Revista Mexicana de Astronomía y Astrofísica, **45**, 191–203 (2009)

PHYSICAL PARAMETERS OF SEVEN FIELD RR LYRAE STARS IN BOOTES[1]

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Received 2009 March 9; accepted 2009 July 27

RESUMEN

Se reporta fotometría $uvby-\beta$ para las estrellas tipo RR Lyrae AE, RS, ST, TV, TW, UU, y XX en Bootes. Se calculan los parámetros físicos $M/M_{\odot}$, $\log(L/L_{\odot})$, M_V, log $T_{\rm eff}$ y [Fe/H] a partir de la descomposición de Fourier de las curvas de luz y de calibraciones empíricas desarrolladas para este tipo de estrellas, y se discute la confiabilidad de estos valores. Los valores de [Fe/H] obtenidos se comparan con aquellos calculados a partir del índice ΔS para algunas estrellas de la muestra. Se encontró que el enrojecimiento de la zona es despreciable, comparándolo con el mostrado por diversos objetos en la misma región del cielo. Por tanto, se calcularon las distancias a estos objetos. La variación, a lo largo del ciclo de pulsación, de los índices fotométricos desenrojecidos $(b-y)_0$ y c_1 permite la comparación con mallas teóricas y por lo tanto la estimación independiente de log $T_{\rm eff}$ y log g.

ABSTRACT

Strömgren $uvby-\beta$ photometry is reported for the RR Lyrae stars AE, RS, ST, TV, TW, UU, and XX in Bootes. The physical parameters $M/M_{\odot}$, $\log(L/L_{\odot})$, M_V, log $T_{\rm eff}$ and [Fe/H], have been estimated from the Fourier decomposition of the light curves and the empirical calibrations developed for this type of stars. The obtained [Fe/H] values are compared with those calculated from the ΔS index for some sample stars. It was found that reddening in the zone is negligible compared to that shown by several objects in the same sky zone. From that, distance to the stars was calculated. The variation of the unreddened indexes $(b-y)_0$ and c_1 along the pulsational cycle allows the direct comparison with the theoretical grids and, hence, an independent determination of $T_{\rm eff}$ and log g.

Key Words: stars: variables: RR Lyrae — techniques: photometric

1. INTRODUCTION

It is known that the Horizontal Branch (HB) morphology in globular clusters of similar metallicities varies and the identification of a parameter, other than metallicity, responsible for this has originated much discussion in the astronomical literature. Recent findings that more massive clusters tend to have HBs extended further into higher temperatures led Recio-Blanco et al. (2006) to suggest the cluster's total mass may be the "second parameter". However since the HB morphology varies due to stellar evolution, age is still a strong candidate for the second parameter status (e.g., Stetson et al. 1999; Catelan 2000, and references therein).

RR Lyrae stars are outstanding stars in the HB whose absolute magnitudes and iron abundances have been carefully calibrated (Clementini et al. 2003; Layden 1994) and therefore play an important role in studies of the structure of the Galaxy. The estimation of individual physical parameters of RR Lyrae stars in globular clusters not only provides relevant insights into the distance and iron abundances of their parent clusters (e.g. Arellano Ferro et al.

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2008a,b), but also imposes constraints on the structure of the HB and on the stellar evolution in this phase (see e.g. Zinn 1993; van den Bergh 1993; Zinn 1996).

Determination of the physical parameters in RR Lyrae stars can be attained from the analysis of data obtained from fundamental techniques in astronomy such as photometry or spectroscopy. High dispersion spectroscopy produces accurate estimates for some of the atmospheric physical parameters, i.e., temperature, surface gravity and metallicity, that have been used to calibrate low resolution methods such as the ΔS method (Preston 1959). However this approach is limited mostly to brighter objects since the large amount of telescope time required to study fainter objects is very competitive and hence scarce. However, isolated examples of accurate spectroscopy of objects around $V \sim 16-17$ mag do exist (e.g. James et al. 2004; Gratton et al. 2005; Yong et al. 2005; Cohen & Meléndez 2005).

Alternative approaches to physical parameter estimations are theoretical and semi-empirical calibrations of photometric indices, such as the synthetic color grids from atmospheric models (e.g. Lester, Gray, & Kurucz 1986) or, for RR Lyrae stars, the decomposition of light curves in Fourier harmonics and the calibration of the Fourier parameters in terms of key physical parameters (e.g. Simon & Clement 1993; Kovács 1998; Jurcsik 1998; Morgan, Wahl, & Wieckhorst 2007). Kovács and his collaborators, in an extensive series of papers (e.g. Kovács & Jurcsik 1996, 1997; Jurcsik & Kovács 1996; Kovács & Walker 2001), have developed purely empirical relations from the Fourier analysis of the light curves. The basic stellar parameters are based on the assumption that the period and the shape of the light curve are directly correlated with the physical parameters (or quantities related to them) such as the iron abundance [Fe/H], absolute magnitude M_V and effective temperature $T_{\rm eff}$.

In this paper we shall use the Strömgren photometry of seven RR Lyrae stars in Bootes to estimate their physical parameters. Furthermore, since we have obtained simultaneous $uvby-\beta$ photoelectric photometry, the precise variation of the stellar brightness and colors along the cycle of pulsation for each star can also be used to provide rigid constraints on the physical parameters.

2. OBSERVATIONAL MATERIAL AND REDUCTIONS

Some of the stars included in the present work were part of the observing program in previous studies as secondary targets (Lampens et al. 2005; Peña et al. 2003). Other seasons were devoted entirely to some of these RR Lyrae stars. The log of the various seasons is given in Table 1. The observations were obtained with the 1.5 m telescope at the Observatorio Astronómico Nacional of San Pedro Mártir, Mexico. The telescope was equipped with a six channel spectrophotometer described in detail by Schuster & Nissen (1988). All the data reductions were performed using the NABAHOT package (Arellano Ferro & Parrao 1989).

TABLE 1

LOG OF THE OBSERVING SEASONS

Star	Initial date	Final date
AE Boo	Jun 08, 2001	Jun 11, 2001
TV Boo	Feb 04, 2002	Feb 11, 2002
RS Boo & TV Boo	Apr 08, 2004	Apr 11, 2004
Full sample	May 27, 2005	Jun 16, 2005

In those seasons in which the present RR Lyrae stars were supplementary objects, fewer data points were obtained each night. The accumulated time span including all the seasons, however, was large enough to cover the whole cycle of all the stars in our sample. In each season the same observing routine was employed: a multiple series of integrations was carried out, often five 10 s integrations of the star, to the average of which a one 10 s integration of the sky was subtracted. A series of standard stars taken from the Perry, Olsen, & Crawford (1987) and Olsen (1983) was also observed with the same procedure on each night to transform the data into the standard system. The seasonal transformation coefficients, except for the 2002 season, are reported in Table 2. The indicated coefficients are those in the following equations (Gronbech, Olsen, & Strömgren 1976).

$$V = A + B\ (b-y)_{\rm st} + y_{\rm inst} \quad (1)$$

$$(b-y)_{\rm st} = C + D\ (b-y)_{\rm inst} \quad (2)$$

$$m_{1\ \rm st} = E + F m_{1\ \rm inst} + G\ (b-y)_{\rm inst} \quad (3)$$

$$c_{1\ \rm st} = H + I m_{1\ \rm inst} + J\ (b-y)_{\rm inst} \quad (4)$$

$$\beta_{1\ \rm st} = K + L\ \beta_{1\ \rm inst}\,. \quad (5)$$

For the 2002 season the transformation was made by filter instead of by color indices. In this season the relations between the standard and the instrumental values for each filter were the following:

$$V = y_{\rm st} = a + b\ y_{\rm inst} \quad (6)$$

$$b_{\rm st} = c + d\ b_{\rm inst} \quad (7)$$

TABLE 2

TRANSFORMATION COEFFICIENTS OF EACH SEASON

Season	A	B	C	D	E	F	G	H	I	J	K	L
Jun, 01	18.076	+0.020	1.354	0.991	−1.437	1.012	−0.002	−0.623	1.001	0.148	···	···
Apr, 04	18.839	−0.010	1.539	0.993	−1.231	0.949	0.024	−0.429	0.988	0.068	···	···
May, 05	19.000	−0.007	1.531	0.983	−1.309	1.068	0.020	−0.315	1.031	0.153	2.626	−1.348
	a	b	c	d	e	f	g	h				
Feb, 02	19.1565	1.0054	20.9731	1.0236	21.0131	1.0101	20.5204	1.0025				

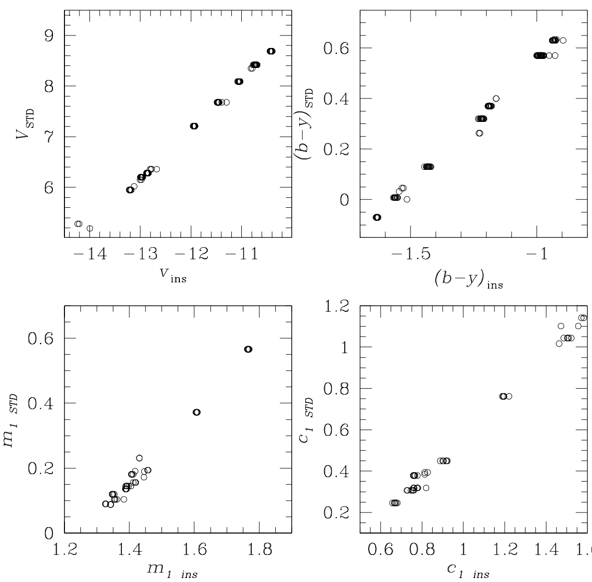

Fig. 1. Transformations from the instrumental to the standard *uvby* system for the magnitude and colors. See equations (1–5).

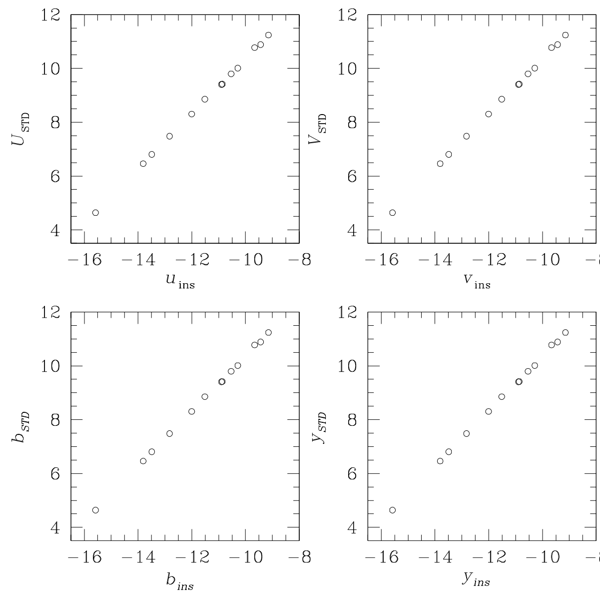

Fig. 2. Transformations from the instrumental *uvby* magnitudes to the standard system. See equations (6–9).

$$v_{\rm st} = e + f\, v_{\rm inst} \quad (8)$$
$$u_{\rm st} = g + h\, u_{\rm inst}\,. \quad (9)$$

The coefficients are reported in the bottom part of Table 2. The mean dispersions of the transformation equations are 0.028, 0.016, 0.015, 0.010 mag in u, v, b and y respectively. It was after this that the color indices were calculated. In Figure 1 the instrumental y magnitude and color transformations are shown. In Figure 2 the relationships between the instrumental and the standard magnitudes in the 2002 season are shown.

The final $uvby-\beta$ photometry is available in electronic form from the Centre de Données Astronomiques, Strasbourg, France. The V light curves are shown in Figure 3. In the following section a brief description of the light and color curves for each star is given.

3. NOTES ON INDIVIDUAL STARS

Due to the fact that in most of the seasons the RR Lyrae stars were not the primary target, the sampling of these stars was not homogeneous. For example, AE Boo was observed for four nights in 2001 for a relatively short time on each night, although the phase coverage was almost complete. In the 2005 season, only a couple of points per night were obtained for most RR Lyrae stars although the time span of the observations covered seventeen nights. For AE Boo around 180 points in V were gathered.

The other amply observed star was TV Boo which was measured in 2002 (250 data points), in 2004 (5 points) and in 2005 (13 points). According to Wils, Lloyd, & Bernhard (2006) TV Boo shows a Blazkho period of 10 d. Given the time span of our observations on this star and the uncertainties

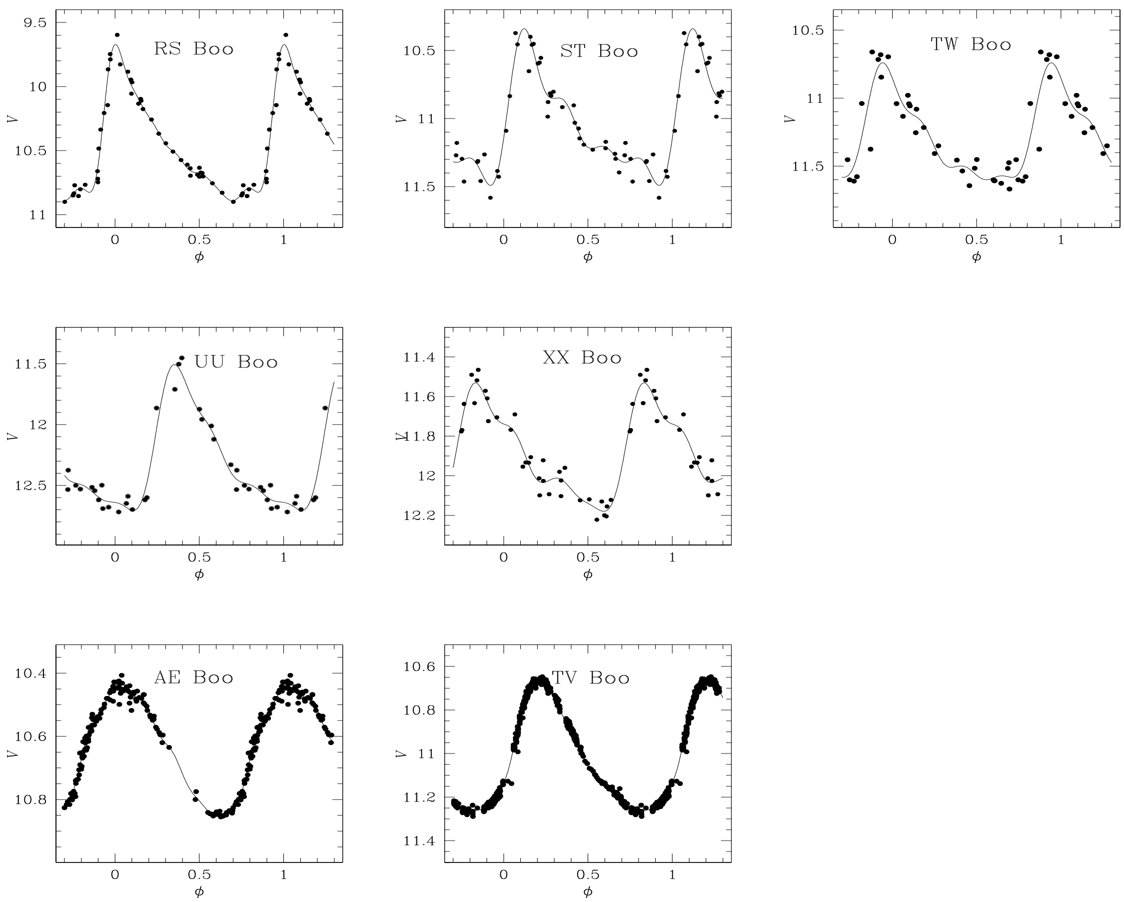

Fig. 3. V light curves of the sample stars.

in the V magnitude (0.03 mag) we can neither verify nor contradict this assertion. We call attention to the fact that the time of maximum light of this star does not coincide with a phase value of zero implying a secular variation of the period. We did not pursue this possibility.

The last star which was observed in more than one season was RS Boo. The data cover the full cycle and the maximum light coincides with phase zero. Of the remaining stars, ST Boo, TW Boo, UU Boo and XX Boo, all observed in the 2005 season at the rate of a couple of points per night and for seventeen nights, all show a maximum light shifted relative to phase zero at least with the light elements reported in Table 3. For each of these stars a sample of around thirty-five data points was gathered.

4. IRON ABUNDANCE ESTIMATION

4.1. [Fe/H] *from the Fourier light curve decomposition*

As a first approach, the V light curves were decomposed into their harmonics and the Fourier coefficients used to estimate the iron abundance via semiempirical calibrations. In order to do that, the light curves were phased with periods and the epochs listed in Table 3, which were adopted from Kholopov et al. (1985).

Given the ephemerides, the light curves in Figure 3 were fitted by the equation:

$$m(t) = A_0 + \sum_{k=1}^{N} A_k \cos(\frac{2\pi}{P} k(t-E) + \phi_k) , \quad (10)$$

where k corresponds to the $k-th$ harmonic of amplitude A_k and displacement ϕ_k.

The Fourier coefficients, defined as:

$$\begin{aligned} \phi_{ij} &= j\phi_i - i\phi_j , \\ R_{ij} &= A_i/A_j , \end{aligned}$$

have been calibrated in terms of physical parameters. The Fourier coefficients corresponding to the solid curve in Figure 3 and the number of harmonics used to produce the best possible fit, are given

TABLE 3

EPHEMERIDES AND FOURIER COEFFICIENTS FOR THE SAMPLE STARS

ID	HJD 2400000.0+	P (days)	A_0 σ_{A_0}	A_1 σ_{A_1}	A_2 σ_{A_2}	A_3 σ_{A_3}	A_4 σ_{A_4}	ϕ_{21} $\sigma_{\phi_{21}}$	ϕ_{31} $\sigma_{\phi_{31}}$	ϕ_{41} $\sigma_{\phi_{41}}$	N	Type
RS Boo	41770.4900	0.37733896	10.470	0.447	0.207	0.105	0.084	4.088	1.881	5.858	8	ab
			0.014	0.019	0.018	0.020	0.018	0.129	0.215	0.287		
ST Boo	19181.4860	0.62229069	11.044	0.383	0.194	0.133	0.097	3.979	2.129	6.112	4	ab
			0.017	0.023	0.024	0.024	0.024	0.177	0.268	0.358		
TW Boo	26891.2680	0.53227315	11.309	0.359	0.138	0.069	0.058	3.782	1.827	5.438	4	ab
			0.027	0.037	0.039	0.035	0.039	0.359	0.703	0.772		
UU Boo	36084.4100	0.45692050	12.251	0.504	0.203	0.086	0.055	3.905	2.074	6.078	4	ab
			0.022	0.030	0.032	0.035	0.035	0.212	0.431	0.652		
XX Boo	29366.6460	0.58140160	11.914	0.257	0.114	0.051	0.039	4.087	2.726	6.189	4	ab
			0.012	0.018	0.015	0.017	0.019	0.201	0.387	0.476		
AE Boo	30388.2500	0.31489240	10.646	0.202	0.027	0.008	0.008	4.903	3.722	2.377	6	c
			0.004	0.006	0.006	0.006	0.006	0.223	0.730	0.670		
TV Boo	24609.5150	0.31255936	11.020	0.282	0.078	0.020	0.015	3.915	1.666	6.043	7	c
			0.001	0.001	0.001	0.001	0.001	0.017	0.051	0.074		

in Table 3. The number of significant harmonics depends on the dispersion of the light curve. Only significant harmonics were retained; their influence on [Fe/H] estimated from the Fourier decomposition will be discussed at the end of this section.

For the RRab stars, [Fe/H] was estimated using the calibration of Jurcsik & Kovács (1996);

$$[\mathrm{Fe/H}]_{\mathrm{J}} = -5.038 \; - \; 5.394 \; P \; + \; 1.345 \; \phi_{31}^{(s)} \,, \quad (11)$$

The standard deviation in the above equation is 0.14 dex. In equation (11), the phase $\phi_{31}^{(s)}$ is calculated from a sine series. To convert the cosine series based $\phi_{jk}^{(c)}$ into the sine series $\phi_{jk}^{(s)}$, one can use $\phi_{jk}^{(s)} = \phi_{jk}^{(c)} - (j-k)\frac{\pi}{2}$. The metallicity $[\mathrm{Fe/H}]_{\mathrm{J}}$ from equation (11) can be converted to the metallicity scale of Zinn & West (1984) (ZW) via $[\mathrm{Fe/H}]_{\mathrm{J}} = 1.43\,[\mathrm{Fe/H}]_{\mathrm{ZW}} + 0.88$ (Jurcsik 1995). The values of $[\mathrm{Fe/H}]_{\mathrm{ZW}}$ are given in Table 4.

Before applying equation (11) to the five RRab stars in our sample, we calculated the *compatibility condition parameter* D_m which, according to Jurcsik & Kovács (1996) and Kovács & Kanbur (1998), should be smaller that 3.0. The values of D_m for the five RRab stars are given in Table 4. It is worth commenting that the D_m parameter calculated for the RRab stars does not seem to correspond to the quality and/or density of the light curve. For instance, from Figure 3 the curve of RS Boo is clearly the best and that of ST Boo is among the worst. However, the corresponding D_m values are 4.1 and 1.2 respectively, which is contrary to what would be expected. Thus we decided to relax this criterion somewhat and apply the decomposition to the five RRab stars in our sample. We shall discuss the uncertainties in [Fe/H] due to the number of harmonics used to fit the data later in this section.

TABLE 4

$[\mathrm{Fe/H}]_{\mathrm{ZW}}$ FOR THE SAMPLE OF RR LYRAES

ID	$[\mathrm{Fe/H}]_{\mathrm{ZW}}$	D_m	Type
RS Boo	−0.84	4.1	ab
ST Boo	−1.53	1.2	ab
TW Boo	−1.47	1.9	ab
UU Boo	−0.95	5.9	ab
XX Boo	−0.81	5.1	ab
AE Boo	−1.30		c
TV Boo	−2.04		c

For the RRc stars we have calculated [Fe/H] using the recent calibration of Morgan et al. (2007), which provides [Fe/H] in the ZW scale;

$$[\mathrm{Fe/H}]_{\mathrm{ZW}} = 52.466P^2 - 30.075P + 0.131(\phi_{31}^{(c)})^2 + 0.982\phi_{31}^{(c)} - 4.198\phi_{31}^{(c)}P + 2.424\,. \quad (12)$$

The results are listed in Table 4.

Equation 11 is calibrated using the compilation of ΔS values of Suntzeff, Kraft, & Kinman (1994)

TABLE 5

ΔS VALUES FOR THE SAMPLE STARS

ID	Type	Spectra	ΔS^1	ΔS^2	ΔS^3	Px (mas)
RS Boo	ab	A7-F5	1.36[a,b]			0.11 ± 1.40
ST Boo	ab	A7-F7		8.9[a]		1.19 ± 1.61
TW Boo	ab	F0-F8	2.3	6.2,8.3[b]		-0.28 ± 1.63
UU Boo	ab					
XX Boo	ab			7.0[c],10.1[c]		
AE Boo	c	F2				0.32 ± 2.00
TV Boo	c	A7-F2	11.6[a,b]		12.1	-0.07 ± 1.60

[1]Suntzeff et al. (1994). [2]Smith (1990). [3]Liu & Janes (1990). [a]Average of multiple measurements. [b]Values from near minimum light. [c]Unknown phase.

transformed into [Fe/H] and the spectroscopic values $[Fe/H]_K$ of Layden (1994). On the other hand, equation (12) is calibrated using the [Fe/H] values of Zinn & West (1984) and Zinn (1985) which are the weighted average of iron abundances obtained from an assortment of methods, including the ΔS and the Q_{39} indices. Therefore, the values of [Fe/H] derived from the Fourier approach for both RRab and RRc stars are not completely independent from the values of [Fe/H] estimated from ΔS. However, a discussion of ΔS and [Fe/H] values found in the literature for the stars in our sample is of interest and we present it in the following section.

Perhaps the most relevant source of uncertainty in the physical parameters estimated from the Fourier decomposition approach is the quality and density of the light curve and hence its mathematical representation, i.e., the number of harmonics needed to fit the observed data. For the four RRab stars ST Boo, TV Boo, UU Boo and XX Boo, whose light curves are scattered and/or not very dense, the number of significant harmonics 3 or 4 is rather undistinguishable. It can be noted that the values of $T_{\rm eff}$ and M_V vary 25–90 K and 0.01–0.07 mag. respectively, while the most striking variation takes place in [Fe/H] where variations on average were ± 0.19 dex. This value is marginally larger than the declared mean standard deviations of equations (11) and (12) (0.14 mag.) Thus we estimate an uncertainty in the values of $[Fe/H]_{ZW}$ of about ± 0.17 dex. We have retained the cases with the smaller standard deviation from the fit and exclusively with significant harmonics in Table 3.

4.2. [Fe/H] *from the* ΔS *parameter*

It is a common practice to estimate the value of [Fe/H] for RR Lyrae stars using the ΔS metallicity parameter, Preston's (1959) method. In an extensive study of the ΔS parameter on RR Lyrae variables in the Galactic halo, Suntzeff et al. (1994) provided average values of ΔS with errors of 0.3 units, which correspond to 0.05 dex in [Fe/H]. They reported ΔS values for three stars of the present study: RS Boo, TW Boo and TV Bootes with values 1.36, 2.3 and 11.6. For RS Boo and TV Bootes they give numerous estimations of ΔS, but since they correspond to similar phases, we have taken the averages and marked them with asterisks in Table 5. ΔS values for ST Boo, TW Boo and XX Boo are found in the paper by Smith (1990), the average or individual values are given in Table 5. We note here the discrepant values for TW Boo from Suntzeff et al. (1994) and Smith (1990) despite their corresponding to similar phases of 0.12 and 0.17 respectively. Also, two discrepant values for XX Boo but for very different phases are found in Smith (1990). A value for TV Boo is given by Liu & Janes (1990) as 12.1 which was measured near maximum light. A summary of the ΔS values for our sample stars is given in Table 5.

While converting ΔS into [Fe/H] is a current practice, one should bear in mind that the determination of ΔS is subject to numerous sources of uncertainty, as have been discussed for example by Butler (1975), Smith (1986, 1990) and Suntzeff et al. (1994), e.g. ΔS estimated during the rising light can differ greatly from that estimated during the declining brightness. The correction to minimum phase is done using empirical curves on the $\Delta S - SpT(H)$-plane with considerable dispersion and personal judgment in their final definition (Smith 1986, 1990), and the finally reported weighted mean value of ΔS is often obtained from a few measurements randomly distributed along the pulsation cycle of the

TABLE 6

PHYSICAL PARAMETERS FROM THE CALIBRATIONS FOR THE RR LYRAE STARS STUDIED

ID	$[Fe/H]_{ZW}$	$[Fe/H]_{\Delta S}$	$[Fe/H]_K$	T_{eff}	M_V	BC	$\log L/L_\odot$	d (pc)	N
RS Boo	-0.84	-0.81	-0.32 ± 0.09	6905	0.79	0.000	1.58	862	8
ST Boo	-1.53	-1.82	-1.86 ± 0.14	6454	0.48	-0.050	1.73	1295	4
TW Boo	-1.47	$-0.90, -1.46, -1.74$	-1.41 ± 0.09	6581	0.58	-0.050	1.69	1396	4
UU Boo	-0.95		-1.92 ± 0.20	6818	0.56	-0.010	1.68	2284	4
XX Boo	-0.81	$-1.56, -1.98$		6675	0.62	-0.030	1.67	1818	4
AE Boo	-1.30			7384	0.58	0.026	1.74	1032	6
TV Boo	-2.04	-2.18		7199	0.59	0.026	1.75	1218	7

star. Some of the above considerations may explain the large differences in the existing values of ΔS for a given star (see Table 5).

Beside the above sources of uncertainty in ΔS, we have to note that the transformation of this observational parameter into [Fe/H] encounters serious problems: the high dispersion abundance determinations in RR Lyrae are old (e.g. Butler 1975; Butler & Deeming 1979; Butler et al. 1982) while modern high signal-to-noise, high dispersion digital data, analyzed with modern synthetic codes are non-existent. According to Manduca (1981), conversion of ΔS to [Fe/H] has calibration problems for the metal-rich and metal-poor domains and in fact his theoretical calibration is not linear. We have chosen however, to use the more recent empirical linear transformation of Jurcsik (1995); $[Fe/H]_J = -0.190\,\Delta S - 0.027$, which is valid for $[Fe/H]_J$ between 0.0 and -2.3 dex and for field and cluster RR Lyraes, to transform the ΔS parameter into iron abundances on the ZW scale. We have first used the ΔS values in Table 5 and the formula of Jurcsik (1995), to calculate $[Fe/H]_J$, and then brought this into the ZW scale as discussed in § 4.1; we have labeled these iron values as $[Fe/H]_{\Delta S}$ and they are listed in Table 6.

For the sake of comparison, in Columns 2–4 of Table 6 we have listed the values of the Fourier estimations of the metallicity, $[Fe/H]_{ZW}$, the values from ΔS, $[Fe/H]_{\Delta S}$ and the independent estimations from Layden (1994) $[Fe/H]_K$. $[Fe/H]_K$ values are based on the strength of the Ca II K line and are in the ZW scale.

For RS Boo $[Fe/H]_K$ is rather discrepant but the agreement between $[Fe/H]_{ZW}$ and $[Fe/H]_{\Delta S}$ is excellent. For ST Boo and TW Boo the agreement between the three estimations is very good especially considering the implicit uncertainties in the values of ΔS. Large discrepancies are noted for XX Boo and TV Boo. In the case of XX Boo one may argue that the coverage of the light curve can be considerably improved and hence the Fourier value could be expected to improve. However, in the case of TV Boo, whose light curve is well defined and very dense, and whose ΔS values are not scattered, we do not have an explanation handy. Nevertheless, as an overall comparison of approaches to the determination of the iron abundance in RR Lyrae stars, and considering the uncertainties involved in the estimation of the ΔS parameter discussed above and the fact that in the Fourier solution the full shape of the light curve is included, we consider the Fourier decomposition a more solid approach.

4.3. *The effective temperature* T_{eff}

The effective temperature can also be estimated from the Fourier coefficients. For the RRab stars we used the calibrations of Jurcsik (1998)

$$\log T_{eff} = 3.9291 - 0.1112\,(V-K)_o - 0.0032\,[Fe/H]\,, \quad (13)$$

with

$$(V-K)_o = 1.585 + 1.257\,P - 0.273\,A_1 - 0.234\,\phi_{31}^{(s)} + 0.062\,\phi_{41}^{(s)}\,. \quad (14)$$

Equation (13) has a standard deviation of 0.0018 (Jurcsik 1998), but the accuracy of $\log T_{eff}$ is mostly set by the color equation (14). The error estimate on $\log T_{eff}$ is 0.003 (Jurcsik 1998).

For the RRc stars we have used the calibration of Simon & Clement (1993);

$$\log T_{eff} = 3.7746 - 0.1452 \log P + 0.0056\,\phi_{31}^{(c)}\,. \quad (15)$$

The values obtained from the above calibrations are reported in Column 5 of Table 6.

TABLE 7

$uvby - \beta$ PHOTOELECTRIC PHOTOMETRY AND REDDENING OF δ SCUTI STARS IN BOOTES

Star	V	$b-y$	m_1	c_1	β	$E(b-y)$	$uvby-\beta$ source
ι Boo	4.75	0.128	0.198	0.834	2.817	0.001	1
γ Boo	3.03	0.116	0.191	1.008	2.817	0.006	1
κ^2Boo	4.54	0.125	0.187	0.951	2.806	0.001	1
CN Boo	5.98	0.162	0.201	0.748	2.770	0.000	1
YZ Boo	10.57	0.151	0.178	0.868	2.768	0.000	1
YZ Boo	10.466	0.184	0.136	0.681	2.723	0.000	2

[1]Rodríguez et al. (1994).

[2]Peña et al. (1999).

5. DISTANCE TO THE FIELD RR LYRAE

5.1. M_V from the Fourier parameters

Absolute magnitudes of RR Lyrae can also be estimated from the Fourier parameters. For the RRab one can use the calibration of Kovács & Walker (2001);

$$M_V(K) = -1.876 \log P - 1.158 A_1 + 0.821 A_3 + K \,. \tag{16}$$

The standard deviations in the above equation is 0.04 mag. The zero point of equation (16), K=0.43, has been calculated by Kinman (2002) using the prototype star RR Lyrae as calibrator, adopting the absolute magnitude $M_V = 0.61 \pm 0.10$ mag for RR Lyrae, as derived by Benedict et al. (2002) using the star parallax measured by the HST. Kinman (2002) finds his result to be consistent with the coefficients of the M_V-[Fe/H] relationship given by Chaboyer (1999) and Cacciari & Clementini (2003). All these results are consistent with the distance modulus of the LMC of 18.5 ± 0.1 (Freedman et al. 2001; van den Marel et al. 2002; Clementini et al. 2003). Catelan & Cortés (2008) have argued that the prototype RR Lyrae has an overluminosity due to evolution of 0.064 ± 0.013 mag relative to HB RR Lyrae stars of similar metallicity. This would have to be taken into account if RR Lyrae is used as a calibrator of the constant K in equation (11). Considering this, Arellano Ferro et al. (2008a) have estimated a new value of $K = 0.487$.

For the sake of homogeneity and better comparison with previous results on luminosities of RR Lyrae stars (e.g. Arellano Ferro et al. 2008a,b), in the following we have adopted $K = 0.43$.

For the RRc stars, the calibration of Kovács (1998) was used;

$$M_V(K) = -0.961\, P - 0.044\, \phi_{21}^{(s)} + 4.447\, A_4 + 1.261 \,. \tag{17}$$

The standard deviation in the above equation is 0.042 mag. In fact, we have propagated the errors, given in Table 3, in the amplitudes A_1 and A_3 in equation (16) and in $\phi_{21}^{(s)}$ and $\phi_{41}^{(s)}$ in equation (17) and found that they produce uncertainties in $M_V \sim 0.04$ mag. Cacciari, Corwin, & Carney (2005) have pointed out that in order for the equation (17) to deliver absolute magnitudes in agreement with the mean magnitude for the RR Lyrae stars in the LMC, $V_0 = 19.064 \pm 0.064$ (Clementini et al. 2003), the zero point of the above equation should be decreased by 0.2 ± 0.02 mag. After this correction, we found the M_V values for the RRc stars AE Boo and TV Boo reported in Table 6. These values of M_V have been converted into $\log L/L_\odot$ (Column 5). The bolometric corrections for the average temperatures of RRab and RRc stars given in Column 7 of Table 6, were estimated from the $T_{\rm eff}$-BC$_V$ from the models of Castelli (1999) as tabulated in Table 4 of Cacciari et al. (2005).

5.2. Reddening

In order to determine distance and detailed variations of the physical parameters along the cycle of pulsation of each star, it is necessary to first estimate the reddening. For field stars however, a proper determination of reddening is complex and no direct method is known to us. In view of this, we have determined the reddening of different objects in the same direction of the sky. We have chosen five δ Scuti stars and the globular cluster NGC 5466 in Bootes and M3 which is very near NGC 5466, as indicators of reddening in that direction of the sky. Despite the large distance to M3 and NGC 5466 (10.4 and 15.9 kpc respectively), $E(b-y) \sim 0.0$ as estimated from

TABLE 8

COMPARISON OF $M_V(F)$ OBTAINED FROM THE FOURIER APPROACH WITH TWO THEORETICAL CALIBRATIONS FROM CORTÉS & CATELAN (2008)

Star	[Fe/H]$_{\rm ZW}$	(c_1)	$M_y(c1)^1$	$M_y(Z)^2$	$M_V(F)^3$	$M_V(F)$-$M_y(c1)$
RS Boo	−0.84	0.928	0.884	0.765	0.79	−0.093
		±0.012	±0.016	±0.063		
ST Boo	−1.53	0.922	0.319	0.526	0.48	+0.163
		±0.010	±0.013	±0.030		
TW Boo	−1.47	0.893	0.537	0.577	0.58	+0.045
		±0.017	±0.022	±0.039		
UU Boo	−0.95	0.941	0.659	0.665	0.56	−0.098
		±0.017	±0.023	±0.051		
XX Boo	−0.81	0.814	0.678	0.776	0.62	−0.057
		±0.011	±0.018	±0.063		
AE Boo	−1.30	1.002	0.880	0.619	0.58	−0.298
		±0.003	±0.004	±0.044		
TV Boo	−2.04	1.133	0.625	0.481	0.59	−0.034
		±0.006	±0.006	±0.018		

[1]From C_0, Z and P through equation (1) of Cortés & Catelan (2008).

[2]From Z through equation (5) of Cortés & Catelan (2008).

[3]From the Fourier decomposition approach in this work, Table 6.

the $E(B-V)$ values listed by Harris (1996). In the case of the δ Scuti stars we have used the expressions derived by Crawford (1975, 1979) for F- and A-type stars respectively, with the zero point correction suggested by Nissen (1988), to estimate the intrinsic color $(b-y)_0$ for stars near the main sequence. Two sources of $uvby-\beta$ were considered; Rodríguez et al. (1994) and Peña, González, & Peniche (1999). These δ Scuti stars, their magnitude-weighted mean colors and $E(b-y)$ are listed in Table 7. From these results it seems reasonable to conclude that the reddening for the sample RR Lyrae stars in Bootes is negligible and we shall assume a value of zero for all the stars in our sample.

5.3. *Distances*

The implied distances given by the Fourier absolute magnitudes, and the reddenings and bolometric corrections discussed in § 5.1 and 5.2 are listed in Column 9 of Table 6. These distances can be compared with estimations from the parallaxes, for those stars which have parallaxes determined by the new reductions of the Hipparcos data by van Leeuwen (2007). For the stars RS Boo, ST Boo, TW Boo, AE Boo and TV Boo the parallaxes and their errors are listed in Table 5. It should be noted that their errors are very large and hence the parallaxes for these stars very uncertain. Except for ST Boo, those numerical values lead to distances much larger than the values derived from the Fourier approach in Table 6, and if these distances are used to calculate the corresponding absolute magnitude they lead to absurd results.

6. M_V FROM THE STRÖMGREN c_1 INDEX

The Strömgren c_1 index is a gravity (hence luminosity) sensitive parameter for late-A to F type stars (Strömgren 1966), and therefore it is thought to be useful in determining the absolute magnitude M_V for RR Lyrae stars if properly calibrated. In fact, in a recent paper Cortés & Catelan (2008) have offered such calibration for RR Lyrae stars. These authors have employed the pseudocolour $c_0 \equiv (u-v)_0 - (v-b)_0$, the fundamental period P and the metallicity Z for a large number of synthetic RR Lyrae stars to calibrate M_y and the colors $b-y$, $v-u$ and $u-v$ (their equation 1).

We have fitted the c_1 curves with a curve of the form of equation (10) to estimate the magnitude-weighted means (c_1) given in Column 3 of Table 8. These magnitude-weighted means differ from the intensity weighted means $<c_1>$ by about 0.01 mag. (Catelan & Cortés 2008), which are smaller than the uncertainties for c_1 of our observations as derived from the standard stars (see Table 3 in Peña et al. 2007). Since $E(b-y)=0$ for the sample stars (§ 5.2)

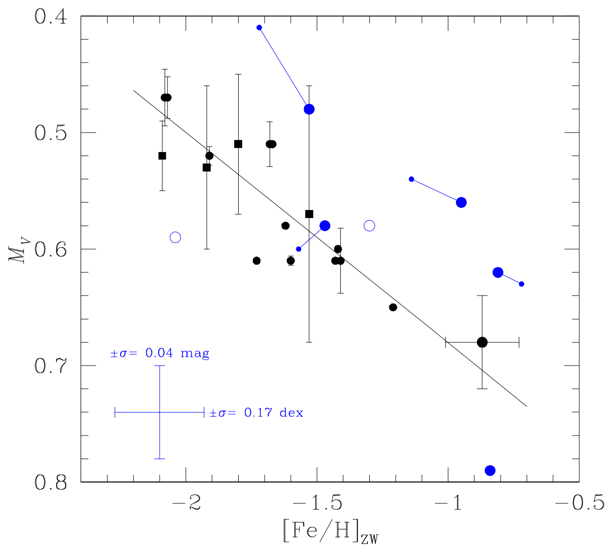

Fig. 4. M_v − [Fe/H] relation derived from cluster RR Lyrae stars (Arellano Ferro et al. 2008a). Blue points are the five RRab stars in our sample. When available, two positions are given; the larger points are the finally adopted solution while the smaller points correspond to similar solutions with smaller number of harmonics. Open blue circles correspond to the two RRc stars in our sample. The uncertainties in the values of the blue symbols are discussed in the text and indicated by the blue error bars. See electronic version for the color coding.

we have taken $c_0 = c_1$. For the conversion of [Fe/H] to Z we have used log Z = [M/H]−1.765; M/H = [Fe/H] + log (0.638 f + 0.362) and $f = 10^{[\alpha/\mathrm{Fe}]}$ (Salaris, Chieffi, & Straniero 1993). We adopted $\alpha = 0.31$ as an appropriate value for halo population stars. The predicted values of the absolute magnitude from c_1, $M_y(c1)$, are given in Column 4 of Table 8.

In their equation (5) Cortés & Catelan (2008) have calculated a quadratic calibration of the M_y–log Z relationship which can be used to calculate $M_y(Z)$ given in Column 5 of Table 8. As argued by Cortés & Catelan (2008) $M_y(c1)$ refers to the magnitude of an individual star whereas $M_y(Z)$ is the absolute magnitude of a star of similar metallicity. Thus we shall compare the Fourier approach results of each star, $M_V(F)$, with their corresponding $M_y(c1)$. This comparison is made in Column 7. The uncertainties of $M_y(c1)$ are the result of propagating the uncertainties in $<c_1>$ throught the equation (1) of Cortés & Catelan (2008). The uncertainties $M_y(Z)$ are estimated by propagating the uncertinty in $\sigma_{[\mathrm{Fe/H}]} = 0.17$ dex (§ 4.1). One can see that the dispersion in the calibration of equation (1) in Cortés & Catelan (2008) is ≤ 0.01 mag (their Figure 3) and we recall that the uncertainties in $M_V(F)$ is ±0.04 mag (§ 5.1). Therefore the differences in Column 7 seem to be somewhat on the large side. It should be noted, however, that no systematics can be seen and that if the $M_y(c1)$ values are plotted in Figure 4 the dispersion of the field stars about the cluster M_V − [Fe/H] relationship becomes very large. Also, it can be seen that the values of $M_y(c1)$ for the two RRc stars, AE Boo and TV Boo are the most discordant despite of having the most densely covered light curves and hence the smallest uncertainties.

7. M_V − [Fe/H] RELATION

A recent linear version of the M_V − [Fe/H] relationship for RR Lyrae stars has been calculated by Arellano Ferro et al. (2008b) based on the light curve decomposition technique of RR Lyrae stars in a group of globular clusters with a large range of metallicities. This relationship, reproduced in Figure 4, has been amply discussed by Arellano Ferro et al. (2008a) who found it to be consistent with independent empirical linear versions and with theoretical non-linear versions after evolution from the Red Giant Branch is taken into account. To check the consistency of our present results for the field stars in Bootes with those of RR Lyrae stars in globular clusters, we have plotted in Figure 4 the seven stars in our sample using the Fourier $[\mathrm{Fe/H}]_{\mathrm{ZW}}$ and M_V reported in Table 6. It can be seen that with some larger scatter the Bootes stars distribution, given the uncertainties, follow the trend of the globular cluster RR Lyrae. The error bars correspond to the uncertainties in the Fourier calibrations of $[\mathrm{Fe/H}]_{\mathrm{ZW}}$ and M_V, i.e., those of equations (11), (12), (16) and (17).

8. DETERMINATION OF PHYSICAL PARAMETERS ALONG THE PULSATIONAL CYCLE

Given the simultaneity in the acquisition of the data in the different color indices, once the reddening has been inferred, it is possible to determine the variation of the physical parameters of the star along the cycle. This can be accomplished with the models developed particularly for $uvby-\beta$ photometry by Lester, Gray, & Kurucz (1986, hereafter LGK86). The models have been built taking into account that the $uvby-\beta$ system is well designed to measure key spectral signatures that can be used to determine basic stellar parameters. The theoretical calibrations have the advantage of relating the photometric indices to the effective temperature, surface gravity,

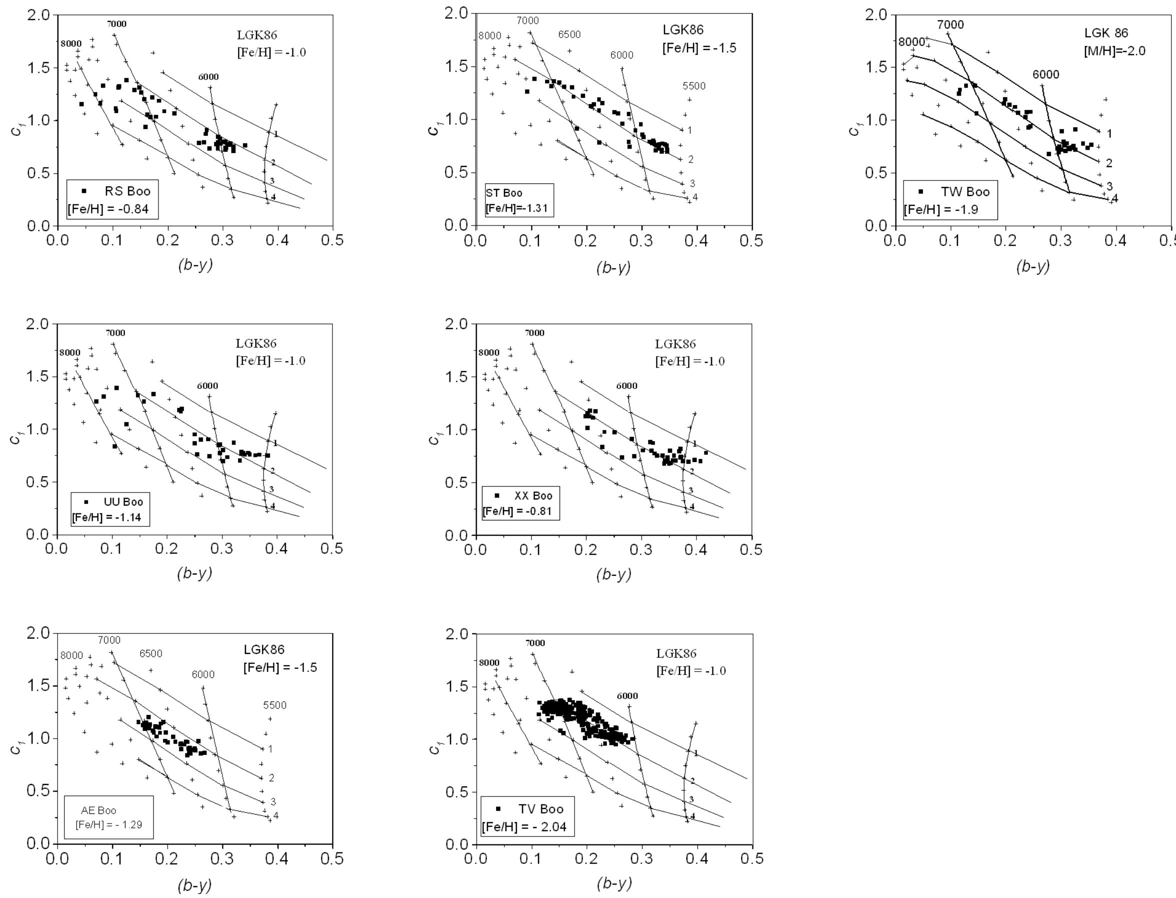

Fig. 5. Model grids on the $(b-y)_0 - c_0$ plane along the stellar variations through the cycle.

TABLE 9

$T_{\rm eff}$ AND log g VARIATION RANGES FROM THE THEORETICAL GRIDS OF LGK86

Star	[Fe/H] adopted	$T_{\rm eff}$ Fourier	$T_{\rm eff}$ min	$T_{\rm eff}$ max	$\Delta T_{\rm eff}$ K	log g min	log g max	Δ log g dex	ΔV mag
RS Boo	−1.0	6569	5700	8000	2300	2.2	3.5	1.2	0.82
ST Boo	−1.5	6141	5700	7500	1800	1.5	3.0	1.5	0.73
TW Boo	−2.0	6262	5500	7000	1500	1.5	3.0	1.5	0.85
UU Boo	−1.0	6486	5500	8000	2500	1.5	3.0	1.5	0.63
XX Boo	−1.0	6350	5000	6800	1300	1.5	3.0	1.5	0.77
AE Boo	−1.5	7384	6100	7200	1100	1.9	2.8	0.9	0.48
TV Boo	−1.0	7199	6000	7500	1500	1.3	2.5	1.2	0.79

and metallicity. LGK86 provide grids, on the plane $(b-y) - c_1$, of constant $T_{\rm eff}$ and log g, for a large range of [Fe/H] values. Based on the Fourier value $[\mathrm{Fe/H}]_{\rm ZW}$, a model with the nearest [Fe/H] value was chosen for each star. In Figure 5 the cycle variation of each star on its corresponding grid is illustrated. This allows an estimation of the $T_{\rm eff}$ and log g variation ranges during the pulsation cycle and a comparison with the estimated temperature from the Fourier approach.

It is interesting to note that the studied stars have different effective temperatures and surface gravity limits, as well as different ranges which cannot be determined in detail with only the Fourier techniques. These results have been summarized in Table 9 in which we have also included those determined through the empirical calibrations from the Fourier coefficients. As can be seen, both methods give analogous results. In Table 9 the variation ranges in V, $T_{\rm eff}$ and $\log g$ are also indicated.

9. CONCLUSIONS

From data acquired in several photometric campaigns we have obtained extended $uvby-\beta$ photometry over a relatively large time span for two stars and data that adequately cover the cycle of pulsation for all of them. The V light curves have been Fourier decomposed and the corresponding Fourier parameters from their harmonics were used to calculate the iron abundance and luminosity of each star. The reddening was estimated by considering different objects in the same direction. The unreddened Strömgren indices c_0 and $(b-y)_0$ served to determine the variation along the cycle of the effective temperature and surface gravity.

The iron abundance $[\mathrm{Fe/H}]_{\rm ZW}$ was calculated first from the Fourier decomposition of the light curve and the calibration proposed by Kovács and co-workers and described with detail in § 4.1. We also utilized the ΔS parameter to estimate the metallicity. In § 4.2 we amply discuss the uncertainties and the limitations of this technique. We conclude that the Fourier decomposition approach gives more reliable results.

Since RR Lyrae stars are distance indicators, the individual estimations of the absolute magnitude for field stars from independent methods are of interest. The absolute magnitude, $M_V(F)$, predicted from Fourier decomposition (§ 5.1), is reported along with two other determinations. Once the iron abundance and the reddening are determined, an independent estimate of $M_V(c_1)$ can be made from the c_1 index and the pseudocolor C_0 (Cortés & Catelan 2008). This is described in § 6. Also in that section we calculate $M_V(Z)$ from the metallicity alone making use of a theoretical quadratic M_V-Z relationship offered by Cortés & Catelan (2008). All these results were compiled in Table 8. It was pointed out that, although in some individual cases the differences between $M_V(F)$ and for example $M_V(c_1)$ were larger than the expected from the uncertainties of the methods involved, no systematic trends could be seen and in some cases the agreement was fairly good. The agreement does not seem to be related to the quality or the density of the V light curve, since the disagreement is largest for the two RRc stars which also have the best light curves in our sample.

The $M_V-[\mathrm{Fe/H}]$ relationship for RR Lyrae stars has been amply discussed in the literature. A recent version of it calculated exclusively from the Fourier decomposition approach of RRab and RRc stars in globluar clusters (Arellano Ferro et al. 2008a) was used to confront our present results for the field stars (Figure 4). It can be seen that with some larger scatter the distribution of the Bootes stars follow the same trend of the globular cluster RR Lyrae stars and it was noted that if the alternative results for the absolute magnitude, $M_V(c_1)$ or $M_V(Z)$ were used the quality of the comparison would remain.

With $M_V(F)$ and the reddening we have reported the resulting distances for the sample stars (Table 8). It would be desirable to compare these distances with the derived distances from an independent technique. Unfortunately, those determined from the new reductions of the Hipparcos catalogue (van Leeuwen 2007) have such large errors that the comparison is impossible.

We would like to thank the following people for their assistance: the staff of the OAN, M. Sánchez and V. Alonso at the telescope during some of the 2005 observations and A. Pani for those of 2002, and L. Parrao for the reduction of the 2002 season. We are grateful to an anonymous referee for useful corrections and suggestions. This paper was partially supported by PAPIIT-Universidad Nacional Autónoma de México IN108106 and IN114309. J. Miller and J. Orta did the proofreading and typing, respectively. This article has made use of the SIMBAD database operated at CDS, Strasbourg, France and ADS, NASA Astrophysics Data Systems hosted by Harvard- Smithsonian Center for Astrophysics.

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