same properties for the case of a generalised rotating dilaton of the type derived, without rotation, by Garfinkle et al. (1991, 1992). We show that there is, as far as it refers only to the energy and angular momentum, an interaction among the fields, so that, the gravitational and electromagnetic fields may be obscured by the strength of the scalar field.

Key Words: black hole physics — gravitation — magnetic fields — relativity

1. INTRODUCTION

Astrophysical black holes appear in a wide variety of environments. We find them as a result of stellar evolution (X-ray binaries, supernovas, collapsars, etc.), and also some of intermediate mass or supermassive ones, like those that lie at the centre of different galaxies. (see, for instance Noyola, Gebhardt, & Bergmann 2008; Eckart, Straubmeier, & Schödel 2005; Lee & Park 2002; and Kreitler 2006a,b,c, and references therein).

Astrophysical black holes always have a certain amount of angular momentum associated to them. It is perhaps this angular momentum which gives rise to the energetic jets that are observed in many astrophysical systems (see for instance, Falcone et al. 2008; Pope et al. 2008) such as X-ray binaries, long gamma-ray bursts and quasars.

For the benefit of readers that are astronomers, we introduce now the concept of scalar fields and dilatons in a more or less historical perspective. There are two different situations in which a scalar field has importance: first, as time-varying gravitational and cosmological "constants". The first gravitational theory of that kind was Brans & Dicke (1961). Second, in the context of string theory. In the latter, the "*dilaton*" is related to the graviton; in the former, the scalar field is related to the Machian concept, whereby there is a causally related inertial phenomenon in local physics due to the overall distribution of mass in the Universe. In fact, nowadays it is clear that the gravitational scalar field is very close to the strings' dilatons. Let us give a trivial example. Wald (1994), suggests that the problem of black-hole evaporation could be approached by considering a *dilaton* scalar field in lower dimensional general relativity, with a "string-inspire" action

$$S = \frac{1}{2}\pi \int d^2x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4\nabla a\Phi\nabla^a\Phi + 4\Lambda^2\right) - \frac{1}{2}\nabla_a\phi\nabla^a\phi\right],$$

where Λ, Φ and ϕ stand for the cosmological constant, the *dilaton* field, and a scalar one, respectively. Classically, the field equations describe the gravitational collapse of the matter field ϕ , which yields a black hole. It could appear to provide testing ideas for the quantum behavior of black holes, for instance treating ϕ as a quantum field propagating in a classical background spacetime, corresponding to the formation of a black hole by gravitational collapse.

Dilaton field-black holes, were studied recently by Vagenas (2003) (see also Xulu 1998). Scalar fields, defined by a cosmological constant, plus electric charge and gravitation, were also treated in a recent paper by Martínez & Troncoso (2006), with important cosmological applications (Halpern 2008).

Scalar fields may alter our view of the Universe. Kaluza-Klein theory, contains a scalar field arising from the pentadimensional 5–5 component of the metric tensor (Wesson 1999, 2006). Such scalar fields, generally named as *dilatons,* were also identified with inflationary model's *inflaton* (Collins, Martin, & Squires 1989). String and brane theories deal with *dilatons* which play rôles similar to the gravitons. String theories have compactified internal space, whose size arises as a *dilaton*, or scalar field. Altogether, it has been claimed that gravitons interact among themselves and may have also scalar field companions. Scalar fields disguised under a cosmological "constant" term, provide clues to dark energy and dark matter models, in addition to the inflationary ones, plaguing astrophysical and cosmological literature. Scalar fields may add a little complexity to the vacuum. The four dimensional energy of the vacuum is a measure of the five dimensional scalar field (Wesson 2006). For the energy of the vacuum, in connection with gravity and scalar fields, see, for instance, Berman (2007a,b,c, 2008), Faraoni (2004), Fujii & Maeda (2003).

The calculation of energy and angular momentum of black-holes, has, among others, an important astrophysical rôle, because such objects remain the ultimate source of energy in the Universe, and the amount of angular momentum is related to the possible amount of extraction of energy from the b.h. (Levinson 2006; Kreitler 2006a,b,c). The consequences for jet production are also of astrophysical interest.

Therefore, Berman (2007a) checked whether the calculation of the energy and angular momenta contents for a K. N. black hole given by Virbhadra (1990a,b,c), and Aguirregabiria, Chamorro, & Virbhadra (1996), included the gravitomagnetic contribution. It was seen that this did not occur. Berman recalculated the energy and angular momenta formulae, so that gravitomagnetism was included in the scenario. In the present text, we advance the theoretical framework by studying the effect of a *dilaton*, or scalar field, within charged rotating black holes.

2. REVIEW OF PREVIOUS RESULTS

Chamorro & Virbhadra (1996) have calculated the energy of a spherically symmetric charged non-rotating *dilaton* black hole, which obeys the metric

$$ds^2 = A^{-1}dt^2 - Adr^2 - Dr^2(d\Omega^2), \tag{1}$$

Garfinkle, Horowitz, & Strominger (1991, 1992) departed from a variational principle which included a scalar field Φ, and the electromagnetic tensor $F_{\alpha\beta}$, in addition to the Ricci scalar R, to wit,

$$\delta \int \left[-R + 2\left(\nabla\Phi\right)^2 + e^{-2\beta\Phi}F^2\right]\sqrt{-g}d^4x = 0. \tag{2}$$

The resultant field equations are:

$$\begin{aligned}
\nabla_j\left(e^{-2\beta\Phi}F^{jk}\right) &= 0, \\
\nabla^2\Phi + \frac{\beta}{2}e^{-2\beta\Phi}F^2 &= 0, \\
R_{ij} &= 2\nabla_i\Phi\nabla_j\Phi + 2e^{-2\beta\Phi}F_{ia}F^a_j - \frac{1}{2}g_{ij}e^{-2\beta\Phi}F^2,
\end{aligned}$$

and, the *dilaton* was described by the following solution:

$$e^{-2\Phi} = \left[1 - \frac{r_-}{r}\right]^{\frac{1-\sigma}{\beta}}. \tag{3}$$

The sign of β only influences the sign of Φ; we are going therefore to take $\beta > 0$.

The usual Coulomb interaction is given by

$$F_{0r} = \frac{Q}{r^2}. \tag{4}$$

The metric coefficients are

$$A^{-1} = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{\sigma}, \tag{5}$$

and

$$D = \left(1 - \frac{r_-}{r}\right)^{1-\sigma}. \tag{6}$$

In the above, we have made use of the following constraints and/or definitions,

$$\sigma = \frac{1-\beta^2}{1+\beta^2}. \tag{7}$$

$$r_+ + \sigma r_- \equiv 2M. \tag{8}$$

$$r_+ r_- \equiv Q^2\left(1+\beta^2\right). \tag{9}$$

It can be seen that β rules the relative intensity among the three fields, gravitational, electromagnetic and scalar.

In determining the mass and angular momenta of a given asymptotically flat space-time, there are in the literature a number of specific procedures, such as the ADM-Mass, or related pseudo-tensor and gravitational superpotential theories. Some complexes (Landau-Lifshitz, Papapetrou, Weinberg, etc) are usually employed. An important step towards the freedom on their use has been the calculation of Aguirregabiria et al. (1996), showing that most of them yield the same result when applied to a large class of metrics.

The lesson given by Berman (2007a) was that when energy or momentum were calculated, it sufficed to take the charge contribution, leaving $M = 0$, and, at the end of pseudotensorial calculation, making the following transformation:

$$Q^2 \to \left[Q^2 + M^2 + P^2\right], \tag{10}$$

where P stands for the magnetic charge (if there is some).

Of course, room should be made for the inertial content, Mc^2 in the case of the energy, and aM, in the case of rotating black hole's angular momentum: these two terms are the total energy or momentum, when $r \to \infty$.

For instance, if the electric energy of Reissner-Nordström's black hole were given by $-Q^2/2r$, the total contributions for the energy content would be written as,

$$E_{RN} = Mc^2 - \frac{\left[Q^2 + M^2 + P^2\right]}{2r}. \tag{11}$$

When a scalar field of the above form enters into the scene, Chamorro & Virbhadra (1996) found, by pseudo-tensor calculations, for the electric contribution, the term, $-Q^2/2r\left(1-\beta^2\right)$. Therefore, by means of our rule, we have the complete formula as given by,

$$E = Mc^2 - \frac{\left[Q^2 + M^2 + P^2\right]}{2r}\left(1-\beta^2\right). \tag{12}$$

We now turn our attention to the rotating charged situation. By analogy with the above case, consider that, for a K. N. black hole, the metric may be given in Cartesian coordinates by:

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 - \frac{2\left[M - \frac{Q^2}{2r_0}\right] r_0^3}{r_0^4 + a^2 z^2} \cdot \bar{F}^2, \tag{13}$$

while

$$\bar{F} = dt + \frac{Z}{r_0}dz + \frac{r_0}{\left(r_0^2 + a^2\right)}\left(xdx + ydy\right) + \frac{a\left(xdy - ydx\right)}{a^2 + r_0^2}, \tag{14}$$

$$r_0^4 - \left(r^2 - a^2\right) r_0^2 - a^2 z^2 = 0, \tag{15}$$

and

$$r^2 \equiv x^2 + y^2 + z^2. \tag{16}$$

In the above, M, Q and "a" stand respectively for the mass, electric charge, and the rotational parameter, which has been shown to be given by:

$$a = \frac{J_{\mathrm{TOT}}}{M}, \tag{17}$$

where J_{TOT} stands for the total angular momentum of the system, in the limit $R \to \infty$. As Berman (2007a) described in his recent paper, we may keep the electric energy calculations by Virbhadra (1990a,b,c) and Aguirregabiria et al. (1996), and, by applying the transformation (10), obtain, for the energy and angular momenta, the formulae of Berman (2007a):

$$(P_0)_{KN} = M - \left[\frac{Q^2 + M^2 + P^2}{4\varrho}\right]\left[1 + \frac{\left(a^2 + \varrho^2\right)}{a\varrho} arctgh\left(\frac{a}{\varrho}\right)\right], \tag{18}$$

$$P_1 = P_2 = P_3 = 0. \tag{19}$$

Likewise, if we apply:

$$J^{(3)} = \int \left[x^1 p_2 - x^2 p_1\right] d^3x,$$

we find

$$\left(J^{(3)}\right)_{KN} = aM - \left[\frac{Q^2 + M^2 + P^2}{4\varrho}\right] a \left[1 - \frac{\rho^2}{a^2} + \frac{\left(a^2 + \varrho^2\right)^2}{a^3\varrho} \,\mathrm{arctgh}\left(\frac{a}{\varrho}\right)\right]. \tag{20}$$

$$J^{(1)} = J^{(2)} = 0. \tag{21}$$

3. ROTATING KERR-NEWMAN DILATON ENERGY-MOMENTUM

We now are able to write the corresponding result, for a *dilaton* Kerr-Newman black hole's energy and momenta, where, the linear momentum densities are given by:

$$p_1 = -2\left(1 - \beta^2\right)\left[\frac{\left(Q^2 + M^2 + P^2\right)\rho^4}{8\pi(\rho^4 + a^2z^2)^3}\right] ay\rho^2,$$

$$p_2 = -2\left(1 - \beta^2\right)\left[\frac{\left(Q^2 + M^2 + P^2\right)\rho^4}{8\pi(\rho^4 + a^2z^2)^3}\right] ax\rho^2,$$

$$p_3 = 0,$$

while the energy density is given by:

$$\mu = \left(1 - \beta^2\right)\left[\frac{\left(Q^2 + M^2 + P^2\right)\rho^4}{8\pi(\rho^4 + a^2z^2)^3}\right]\left(\rho^4 + 2a^2\rho^2 - a^2z^2\right).$$

The energy and angular momenta are then,

$$(P_0)_{dilaton} = M - \left[\frac{Q^2 + M^2 + P^2}{4\varrho}\right]\left[1 + \frac{\left(a^2 + \varrho^2\right)}{a\varrho} arctgh\left(\frac{a}{\varrho}\right)\right](1 - \beta^2), \tag{22}$$

$$P_1 = P_2 = P_3 = 0, \tag{23}$$

$$\left(J^{(3)}\right)_{dilaton} = aM - \left[\frac{Q^2 + M^2 + P^2}{4\varrho}\right] a \left[1 - \frac{\rho^2}{a^2} + \frac{\left(a^2 + \varrho^2\right)^2}{a^3\varrho} arctgh\left(\frac{a}{\varrho}\right)\right]\left(1 - \beta^2\right), \tag{24}$$

and

$$J^{(1)} = J^{(2)} = 0. \tag{25}$$

In the above, we define ρ as the positive root of equation

$$\frac{x^2+y^2}{\varrho^2+a^2} + \frac{z^2}{\varrho^2} = 1. \tag{26}$$

Relations (23) and (25), "validate" the coordinate system chosen for the present calculation: this is tantamount to the choice of a center-of-mass coordinate system in Newtonian Physics, or the use of comoving observers in Cosmology.

We made the following transformation:

$$Q^2 \to \left[Q^2 + M^2 + P^2\right]\left(1-\beta^2\right), \tag{26a}$$

which takes us from the black hole charge contribution to the total scalar field —electromagnetic charges— gravitation field contributions, which constitute the *dilaton* Kerr-Newman black hole!!!

That our method "works" is a question of applying, say, some superpotential calculations. In Berman (2007a), we have supported this method for the case $\beta = 0$. There is no reason not to generalise it to $\beta \neq 0$ cases. But, again, we can be sure that our formulae keep intact the following physical good properties:

(1) gravitomagnetic effects are explicit;

(2) the triple interaction, among scalar, gravitational and electromagnetic fields becomes evident, as far as energy and angular momenta are concerned; and

(3) when $\beta = 1$, the scalar field neutralizes the other interactions; if $\beta < 1$, the neutralization is only partial.

By considering an expansion of the arctgh (a/ϱ) function in terms of increasing powers of the parameter "a", and by neglecting terms $(a/\rho)^{3+n} \simeq 0$, (with $n = 0, 1, 2, \ldots$) we find the energy of a slowly rotating *dilaton* Kerr-Newman black-hole

$$E \simeq M - \left[\frac{Q^2+M^2+P^2}{R}\right]\left[\frac{a^2}{3R^2}+\frac{1}{2}\right]\left(1-\beta^2\right), \tag{27}$$

where $\varrho \to R$, if $a \to 0$, according to (26).

We can interpret the terms $(Q^2+P^2)a^2(1-\beta^2)/3R^3$ and $M^2a^2(1-\beta^2)/3R^3$ as the magnetic and gravitomagnetic energies caused by rotation.

Expanding the arctgh function in powers of (a/ϱ), and retaining up to third power, we find the slow rotation angular momentum:

$$J^{(3)} \cong aM - 2\left[Q^2+M^2+P^2\right]a\left[\frac{a^2}{5R^3}+\frac{1}{3R}\right]\left(1-\beta^2\right). \tag{28}$$

In the same approximation, we would find:

$$\mu \cong \left[\frac{Q^2+M^2+P^2}{4\pi R^4}\right]\left[\frac{a^2}{R^2}+\frac{1}{2}\right]\left(1-\beta^2\right). \tag{29}$$

The above formula could be also found by applying directly the definition,

$$\mu = \frac{dP_0}{dV} = \frac{1}{4\pi R^2}\frac{dP_0}{dR}. \tag{30}$$

We further conclude that we may identify the gravitomagnetic contribution to the energy and angular momentum of the *dilaton* K.N. black hole, for the slow rotating case, as:

$$\Delta E \cong -\frac{M^2a^2}{3R^3}\left(1-\beta^2\right), \tag{31}$$

and

$$\Delta J \cong -2M^2\left[\frac{a^3}{5R^3}+\frac{a}{3R}\right]\left(1-\beta^2\right) \approx -\frac{2M^2a}{3R}\left(1-\beta^2\right), \tag{32}$$

as can be easily checked by the reader.

4. THE METRIC ELEMENT FOR THE K-N DILATON

We may succeed in obtaining the correct metric for the K-N *dilaton* black hole, by requiring that:

1. when $\beta = 0$, we must retrieve back K-N original metric (13);
2. when $a = 0$, we should obtain Garfinkle et al.'s metric (1);
3. when $\beta = a = 0$, we should reduce to Reissner-Nordström's metric;
4. in the reversed order, we must keep the same relationship, either between Reissner-Nordström's and K-N metric's, or, between Garfinkle et al.'s and our new metric, to be presented below;
5. Chamorro & Virbhadra's result, should be derived from our new result, provided that we take care of transformation (26a);
6. the metric to be found, should be the simplest one to obey the above requirements.

We now present the metric:

$$ds^2 = Rdt^2 - Sdr^2 - Dr^2 d\Omega^2 - \frac{2\left[M\sqrt{1-\beta^2} - \frac{\left[Q^2\right](1-\beta^2)}{2r_0}\right] r_0^3}{r_0^4 + a^2 z^2} \cdot \bar{F}^2, \tag{33}$$

where, $\bar{F}$ is given by relations (14), (15) and (16), and,

$$R \equiv A^{-1} + \Gamma, \tag{34}$$

$$S \equiv A + \frac{\Gamma}{\Gamma - 1}, \tag{35}$$

$$\Gamma \equiv \frac{2M}{r}\sqrt{1-\beta^2} - \frac{\left[Q^2\right]\left(1-\beta^2\right)}{r^2}. \tag{36}$$

The above metric represents our *dilaton*.

5. CONCLUDING REMARKS

It is important to notice that the contributed energy, due to the scalar field is given by the term

$$\frac{\beta^2}{2r}\left[M^2 + Q^2 + P^2\right] > 0, \tag{37}$$

but the corresponding energy density is negative, given by

$$-\left[\frac{\beta^2\left[M^2 + Q^2 + P^2\right]}{8\pi R^4}\right]. \tag{38}$$

This negative energy density is the trademark of the scalar field. It must be remarked that all of our results do not match with Chamorro & Virbhadra's, except in the particular case when $M = P = 0$. Of course, those authors only examined the Reissner-Nordström's *dilaton*, a non-rotating black hole. We also found that there is a relative interaction between matter, charges and the scalar field.

We remember that the terms Mc^2 and aM which appear respectively, in the energy and momentum formulae, refer to inertia and not to gravitation: thus, they refer to Special Relativity. We have found also that the scalar field reduces the self-energies, of gravitation and electromagnetic origin, by a factor $\left(1-\beta^2\right)$. This fact remains an important feature of the present derivations, since we may think of a kind of new Equivalence Principle under the possibility that not only acceleration is equivalent to a gravitational field, but the kind of neutralization we have studied points to a way of eliminating gravity at small scales, at least, by means of a scalar field. We have been accused of a very "sloppy" use of the neutralization property cited above; however, we must take care, because we are only dealing with the energy momentum concept, and the physics of the problem has a lot more to say, in addition to energy considerations. For instance, the β parameter has been making a shift in the location of the horizons but may not be the only ruler of the intensity among the three fields, when we are dealing with other physically related properties.

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REFERENCES

Aguirregabiria, J. M., Chamorro, A., & Virbhadra, K. S. 1996, Gen. Relativ. Gravitation, 28, 1393
Berman, M. S. 2007a, RevMexAA, 43, 297
__________. 2007b, Introduction to General Relativity and the Cosmological Constant Problem (New York: Nova Science)
__________. 2007c, Introduction to General Relativistic and Scalar-Tensor Cosmologies (New York: Nova Science)
__________. 2008, A Primer in Black Holes, Mach's Principle and G ravitational Energy (New York: Nova Science)
Brans, C., & Dicke, R. H. 1961, Phys. Rev., 124, 925
Chamorro, A., & Virbhadra, K. S. 1996, Int. J. Mod. Phys. D, 5, 251
Collins, P. D. B., Martin, A. D., & Squires, E. J. 1989, Particle Physics and Cosmology (New York: Wiley-Interscience)
Eckart, A., Straubmeier, C., & Schödel, R. 2005, The Black Hole at the Center of the Milky Way (Singapore: World Scientific)
Falcone, A. D., et al. 2008, in AIP Conf. Proc. 1000, Gamma Ray Burst, ed. M. Galassi, D. Palmer, & E. Fenimore (New York: AIP), 91
Faraoni, V. 2004, Cosmology in Scalar Tensor Gravity (Dordrecht: Kluwer)
Fujii, Y., & Maeda, K.-I. 2003, The Scalar Tensor Theory of Gravitation (Cambridge: Cambridge Univ. Press)
Garfinkle, D., Horowitz, G. T., & Strominger, A. 1991, Phys. Rev. D, 43, 3140
__________. 1992, Phys. Rev. D, 45, 3888
Halpern, P. 2008, Ap&SS, 313, 357
Kreitler, P. 2006a, Trends in Black-Hole Research (New York: Nova Science)
__________. 2006b, New Developments in Black-Hole Research (New York: Nova Science)
__________. 2006c, Focus in Black-Hole Research (New York: Nova Science)
Lee, H. K., & Park, M. G. 2002, Black Hole Astrophysics 2002: Proceedings of the Sixth APCTP Winter School (Singapore: World Scientific)
Levinson, A. 2006, in Trends in Black Hole Research, ed. P. V. Kreitler (New York: Nova Science), 119
Martínez, C., & Troncoso, R. 2006, Phys. Rev. D, 74, 064007
Noyola, E., Gebhardt, K., & Bergmann, M. 2008, ApJ, 676, 1008
Pope, E., Pittard, J., Hartquist, T., & Falle, S. 2008, MNRAS, 385, 1779
Vagenas, E. C. 2003, Int. J. Mod. Phys. A, 18, 5949
Virbhadra, K. S. 1990a, Phys. Rev. D, 41, 1086
__________. 1990b, Phys. Rev. D, 42, 2919
__________. 1990c, Phys. Rev. D, 42, 1066
Xulu, S. S. 1998, Int. J. Mod. Phys. D, 7, 773
Wald, R. M. 1994, Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics (Chicago: Chicago Univ. Press)
Wesson, P. S. 1999, Space-Time-Matter: Modern Kaluza, Klein Theory (Singapore: World Scientific)
__________. 2006, Five Dimensional Physics (Singapore: World Scientific)

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