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OPTICAL QUANTUM ENTANGLEMENT IN ASTROPHYSICS

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RESUMEN

Las teorías del entrelazamiento cuántico entre dos partículas lejanas, que claramente confirman la naturaleza no local de la Mecánica Cuántica, se aplican a las partículas producidas naturalmente en los objetos astrofísicos. Estudiamos la producción y la recepción del caso de entrelazamiento cuántico más factible de observarse: la transición espontánea de dos-fotones del nivel metastable $2\,^2S_{1/2}$ del hidrógeno, componente conocido del espectro del continuo de las regiones ionizadas. Obtenemos la tasa de emisión de dos-fotones para cuatro objetos astrofísicos: la Nebulosa de Orión, dos nebulosas planetarias cercanas IC 2149 y NGC 7293, y la corona solar. La producción de pares entrelazados por segundo es de 5.80×10^{48}, 9.39×10^{45}, 9.77×10^{44} y 1.46×10^{16}, respectivamente. La distribución de las direcciones de propagación de ambos fotones emitidos no se anula para ningún ángulo; por lo que es posible observar el par entrelazado a ángulos $\theta \approx 0°$. Debido a que el número de coincidencias de dos-fotones va como el cociente entre el tamaño del detector y la distancia al objeto astrofísico elevado a la cuarta potencia, las coincidencias son escasas; para su detección se requiere de receptores de un tamaño mucho mayor que los que existen en la actualidad.

ABSTRACT

The theories of quantum entanglement between two distant particles, which clearly confirm the non-local nature of Quantum Mechanics, are applied to naturally produced particles in astrophysical objects. We study the production and reception of the cases of optical quantum entanglement most feasible to be observed: the two-photon spontaneous transition of the hydrogen $2\,^2S_{1/2}$ metastable level, which is known to be one of the components of the continuous spectra of ionized regions. We obtain the two-photon emission rate for four astrophysical objects: the Orion Nebula, two nearby planetary nebulae IC 2149 and NGC 7293, and the solar corona. The production of entangled pairs per second is 5.80×10^{48}, 9.39×10^{45}, 9.77×10^{44}, and 1.46×10^{16} respectively. The distribution of the propagation directions of both emitted photons does not vanish at any angle; therefore it is possible to observe the entangled pair at angles $\theta \approx 0°$. Because the number of two-photon coincidences goes as the fourth power of the ratio between the detector size and the distance from the astrophysical object, coincidences are scarce; for its detection we require receivers much larger than those currently available.

Key Words: HII regions — planetary nebulae: individual (IC 2149, NGC 7293) — Sun: corona

1. INTRODUCTION

Einstein, Podolski, & Rosen (1935) wrote a crucial paper about the fundamentals of Quantum Mechanics (hereinafter QM), where they presented a quantum entanglement *Gedankenexperiment* to argue about the

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incompleteness of QM. The great impact of the Einstein et al. (1935) paper in questioning the foundations of QM, granted the EPR acronym to the meaning of *a pair of separated entangled particles*; the implications contained in this work are also known as the EPR Paradox. The consequences of this paper opened new points of view about realism and non-locality in QM. The theories in which it is assumed that QM is incomplete and require additional variables to explain the statistical predictions of QM are called *Hidden Variable Theories*. Later, Bohm (1951) approached the problem with a variant of the EPR, using discrete variables only; an approach which Bell (1964) also used (with the assumption of a locality condition) to establish a test to check whether or not the EPR argument is valid. These are his famous inequalities which were deduced only under the assumption of the traditional physical interpretation of realism and locality (without any contribution of QM); these inequalities would be the consequence of the validity of the local *Hidden Variable Theories*. Clauser et al. (1969), Shimony (1971), Clauser & Shimony (1978, and references therein), established the requirements to make a real experiment to disprove the local *Hidden Variable Theories* by confirming the violation of the Bell's inequalities; thus showing that the EPR argument is incorrect. A variety of entangled pairs of particles can be produced in the laboratory by different methods, such as positronium annihilation, proton-proton scattering, decay of $\pi^0 \rightarrow e^+ + e^-$, cascade-photon experiments and two-photon experiments of the hydrogen metastable forbidden transition $2\,^2S \rightarrow 1\,^2S$ (e.g. Feynmann 1965; Clauser & Shimony 1978; Peres 1995; Kleinpoppen et al. 1998; Mandel & Wolf 1995). Several experimental tests of local hidden-variable theories were performed by Freedman & Clauser (1972), Fry & Thompson (1976), Lamehi-Rachti & Mittig (1976), their results being in good agreement with QM. The conclusive experiment was performed by Aspect, Dalibard, & Roger (1982), who measured the linear polarization correlations of pairs of photons with time-varying analyzers; for their solid angles and polarizer efficiencies, QM predicts $S_{\rm QM} = 0.112$, (the S variable is defined therein). They found that the average of their two runs yields $S_{\rm exp} = 0.101 \pm 0.020$ violating the Shimony-Holt inequality $S \leq 0$ (an equivalent of Bell's inequality) by 5 standard deviations, which effectively disproved the existence of local hidden variables theories. In particular, an experiment to test the EPR non-locality has been performed using the decay of the hydrogen $2\,^2S_{1/2}$ metastable level by Kleinpoppen et al. (1998); their results are also in excellent agreement with QM. Weihs et al. (1998) show an experiment which complies with Bell's requirements for a strict relativistic separation between measurements. For further discussion and analysis of almost three decades of experiments see Aspect (1999). Fedrizzi et al. (2009) show a recent experiment performed between La Palma and Tenerife, in the Canary Islands, with a transmission of entangled photons along 144 km through a turbulent atmosphere and measure de violation of CHSH (Clauser-Horne-Shimony-Holt) Bell's inequality (see Clauser et al. 1969).

The possibility to observe this quantum effect in a naturally occurring astrophysical object seems particularly interesting, since not only it would be observed outside of laboratory conditions, but the distance associated with the non-locality of such an entanglement would be unprecedented.

Of the previously mentioned physical processes, we will focus on the two-photon decay process of the hydrogen $2\,^2S_{1/2}$ metastable level, as this is the most accessible quantum entanglement process from the astrophysical point of view. The possibility of the existence of the two-photon transition was first introduced by Maria Göpert-Mayer (1931). Breit & Teller (1940) obtained the first calculation of the two-photon transition probability for metastable hydrogen, while Spitzer & Greeenstein (1951), considering that the simultaneous two-photon emission could be an important source of the continuous emission of planetary nebulae, provided an accurate value for its transition probability: $A_{2^2S,1^2S} = 8.227\,\mathrm{sec}^{-1}$. Novick (1969) presented an extensive review of the previous work on two-photon productions, and showed (both theoretically and experimentally) that the angular correlation between the propagation directions between both photons is proportional to $(1 + \cos^2\theta)$, which clearly shows that both photons can be emitted in all directions with respect to each other. This is an important result since for astrophysical detection it is convenient that the direction of propagation of the two-photon pair has to be nearly parallel. The main goal of this paper is to make clear that this entangled two-photon decay process of the hydrogen $2\,^2S_{1/2}$ metastable level, well studied in laboratory conditions, occurs naturally and widely in astrophysics, as well as to make a first theoretical study of quantum entanglement in astrophysics to establish the possibility of its detection. In § 2 we present examples of entanglement that could be produced in astrophysical environments, and argue that two-photon production is the one most likely to be detected with a receiver mounted on or near the earth. In § 3 we discuss more specifically the two-photon production from the hydrogen $2^2S \rightarrow 1^2S$ transition, and review its production in different astrophysical environments: H II regions, planetary nebulae (hereinafter PNe), and the Solar Corona; we also discuss here the difficulties of its detection. The causes of annihilation of one of the photons of the two-photon pair of photons due the interstellar medium (ISM) are discussed in § 4, while the results and conclusions are presented in § 5.

2. EXAMPLES OF NON-LOCALITY IN ASTROPHYSICS

Although the EPR phenomena can occur with more than two particles, it is easier to obtain all the relevant variables, for processes where only two particles are produced. The case of two entangled particles of the EPR Bohm's kind, called spin system singlets is discussed by Peres (1995). There are many mechanisms to produce entangled pairs of particles; examples relevant in astrophysics are: (i) the annhilation of the positronium (Feynmann 1965), which produces a pair of entangled γ photons (a process that occurs spontaneously in stellar interiors, stellar coronae, or cosmic rays); (ii) the decay of a spinless particle $\pi^0 \to e^+ + e^-$ (Peres 1995), which happens in some nuclear reactions in stellar interiors or in cosmic rays; (iii) the proton-proton scattering (Clauser & Shimony 1978), which may occur in stellar atmospheres or in the ISM; (iv) the two-photon atomic SPS cascades such as the case of observed $\lambda 5513$ Å and $\lambda 4227$ Å pairs produced in the calcium atom (Clauser & Shimony 1978), which may occur in stellar atmospheres; and (v) the two-photon emission in the hydrogen $2\,^2S_{1/2}$ metastable level that occurs in large quantities in H II regions and PNe, which is observable as part of their whole continuum emission. From all these kinds of EPR systems of entangled particles, the most feasible to be detected is the latter. This is due to the abundance of hydrogen in the universe and to the moderate amount of energy involved in this process.

The EPR phenomena associated to the decay of the hydrogen $2\,^2S_{1/2}$ metastable level have been studied by Kleinpoppen et al. (1998) in experiments that confirm the violation of Bell's inequalities. It is known that in the case of the two-photon singlet state, produced by a SPS atomic cascade (e.g. the hydrogen cascade $3\,^2S_{1/2} \to 2\,^2P_{1/2} \to 1\,^2S_{1/2}$), the initial and final states of the atom have zero total angular momentum and even parity, i.e, $J = 0 \to J = 1 \to J = 0$, while the angular momentum of the combined state can be expressed in terms of the spherical vector function:

$$\Psi = \frac{1}{\sqrt{3}} \left[Y^1_{1,1}(\hat{\eta}_1)\, Y^1_{1,-1}(\hat{\eta}_2) - Y^1_{1,0}(\hat{\eta}_1)\, Y^1_{1,0}(\hat{\eta}_2) + Y^1_{1,-1}(\hat{\eta}_1)\, Y^1_{1,1}(\hat{\eta}_2) \right] , \quad (1)$$

where $Y^1_{j,m}$ is the spherical harmonic vector function with total angular momentum j, magnetic quantum number m, and parity -1 (Clauser & Shimony 1978). This is important because the similarities between the wavefunctions of the atomic cascade and the two-photon emission in the hydrogen $2\,^2S_{1/2}$ metastable level allow the SPS atomic cascade well-known entanglement properties and its theory to be applied to the two-photon hydrogen emission (Biermann, Scully, & Toor 1997).

It is well known that part of the overall continuum radiation from the regions studied in this paper results from the two-photon emission in the hydrogen $2\,^2S$ metastable level, i.e. a pair of two entangled photons. We will denote one of these photons the *signal photon*: Ph_s, and the other one the *idle photon*: Ph_i; adopting the analogy notation of the parametric down-conversion process in Mandel & Wolf (1995). The corresponding frequencies of both photons ν_s, and ν_i are complementary, i.e., $\nu_0 = \nu_s + \nu_i$, by conservation of energy.

3. DISCUSSION OF THE TWO-PHOTON SPONTANEOUS EMISSION

A review of early studies of the two-photon emission in the hydrogen $2\,^2S_{1/2}$ metastable level is presented by Novick (1969); in this paper the relevance of the nondegeneracy of the $2\,^2S_{1/2}$ and $2\,^2P_{1/2}$ hydrogen atomic levels is pointed out, to allow for the metastability of the $2\,^2S_{1/2}$ level as Bethe had concluded. This metastability is used to explain the anomalous behavior of the Balmer absorption lines observed in astrophysics. Breit & Teller (1940) calculated the two-photon transition probability even though they assumed the degeneracy of the $2\,^2S_{1/2}$ and $2\,^2P_{1/2}$ levels; they used the Göpert-Mayer two-photon transition theory to obtain the probability distribution as a function of the frequency of Ph_s:

$$\begin{aligned} A(\nu_s)d\nu_s &= \frac{1024\pi^6 e^4 \nu_s^3 \nu_i^3}{h^2 c^6} \times \\ & \left(\left| \sum_{n''} \left[\frac{\langle n'|r\cdot\hat{\epsilon}_s|n''\rangle\langle n''|r\cdot\hat{\epsilon}_i|n\rangle}{\nu_{n''n}+\nu_i} + \frac{\langle n'|r\cdot\hat{\epsilon}_i|n''\rangle\langle n''|r\cdot\hat{\epsilon}_s|n\rangle}{\nu_{n''n}+\nu_s} \right] \right|^2 \right)_{av} d\nu_s \,, \end{aligned} \quad (2)$$

where n is the initial state of 2^2S, n' is the final state of 1^2S, and n'' is the 2^2P intermediate state. We will denote the frequency and the polarization of the *signal photon* as ν_s and $\hat{\epsilon}_s$ respectively, and the frequency and the polarization of the *idle photon* as ν_i and $\hat{\epsilon}_i$ respectively; r is the position vector of the electron with respect to the nucleus. The spectral distribution is symmetric about $\nu_{nn'}/2$; by conservation of energy we have,

$$\nu_s + \nu_i = \nu_{2\,1} = \frac{E_2 - E_1}{h} = \frac{10.2\text{eV}}{h} = 1.46 \times 10^{15}\ \text{s}^{-1}\,, \tag{3}$$

where the average is taken over all the directions of propagation of the photons and over all their polarizations. Novick (1969) presents theoretical and experimental results for the angular correlation between the propagation directions of both photons, finding for the angular distribution:

$$P(\theta)d\theta d\phi = \frac{1}{6\pi^2}(1 + \cos^2\theta)d\theta d\phi\,. \tag{4}$$

It is commonly known (e.g., Kleinpoppen et al. 1998) that the emitted photons Ph_s, and Ph_i are entangled in their polarizations when the angle between their propagation directions is 180°, i.e., when they are emitted in opposite directions, as is the standard experimental set-up in the Aspect-like experiments to test the Bell inequalities (Duncan et al. 1997); however, for our astrophysical detection purposes we require that both photons are emitted approximately in the same direction, i.e., when the azimuthal angle is $\phi \approx 0°$ in spherical coordinates.

When both entangled photons propagate in the same direction, i.e. $\hat{\eta}_s = \hat{\eta}_i = +\hat{x}$ (both with the same azimuthal angles $\phi_s = \phi_i = 0$, on the plane of the polar angle $\theta = \pi/2$), equation (1) can be expressed as:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|y\rangle_s|y\rangle_i + |z\rangle_s|z\rangle_i\right)\,, \tag{5}$$

where $|y\rangle_s$ and $|z\rangle_s$ represent the polarization states of the *signal photon* in the y and z directions respectively, and $|y\rangle_i$ and $|z\rangle_i$ represent the polarization states of the *idle photon* in the y and z directions respectively. This is the form of the entangled polarization wavefunction for two photons within an angle close to zero.

Regarding the emission of the two-photon radiation from the hydrogen $2\ ^2S_{1/2}$ metastable level we now study three different astrophysical objects with ionized regions: H II regions, PNe, and stellar coronae. As we will see later the efficiency to detect the pairs of photons from a two-photon emission is inversely proportional to the fourth power of their distances; therefore we will focus our attention on some of the closest objects: the Orion Nebula, NGC 7293, the solar corona, and IC 2149, another PN that, though not as close as the other objects, has a well-studied two-photon spectrum (Gurzadyan 1976).

In the following sections we will estimate for our selected astrophysical examples: $R_{\rm PP}$, the production rate of entangled pairs of photons; $Q(2q) = R_{\rm PP} \times 2$, the total number of continuum emitted photons produced by the object via the $2\ ^2S \rightarrow 1\ ^2S$ decay; z_{Ph_s}, the number of signal photons per second per square centimeter that arrive on earth in each case; as well as $C\,(\Theta)$, the number of two-photon coincidences that can be detected with our receiver of diameter D at a distance L from the particular region, which depends on the angle subtended by the receiver:

$$\Theta = \frac{D}{L}\,\text{radians}\,. \tag{6}$$

4. IONIZED REGIONS

We obtain $Q(2q)$ for three different types of ionized regions: H II regions, planetary nebulae and the solar corona. For the first two types, we have selected, in particular, the Orion Nebula, IC 2149 and NGC 7293, since these are some of the closest and brightest objects of their kind. In order to find $Q(2q)$ for the Orion Nebula we need: the number of photoionizing photons per second $Q(H^0)$, (which can be obtained directly from Table 2.3 in Osterbrock & Ferland 2006); and the probability that the recombination results in a two-photon decay (Brown & Matthews 1970):

$$X = \frac{S}{1 + n_e r} + \frac{f_s A_{2^2S,1^2S}\, n_e r(1 - S + n_e r)}{3A(1 + n_e r)^2}\,, \tag{7}$$

TABLE 1

ORION AND PLANETARY NEBULAE DATA

Data	Orion	IC 2149	NGC 7293
L [kpc]	0.414[g]	1.585[a]	0.157[a]
$\log\ F(H\beta)$ [erg cm^{-2}]		-10.55[a]	-9.37[a]
T_e [K]	8500[c]	12000[d]	4600 ± 1200[b]
n_e [cm^{-3}]	3.0×10^{3}[c]	3.2×10^{3}[d]	300[f]
S[e]	0.316	0.332	0.285
r [cm^3][e]	6.62×10^{-5}	6.28×10^{-5}	6.8×10^{-5}
ξ[h]	0.53	0.55	0.57
$\alpha_B/\alpha^{\rm eff}_{H_\beta}$		8.55[c]	8.55[c]
c[†]	0.46[i]	0.43[j]	0.23[j]
$Q(H^0)$ [photons s^{-1}]	2.19×10^{49}	3.41×10^{46}	3.43×10^{45}

[a]Cahn, Kaler, & Stanghellini (1992). [b]Casassus et al. (2003). [c]Osterbrock & Ferland (2006). [d]Gurzadyan (1976). [e]Brown & Mathews (1970). [f]Warner & Rubin (1975). [g]Menten et al. (2007). [h]Equation (8). [i]$c(H_\beta$ method) Costero & Peimbert (1970). [j]$c(Balmer-line$ method) Cahn (1976). [†]c is the reddening logarithmic correction.

where S is the fraction of the recombinations to the excited levels which enter the $n = 2$ level for the first time into the $2\,^2S_{1/2}$ state, n_e is the electron density, $r = q/A_{2^2S,1^2S}$, where q is the total rate coefficient for $2\,^2S_{1/2} \to 2\,^2P_{(1/2,\ 3/2)}$ collisional transitions by both electrons and protons and $A_{2^2S,1^2S} = 8.227\,\mathrm{s}^{-1}$ (Spitzer & Greeenstein 1951), is the total probability per second of a two-photon transition, f_s is the mean number of scatterings that a Lyman-alpha photon experiences in the nebula and $A = 6.265 \times 10^8\,\mathrm{s}^{-1}$ is the probability per second for a Lyman-alpha emission transition. The expression for X is correct to first order in $A_{2^2S,1^2S}/A$; and X is valid, provided $f_s \gg 1$ and $n_e \ll A/q \approx 10^{12}\,\mathrm{cm}^3$.

In order to find out the total number of two-photon emitted photons per second $Q(2q)$ we multiply $Q(H^0)$ by $2X$ (the factor of two is due to the fact we have two photons), where we approximate X to the first order in $1/(1+n_e r)$ and define ξ as $2X$; therefore we take:

$$\xi = \frac{Q(2q)}{Q(H^0)} = \frac{2S}{1+n_e r}\,. \tag{8}$$

We obtain the ratio ξ for the Orion Nebula and the PNe from their corresponding values of S and r in Table 3 from Brown & Mathews (1970) (see also Table 1). In general we will adopt

$$Q(2q) = \xi \cdot Q(H^0)\,. \tag{9}$$

In the case of the PNe, $Q(H^0)$ is proportional to $F(H_\beta)$, the observational $H\beta$ flux measurement.

The number of photoionizations is:

$$Q(H^0) = \frac{\alpha_B(H^0,T)}{\alpha^{\rm eff}_{H_\beta}(H^0,T)} \times \frac{L(H_\beta)}{h\nu_{H_\beta}} = \frac{\alpha_B}{\alpha^{\rm eff}_{H_\beta}} \times \frac{\pi F(H_\beta)}{4\pi L^2 h\nu_{H_\beta}}\,, \tag{10}$$

where α_B is the effective recombination rate to $n = 2$ hydrogen level, $\alpha^{\rm eff}_{H_\beta}$ is the effective H_β photon production rate due to recombinations, which can be obtained from the tables on Osterbrock & Ferland (2006), $L(H_\beta)$ is the total $H\beta$ luminosity and L is the distance to the object.

The number of signal photons z_{Ph_s} which arrive on the earth per cm^2 per second would be:

$$z_{Ph_s} = \frac{Q(2q)}{8\pi L^2}\,\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\,, \tag{11}$$

where L is the distance to the object in cm. The number of signal photons that can be detected by a receiver of diameter D will be:

$$Z_{Ph_s} = z_{Ph_s} \times \frac{\pi D^2}{4} = \frac{Q(2q)}{32 \cdot L^2} \times D^2\,\mathrm{s}^{-1}, \tag{12}$$

The probability that the corresponding *idle photon* to a *signal photon* from a two-photon emission will be emitted within the very small angle $\theta \leq \Theta$, from equation (4) is:

$$P(\Theta) = \frac{\Theta^2}{3}\,; \tag{13}$$

therefore, the number of coincidences per second that arrive at the receiver on the earth will be:

$$C(\Theta) = Z_{Ph_s} \times P(\Theta) = Z_{Ph_s}\frac{\Theta^2}{3}\,, \tag{14}$$

which gives a general expression for the number of two-photon coincidences per second that can be detected:

$$C(\Theta) = \frac{Q(2q)D^2}{8\pi L^2} \times \frac{1}{3}\left(\frac{D}{L}\right)^2 = \frac{Q(2q)}{24\pi} \times \left(\frac{D}{L}\right)^4. \tag{15}$$

For the solar corona we use a different approach to find the total number of emitted signal photons $Q(2q)/2$ per second and the observed two-photon coincidences $C_\odot$. In this case we integrate the local emission coefficient, taking into account the variations of temperature and density as a function of the distance from the surface of the Sun. To calculate the emission coefficient of the solar corona we consider only the Baker & Menzel (1938) Case B. From Kwok (2000) we have:

$$j_\nu^{2p}(L,y) = \frac{1}{4\pi} \times \frac{n_e^2(L)\alpha_B\,[T(L)]}{A_{2^2S,1^2S}}hyA_{2p}(y)\,, \tag{16}$$

where $j_\nu^{2p}(L,y)$ is the local emission coefficient at a distance L from the center of the Sun, with its maximum value when $y = 0.75$, in units of energy per unit volume per unit time per unit solid angle per unit frequency. We assume that hydrogen is completely ionized i.e., $n_p = n_e$. The variable y is the ratio of the photon frequency to the Lyman-alpha frequency $\nu/\nu_{Ly\alpha}$; here $A_{2p}(y)$ is the two-photon transition probability distribution for which Kwok (2000) gives an approximated algebraic expression:

$$A_{2p}(y) = 202.0\mathrm{s}^{-1}\left\{\mathrm{y}(1-\mathrm{y})\left[1-(4\mathrm{y}\,\{1-\mathrm{y}\})^{0.8}\right] + 0.88[\mathrm{y}(1-\mathrm{y})]^{1.53}[4\mathrm{y}(1-\mathrm{y})]^{0.8}\right\}. \tag{17}$$

Its corresponding curve is shown in Figure 1.

We apply the solar corona model for the electron density and temperature from Lang (1999) as a function of the distance L from the solar center with the variable $\zeta = L/R_\odot$, where $R_\odot = 6.96 \times 10^{10}$ cm is the solar radius. We use the following expressions for the solar corona's electron density, for $1 \leq \zeta \leq 3.83$

$$n_e(\zeta) = 1.55 \times 10^8 \zeta^{-6} \times \left(1 + 1.93 \times \zeta^{-10}\right)\mathrm{cm}^{-3}\,, \tag{18}$$

and for $\zeta > 3.83$

$$n_e(\zeta) = 7.2 \times 10^5 \zeta^{-2}\mathrm{cm}^{-3}, \tag{19}$$

according to Lang (1999). The critical electron density can be obtained by using the total collisional transition rate coefficient from Osterbrock & Ferland (2006), $n_{e\,\mathrm{crit}} = 1.645 \times 10^4$ cm^{-3}; for the solar corona's electron temperature we use the following expression:

$$T_e(\zeta) = \zeta^{-2/7} \times 10^6\,\mathrm{K}\,. \tag{20}$$

The total recombination coefficient for case B (Brown & Mathews 1970) is:

$$\alpha_B(t) = 1.627 \times 10^{-13}\,t^{-1/2}(1 - 1.657\log t + 0.584\,t^{1/3})\,\mathrm{cm}^3\,\mathrm{s}^{-1}\,, \tag{21}$$

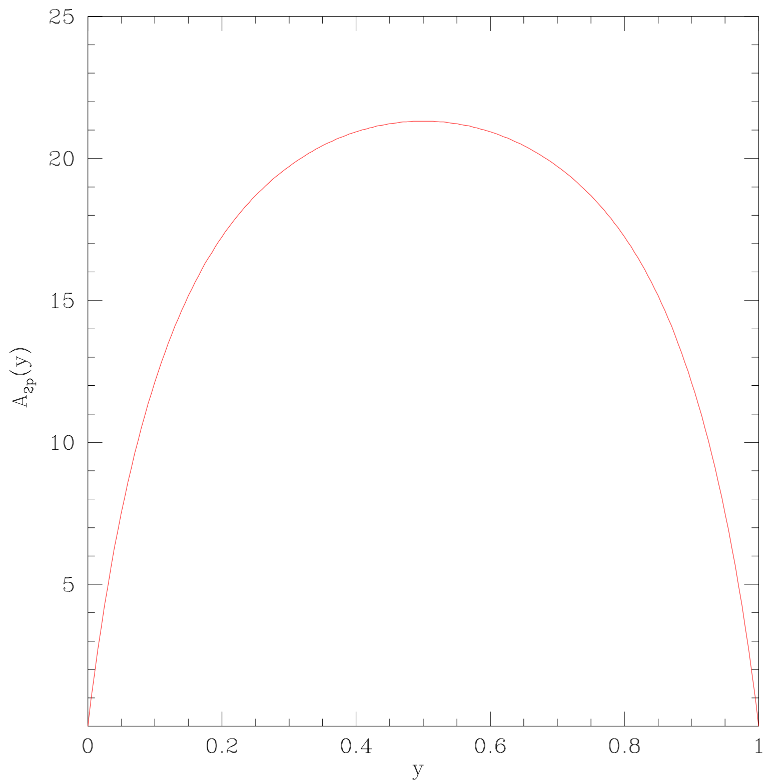

Fig. 1. Two-photon distribution for excited hydrogen. $A_{2p}(y)$ is the two-photon transition probability distribution, while y is the ratio of the photon frequency to the Lyman-alpha frequency.

where $t \equiv 10^{-4} T_e(\zeta)$. We use equation (21), with the help of equations (18), (19), and (20) and obtain

$$\begin{aligned} \alpha_B(\zeta) &= 1.627 \times 10^{-13} (\zeta^{-2/7} \times 10^2)^{-1/2} \times \\ & \left[1 - 1.657 \log_{10}(\zeta^{-2/7} \times 10^2) + 0.584 (\zeta^{-2/7} \times 10^2)^{1/3}\right] \mathrm{cm}^3 \mathrm{s}^{-1} \,. \end{aligned} \tag{22}$$

The number of signal photons produced per second by the solar corona $Q(2q)/2$ can be obtained with the help of equations (16), and (22). We integrate the total contribution of signal photons by using equations (18) and (19) multiplying them by the electron density correction factor $F_e(\zeta) = (n_{e\,\mathrm{crit}} / (n_e(\zeta) + n_{e\,\mathrm{crit}}))$ from $\zeta = 1$(surface of the sun) to a certain value of $\zeta = \zeta_l$. We obtain:

$$Q(2q) = 2 \int_{0.5}^{1} \int_0^{4\pi} \int_{\zeta=1}^{\zeta=\zeta_l} \frac{4\pi R_\odot^3}{4\pi h\nu} \times F_e(\zeta) \times \left(\frac{\zeta^2 n_e^2(\zeta) \alpha_B(\zeta)}{A_{2^2S,1^2S}} \times hy A_{2p}(y) \right) dy d\Omega d\zeta \,. \tag{23}$$

When we integrate up to 1 A.U. ($\approx \zeta = 215$) we obtain $Q(2q) = 2.92 \times 10^{16}$photons $\cdot\, \mathrm{s}^{-1}$. As we can see in Figure 2 the contribution of two-photon emission increases quickly at a distances close from the Sun's surface, at 10 radii from the Sun's surface we have about 95% of the total solar corona two-photon contribution and it slowly approaches the total value of $Q(2q) = 2.78 \times 10^{16}$. Therefore, for practical purposes we will integrate up to the limit value of $\zeta = 10$ in order to consider the solar corona as a distant object similarly as we have treated other ionized regions, and use equation (12) to obtain the minimum diameter of the detector on the surface of the earth.

5. RESULTS

For our calculations, we consider that the main contribution of photoionizing photons in the Orion H II region arises from the dominant O6 star θ^1 Ori C. From Osterbrock & Ferland (2006; see their Table 2.3) we obtain

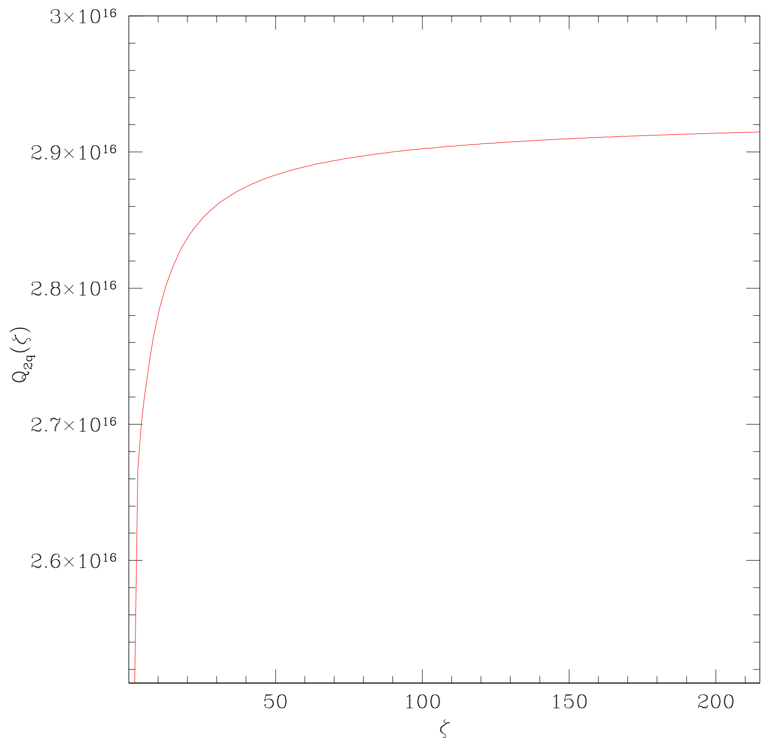

Fig. 2. Contribution of the two-photon emission as a function of ζ, the radii from the Sun's surface.

$Q(H^0) = 2.19 \times 10^{49}$ photoionizing photons s^{-1}. Taking $\xi = 0.53$ from Table 1, we obtain from equation (9), $Q(2q) = 1.16 \times 10^{49}$ photons s^{-1}. Adopting a distance to the Orion H II region of $L = 414\,\mathrm{pc} = 1.28 \times 10^{21}$ cm (Menten et al. 2007), we obtain $z_{Ph_s} = 2.83 \times 10^5 \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$ If, for example, the diameter of the receiver were $D = 1$ km $= 1 \times 10^5$ cm, we would be able to collect $Z_{Ph_s} = 2.22 \times 10^{15}$ signal photons per second. Although the number of detected signal photons per second will increase with the area of the detector, the number of two photon coincidences will decrease with the fourth power of the distance to the object. For the case of the Orion Nebula we estimate, from equation (15) that, in order to detect one two-photon coincidence per year we need a receiver of an approximate diameter of $D = 272$ km.

In the case of the PNe we derive $Q(H^0)$ basically from the observed $H\beta$ flux measurements and the distance to the objects, using equation (10). The fraction ξ is obtained as before, from equation (8) for IC 2149 and NGC 7293. Table 1 shows the relevant astrophysical data for the PNe, and their calculations of $Q(H^0)$, obtained from equation (10), $Q(H^0)$[IC 2149] $= 3.41 \times 10^{46}$ photoionizing photons s^{-1} and $Q(H^0)$[NGC 7293] $= 3.43 \times 10^{45}$ photoionizing photons s^{-1}. Taking ξ[IC 2149] $= 0.55$ and ξ[NGC 7293] $= 0.57$ from Table 1; we obtain from equation (9), $Q(2q)$[IC 2149] $= 1.88 \times 10^{46}$ photons s^{-1} and $Q(2q)$[NGC 7293] $= 1.95 \times 10^{45}$ photons s^{-1}; taking their distances from Table 1 we obtain their number of signal photons z_{Ph_s} which arrive on the earth per cm^2 per second, z_{Ph_s}[IC 2149] $= 3.12 \times 10^1 \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$ and z_{Ph_s}[NGC 7293] $= 3.31 \times 10^2 \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$. For the case of the PNe we estimate, from equation (15) that, in order to detect one two-photon coincidence per year, we need a receiver of an approximate diameters of $D = 5\,200$ km and $D = 906$ km for IC 2149 and NGC 7293 respectively as is shown in Table 2.

From equation (12) is easy to find a diameter of $D = 457$ km for the receiver that we would need to detect one coincidence per year from the solar corona, i.e., $C_\odot(\Theta) = 3.171 \times 10^{-8}$ per sec; this diameter is much larger than the one we have obtained for the Orion H II region (272 km) due to the fact that the Orion Nebula produces about 10^{32} times more two-photon emissions than the solar corona, although the latter is closer to the earth.

TABLE 2

RECEIVER DIAMETERS FOR ONE TWO-PHOTON COINCIDENCE PER HOUR AND PER YEAR

Object	L	$R_{\rm PP}$ pairs $\rm s^{-1}$	$D[1/y]$[a] km	$D[1/y]$[b] km	$D[1/hr]$[b] km
Orion Nebula	414 pc	5.8×10^{48}	272	785	7590
IC 2149	1585 pc	9.39×10^{45}	5200	14000	135000
NGC 7293	157 pc	9.77×10^{44}	906	1540	14900
Solar Corona	1 A.U.	1.46×10^{16}	457	457	4420

[a]No extinction.

[b]See text in § 6.

6. BASIC REQUIREMENTS FOR TWO-PHOTON COINCIDENCE DETECTION

In order to detect the coincidence of a pair of photons arising from a particular two-photon emission it is required that both photons not interact with anything during their travel towards us and that, once they arrive on the receiver, we be able to identify the actual two-photon pair, as distinct from spurious complementary pairs of photons (those which do not arise from the same two-photon pair). There are in total four required elements in the the two-photon detection: (i) production of the two-photon pair, (ii) permanence of the pair of photons, (iii) reception of the two-photon pairs, and (iv) identification of the two-photon pair. In the previous sections we have discussed (i) and (iii). With respect to the permanence of the entanglement of the two-photon pair, we can consider two causes of annihilation of at least one photon from a traveling two-photon pair of photons are: (a) line absorption from the ISM and (b) absorption and scattering by dust by the ISM.

Regarding the line absorption from a continuous emission from $\lambda = 1216$ Å to $\lambda \to \infty$, we can safely conclude by looking into any UV, optical or IR spectrum that the line absorptions from the ISM will not be more than 10% of the total emission intensity. Therefore, it will be an irrelevant effect in the reception of the two-photon pairs. With respect to the absorption and scattering by dust in the ISM, the size of the interstellar grains is, in general, very small and will affect more the UV radiation than other, less energetic, ones. Without going into extreme detail as to the shape of the reddening function (i.e. assuming $A(\lambda) \propto \lambda^{-1}$), the extinction for a 2432 Å photon $A(2432) \approx 2A_V$, where $A_V = -2.5c\, f(V)$ where c is the reddening logarithmic correction, and $f(V)$ is the optical depth function. In order to detect a coincidence both photons are required to "survive" the dust. In a pair where each photon has approximately half the energy each one would face an extinction $A(\lambda) \approx 2A_V$; the probability of "survival" of both is equivalent to having a single photon going through twice the amount of dust, that is: the pair is facing the equivalent of $4A_V$. If we consider an entangled pair of photons with non-identical wavelengths we will find that the total extinction the pair has to face does not change much, remaining close to $4A_V$. Therefore, even a moderate extinction can become important and would increase the required size of the receiver by a factor of 10^c, because $f(V) \approx 1$ (notice that the factor of 4 that multiplies A_V, will cancel with the power of 4 associated with the receiver's diameter). Therefore. the photons that were produced from the Orion Nebula and PNe will be affected by a noticeable amount, while those from the solar corona are largely unaffected. Also, the required sizes for the receivers for the extra solar objects will need to be larger by at least a factor of 2 making the solar corona the best object to detect coincidences.

One great advantage in considering entangled photons is that they do not undergo decoherence and they can travel over free space for long distances keeping their entanglement as is mentioned in Fedrizzi et al. (2009).

We have discussed the production, permanence, and reception of the two-photon pair; what remains to be discussed is the identification of the actual pair. The problem of the identification of the entangled pair, although a difficult task due the huge number of arriving photons, could be solved by a similar set-up (on a larger scale) as shown in Fedrizzi et al. (2009), where they detect the entangled pair of photons that travel long distances in the same direction by using a system of beam splitters, polarizing beam splitters, polarization analyzers, and a charge coupled device (CCD), which allow to measure the coincidences between detectors in order to test the violation of CHSH Bell's inequality that indicates the detection of an entangled pair.

In our case we have the complementarity of frequencies of the photon pair which helps also to identify a genuine entangled pair; therefore we have three stages in the process of entanglement identification: (a) the pair of photons should arrive simultaneously (very narrow time uncertainty), (b) their frequencies should be complementary, which can be analyzed by a system of diffraction gratings (spectrographs), and finally (c) a combination of beam splitters, polarization analyzers, and a CCD should be used to test the violation of CHSH Bell's inequality.

7. CONCLUSIONS

The most feasible kind of quantum entanglement that can be detected in the Universe is the two-photon spontaneous transition of the hydrogen $2\,^2S_{1/2}$ metastable level; the pair of photons keep their entanglement during their travel unless at least one of them is measured or interacts with another atom or dust particle from the ISM.

We have obtained, by different methods, the two-photon emission rate $R_{\rm PP}$ from of four astrophysical objects: the Orion Nebula, two nearby PNe (IC 2149 and NGC 7293), and the solar corona. Their corresponding values are 5.80×10^{48}, 9.39×10^{45}, 9.77×10^{44}, and 1.46×10^{16} pairs of photons per second respectively. The Orion Nebula, even though it is at a distance of 414 pc, would be the best candidate to receive the two-photon coincidences due to the huge amount of photoionizations produced in the hydrogen gas; however, because of the presence of ISM the chances that at least one of the photons of the entangled pair interacts with a particle are higher than in the case of the solar corona for which there is practically no extinction; therefore the solar corona is the best object to detect the entangled pair of photons.

We calculated the the minimum sizes of the receiver diameters D for each of the two-photon coincidence detection astrophysical objects; for one coincidence per year and without extinction we obtained 272 km and 457 km for the Orion Nebula and the solar corona respectively and 906 and 5 200 km for NGC 7293 and IC 2149 respectively. When we include the ISM extinction the diameters are increased by a 10^c factor except for the solar corona due its closeness to the earth and to the low density of the ISM surrounding the earth. Such calculations demonstrate that the best astrophysical object to detect two-photon coincidences is the solar corona. Also the possibility exists of placing receivers closer to the Sun that can observe the object from different angles, which would remove the restriction we have that $\alpha \approx 0°$ for most other astronomical objects. Because the kind of radiation that we are mostly interested is in the UV region, we have considered space receivers. Although we have explored the possibilities of two-photon detection, we do not think that the proposed size level (about 457 km for the solar corona) could be realistically reached even in the foreseeable future.

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