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THE INCLINATION OF THE DWARF IRREGULAR GALAXY HOLMBERG II

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RESUMEN

Damos restricciones al ángulo de inclinación del disco de H I de la galaxia irregular enana Holmberg II (Ho II) a partir de un análisis de la estabilidad gravitacional del disco de gas en las partes externas. Encontramos que se requiere un ángulo de inclinación medio de 27° y, por lo tanto, una velocidad circular en su parte plana de $\approx$60 km s^{-1}, para tener un grado de estabilidad similar al que tienen otras galaxias. Para esa inclinación, Ho II cae en la posición correcta en la relación de Tully-Fisher bariónica y, además, su curva de rotación es congruente con MOND. Sin embargo, el análisis de estabilidad correspondiente indica que esta galaxia podría ser problemática para MOND porque las partes externas de esta galaxia serían marginalmente inestables bajo esa teoría de gravedad. Se requieren simulaciones numéricas de las galaxias enanas ricas en gas para ver la factibilidad de MOND.

ABSTRACT

We provide constraints on the inclination angle of the H I disk of the dwarf irregular galaxy Holmberg II (Ho II) from a stability analysis of the outer gaseous disk. We point out that a mean inclination angle of 27° and thus a flat circular velocity of $\approx$60 km s^{-1}, is required to have a level of gravitational stability similar to that found in other galaxies. Adopting this inclination angle, we find that Ho II lies on the right location in the baryonic Tully-Fisher relation. Moreover, for this inclination, its rotation curve is consistent with MOND. However, the corresponding analysis of the stability under MOND indicates that this galaxy could be problematic for MOND because its outer parts are marginally unstable in this gravity theory. We urge MOND simulators to study numerically the non-linear stability of gas-rich dwarf galaxies since this may provide a new key test for MOND.

Key Words: dark matter — galaxies: individual (Holmberg II) — galaxies: kinematics and dynamics — gravitation

1. INTRODUCTION

The gas rotation curves of spiral galaxies are useful to derive the radial distribution of their dynamical mass and to test modified gravity theories as alternatives to dark matter. Accurate determinations of the rotation curves of low-surface brightness galaxies and dwarf irregular galaxies are very useful to test the predictions of the numerical simulations of structure formation (see de Blok 2010, for a review). In these galaxies, it is possible to derive the dark matter profiles also in their central parts because the contribution of the dark matter to the potential is dominant at any galactocentric radius.

Empirical correlations between the value of the circular velocity in the flat part ($v_{\rm flat}$) and other global properties of the galaxy (e.g., the baryonic mass) provide strong constraints to the theories of formation and evolution of galaxies (e.g., Silk & Mamon 2012). In order to derive the amplitude of the H I rotation curve of a certain galaxy, a good estimate of its inclination angle in the sky, i, is needed to deproject the observed velocity field. The inclination can be determined from the morphology or by modeling the entire two-dimensional radial velocity field using the tilted-ring analysis (e.g. Begeman et al. 1991). In irregular galaxies with large scale asymmetries (e.g., stellar bars or giant holes in the H I surface density distribution) and/or rising rotation curves without a well-defined flat part, inclination is an uncertainty. As an example, consider the dwarf irregular galaxy IC 2574. Inspection of the optical image suggests that this galaxy is highly inclined. Martimbeau et al. (1994) derived an inclination angle of $77° \pm 3°$ from the central isophotes in the R-band

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and a mean inclination of $75° \pm 7°$ from the H I kinematical analysis. Following a similar procedure, Walter & Brinks (1999) confirmed the value of the inclination by Martimbeau et al. (1994). More recently, Oh et al. (2011), using a more sophisticated analysis to minimize the effect of non-circular motions, obtained an average value of the inclination angle of the H I disk of only 53°.

Since the rotation velocity depends on the adopted inclination as $1/\sin i$, uncertainties in the inclination are more important in galaxies with low inclinations (i.e. more face-on galaxies). In this paper, we suggest an indirect method to provide upper limits on i in galaxies with low inclinations, where traditional methods are less precise. We will focus on the dwarf irregular galaxy Holmberg II (Ho II) or DDO 50. From tilted-ring models, Oh et al. (2011) derived a mean inclination angle of 49°. For this inclination, the asymptotic (flat) circular velocity Ho II is 36 km s^{-1}, which is inconsistent with MOND (Sánchez-Salcedo et al. 2013). Motivated by this result, Gentile et al. (2012) re-analyzed the data cubes at the outer parts in Ho II ($R > 4$ kpc) and concluded that an inclination angle between 20° and 35° is more consistent with the observed total H I map (see also McGaugh 2011). Here, we invoke stability arguments to provide constraints on the inclination angle of Ho II and, thus, on the amplitude of its rotation curve.

The paper is organized as follows. § 2 reviews some structural and kinematical parameters of Ho II. In § 3, we compute the Toomre parameter of the gaseous disk and compare it with the values obtained for other galaxies. § 4 shows how the Toomre parameter changes if a different inclination angle is adopted. We suggest that a mean inclination angle of the H I disk of $i \sim 27°$ leads Ho II to a "normal" level of stability, and it is consistent with the baryonic Tully-Fisher relation (e.g, McGaugh 2012), with the Zasov-Smirnova relation (Zasov & Smirnova 2005) and with other empirical trends. The implications of adopting this lower inclination for Modified Newtonian Dynamics (MOND) are discussed in § 5. Conclusions are given in § 6.

2. GLOBAL PROPERTIES OF HO II

Ho II (DDO 50) is a relatively nearby gas-rich dwarf irregular galaxy in the M81 group. Based on the tip of red giant branch method, Karachentsev et al. (2002) derived a distance of 3.4 Mpc, whereas Hoessel et al. (1998) inferred a distance of 3.05 Mpc using Cepheids. To facilitate comparison with the works of Leroy et al. (2008) and Oh et al. (2011), we will adopt a distance of 3.4 Mpc but the implications of using a shorter distance are also discussed in § 4. At a distance of 3.4 Mpc, it has a total absolute B magnitude of -16.9 mag and a radius measured to the $25B$ mag arcsec^{-2} surface brightness level, denoted by R_{25}, of 3.3 kpc (Leroy et al. 2008). The optical inclination, extracted from HYPERLEDA, is 45° (Paturel et al. 2003).

The H I distribution and its kinematics have been studied by Puche et al. (1992), Bureau & Carignan (2002), and by Oh et al. (2011). There is no consensus on the H I inclination angle; ellipse fits to the outer H I disk (at a radius $\approx$7 kpc) suggest an inclination of $\approx$30° (de Blok et al. 2008; McGaugh 2011; Gentile et al. 2012), whereas tilted-ring analysis indicates an inclination (at $\approx$7 kpc), roughly between 40° and 50° (Bureau & Carignan 2002; Oh et al. 2011). For $\langle i \rangle = 49°$, the asymptotic velocity of Ho II is 36 km s^{-1} (Oh et al. 2011).

3. THE TOOMRE STABILITY PARAMETER OF THE GAS DISK IN HO II

There exist several studies aimed to test the importance of the gravitational instability in determining both the sites of massive star formation and the star formation rate in galaxies (Kennicutt 1989; van Zee et al. 1997; Hunter et al. 1998; Martin & Kennicutt 2001; Kim & Ostriker 2001, 2007; de Blok & Walter 2006; Yang et al. 2007; Leroy et al. 2008; Yim et al. 2011; Elmegreen 2011). From observations of nearby Sc galaxies, Kennicutt (1989) and Martin & Kennicutt (2001) found that there exists a gas surface density threshold for star formation. These authors suggested that the threshold depends on the Toomre parameter of the gaseous disk, defined as:

$$Q_g = \frac{\Sigma_c}{\Sigma_g}, \tag{1}$$

where Σ_g is the gas surface density (that is, H I+H_2 gas corrected to include helium and metals), and the critical density for instability is

$$\Sigma_c = \frac{\kappa c_s}{\pi \mathrm{G}}, \tag{2}$$

with κ the epicyclic frequency and c_s the velocity dispersion of the gas. We must emphasize here that Kennicutt (1989) and Martin & Kennicutt (2001) used the velocity dispersion of the gas and not the effective sound speed (see Schaye 2004 for a discussion). Turbulence tends to stabilize the gaseous disk but the magnitude of this effect is small in disks where the H I is the main component (Romeo et al. 2010). In some galaxies, the contribution of the stellar disk to gravitational instability cannot be ignored (Yang et al. 2007). Nevertheless, in the case of gas-rich dwarf galaxies, as it is the case for Ho II, or in the outer parts of spiral galaxies, the inclusion of the stellar contribution is not so relevant (see Figure 5 in Hunter et al. 1998; Meurer et al. 2013).

Recent studies indicate that galactic disks stabilize to a constant stability parameter across the optical galaxy (Meurer et al. 2013; Zheng et al. 2013), lending support to the phenomenon of self-regulation by star formation, as suggested by Quirk (1972). However, although the value of the Q_g-parameter is approximately constant across a galaxy, it varies from galaxy to galaxy, indicating that the 'thermostat' process (i.e. the stellar feedback) is not solely determined by gravitational instabilities (e.g. Zheng et al. 2013). In fact, for a sample of spiral galaxies and dwarf galaxies, Leroy et al. (2008) found that Q_g has a so large scatter from galaxy to galaxy, that there is no a clear evidence of a universal Q_g threshold marking the transition from high star formation efficiency to low star formation efficiency (see also Hunter et al. 1998 and Côté et al. 2000).

Hunter et al. (1998) first noticed that the dwarf irregular galaxy Ho II presents some peculiarities regarding the values of the Toomre parameter when compared to other galaxies. Using an inclination angle of 40°, they found that beyond a galactocentric radius of 2 kpc, Σ_g/Σ_c in Ho II is ≈ 3 times higher than the average peaks in the other irregular galaxies. Only out at ≈ 7 kpc, the ratio between the gas column density and the critical density declines to a value comparable to the peak values in the other irregular galaxies.

Using the updated high-resolution data for the rotation curve and the azimuthally averaged H I surface density (corrected by a factor 1.4 to include helium and metals) for Ho II, as derived in Oh et al. (2011), we have calculated the Toomre parameter Q_g and the critical density Σ_c, for $D = 3.4$ Mpc and $c_s = 6$ km s^{-1}. A value for c_s of 6 km s^{-1} corresponds to the value used by Kennicutt (1989). However, the reason to adopt this value for c_s is more profound. There is growing evidence that suggests that a sound speed of $4 - 6$ km s^{-1} (instead of the observed velocity dispersion), when used to derive the Toomre critical density, gives an optimal description of ongoing star formation regions (de Blok & Walter 2006; Yang et al. 2007). These authors argue that in a multiphase medium, the measured velocity dispersion is the sum of contributions of the dispersions of the cool and warm phases with additional input from star formation and turbulence. As the relevant dispersion value to use in a star formation threshold analysis is that of the cool phase of the ISM, using the second-moment values will likely result in significant overestimates.

Figure 1 shows the radial profile of Q_g and Σ_g/Σ_c in Ho II. We see that $Q_g < 1.5$ in a large part of the galaxy, from 2 to 6 kpc, being minimum at 3.5 kpc. Following Meurer et al. (2013), it is useful to define R_1 and R_2 as the radii enclosing 25% and 75% of the total H I gas, respectively. According to Meurer et al. (2013),

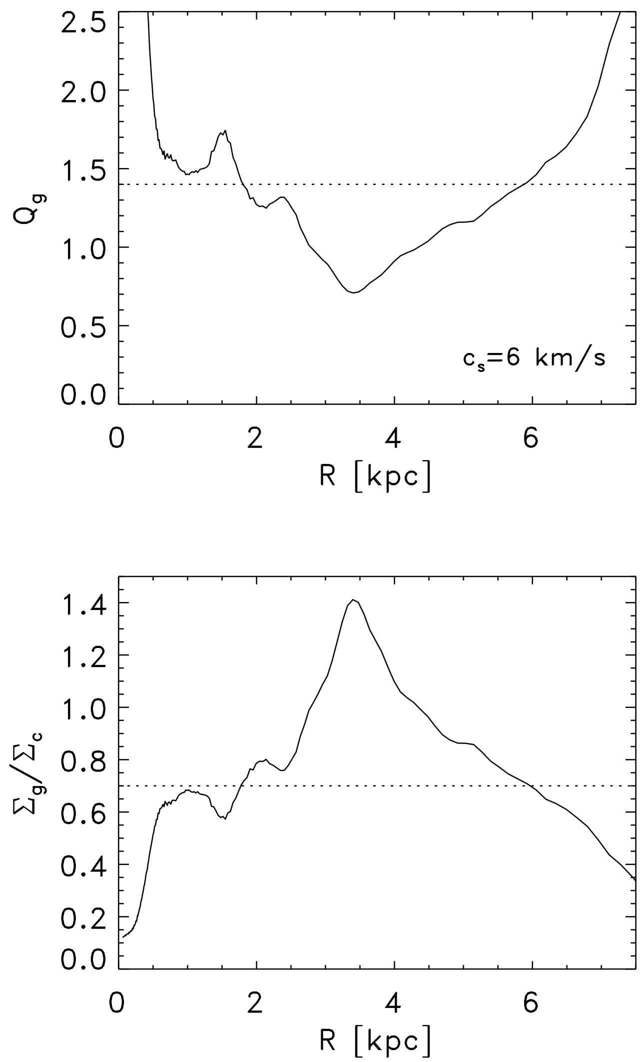

Fig. 1. Radial profile of the gas Toomre parameter (top panel) and Σ_g/Σ_c (bottom panel) in Ho II for c_s = 6 km s^{-1}, D = 3.4 Mpc and $\langle i \rangle$ = 49°. For reference, the horizontal dotted curve corresponds to Q_g = 1.4 or, equivalently, $\Sigma_g/\Sigma_c = 0.7$.

ISM disks do maintain a constant Q_g between R_1 and R_2. In the case of Ho II, we find $R_1 = 2.9$ kpc, $R_2 = 5.5$ kpc, and $\langle Q_g \rangle = 1.0$, where $\langle Q_g \rangle$ is the mean value of Q_g between R_1 and R_2.

From theoretical grounds, one would expect a burst of star formation between $R = 2$ and $R = 6$ kpc because, as Elmegreen (2011) showed, dissipative gaseous disks with $Q_g <$ 2–3 are strongly unstable. Moreover, if Kennicutt's result (1989) is applied, that is, if the Hα emission extends up to the radius where $Q_g \approx 1.4$, then we would predict that the Hα emission should be detected up to $R = 6$ kpc.

However, none of these predictions is correct. To illustrate this, Figure 2 shows the radial distribution for the azimuthal averaged Hα emission from Hunter et al. (1998), and the star formation rate, as derived by

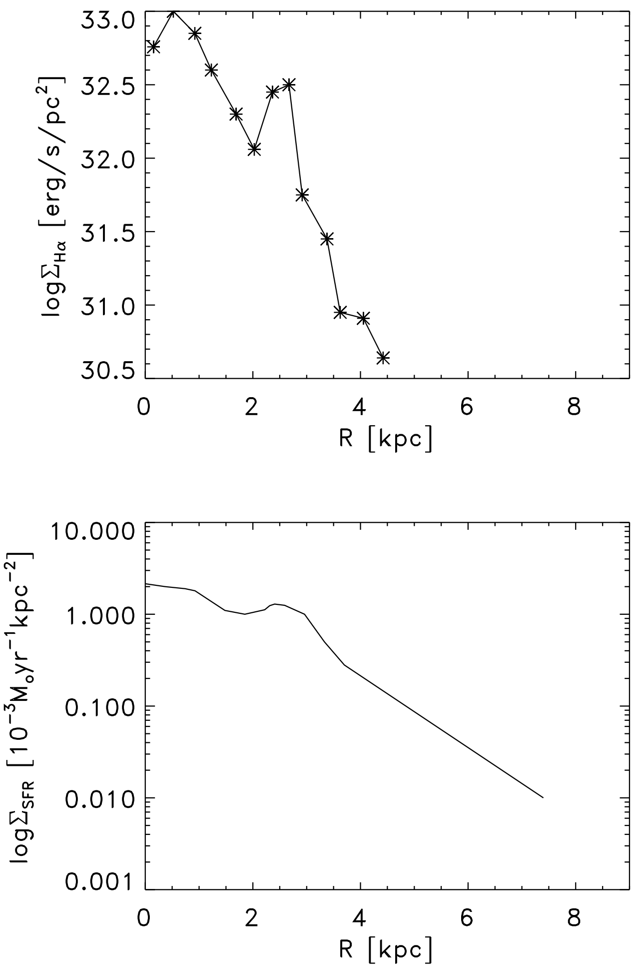

Fig. 2. Top panel: Azimuthally averaged Hα surface brightness as a function of radius in Ho II from Hunter et al. (1998). Bottom panel: star formation rate vs. radius, as derived by Leroy et al. (2008) inside the optical radius R_{25} (i.e. where the B-band magnitude drops below 25 mag arcsec^{-2}) and by Bigiel et al. (2010) between R_{25} and $2R_{25}$.

Leroy et al. (2008) and Bigiel et al. (2010), using far-ultraviolet emission and the 24 μm map. First of all, the current star formation of Ho II is not extraordinary (Hunter et al. 1998). Moreover, the radius at which the furthest Hα is detected, $R_{\rm H\alpha}$, is $\simeq 4.2$ kpc. At $R_{\rm H\alpha}$, and for $c_s = 6$ km s^{-1}, the gaseous disk is still marginally unstable, $\Sigma_g/\Sigma_c \simeq 1.0$. We see that the gas Toomre parameter achieves the threshold value $Q_g = 1.4$ (or, equivalently, $\Sigma_g/\Sigma_c = 0.7$; Kennicutt 1989), at $R = 6$ kpc, where the local star formation rate is very low ($\approx 4 \times 10^{-5} M_\odot\,{\rm yr}^{-1}\,{\rm kpc}^{-2}$). In the following, by comparing with other galaxies, we will see that values for Σ_g/Σ_c close to 1 at $R_{\rm H\alpha}$ are abnormally high.

For a sample of nearby Sc galaxies, Kennicutt (1989) and Martin & Kennicutt (2001) found that $\Sigma_g/\Sigma_c < 0.75$ at $R_{\rm H\alpha}$. Moreover, for those irregular galaxies in Hunter et al. (1998) and in Hunter

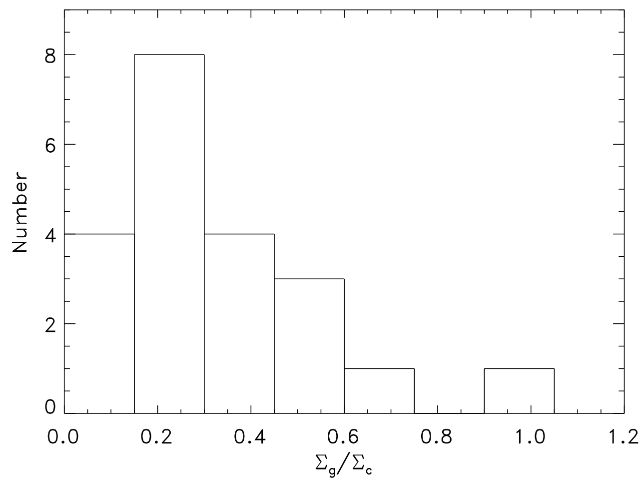

Fig. 3. Histogram of the values of Σ_g/Σ_c at the radius where SFR= $10^{-4} M_\odot\,{\rm yr}^{-1}\,{\rm kpc}^{-2}$ in the 21 galaxies with all the required data presented in Leroy et al. (2008).

et al. (2011) with comfortable inclinations (i.e. $i > 50°$), we obtained $\Sigma_g/\Sigma_c \leq 0.6$ at $R_{\rm H\alpha}$ (using $c_s = 6$ km s^{-1}). Finally, for the sample of Leroy et al. (2008), we have calculated Σ_g/Σ_c at the galactocentric radius where the star formation rate (SFR) is $10^{-4} M_\odot\,{\rm yr}^{-1}\,{\rm kpc}^{-2}$, in the 21 galaxies having all the required data (with $c_s = 6$ km s^{-1}). The histogram of the values of Σ_g/Σ_c is shown in Figure 3. We found a mean value for Σ_g/Σ_c of 0.33. More specifically, 66% of the galaxies have $\Sigma_g/\Sigma_c < 0.35$. Only IC 2574 and HoII have $\Sigma_g/\Sigma_c > 0.6$. Here we must note that IC 2574 presents a solid-like rotation curve. If the star formation rate depends on the rate of collisions between clouds, as suggested by Tan (2000), then galaxies with solid-rotation could exhibit larger values of Σ_g/Σ_c because of their low level of shear and hence a lower rate of collisions between clouds. The level of shear, defined as $|d\ln\Omega/d\ln R|$, is remarkably smaller in IC 2574; at the radius where SFR= $10^{-4} M_\odot\,{\rm yr}^{-1}\,{\rm kpc}^{-2}$, the level of shear is ~ 0.8 in Ho II, whereas it is ~ 0.6 in the case of IC 2574.

In summary, the value of Σ_g/Σ_c in the outer parts of Ho II, from 3.5 to 6 kpc, is larger than the values inferred for other spiral and dwarf irregular galaxies. This fact led us to explore the possibility that the epicyclic frequency κ and thus the critical density Σ_c have been underestimated. This may be possible if Ho II is either more face-on or closer (or both) than adopted.

4. SETTING HO II BACK TO 'NORMALITY'

In order to derive Q_g, we have assumed a distance of $D = 3.4$ Mpc, and an inclination of $\langle i \rangle = 49°$. The distance of Ho II is probably between 3.0 Mpc and

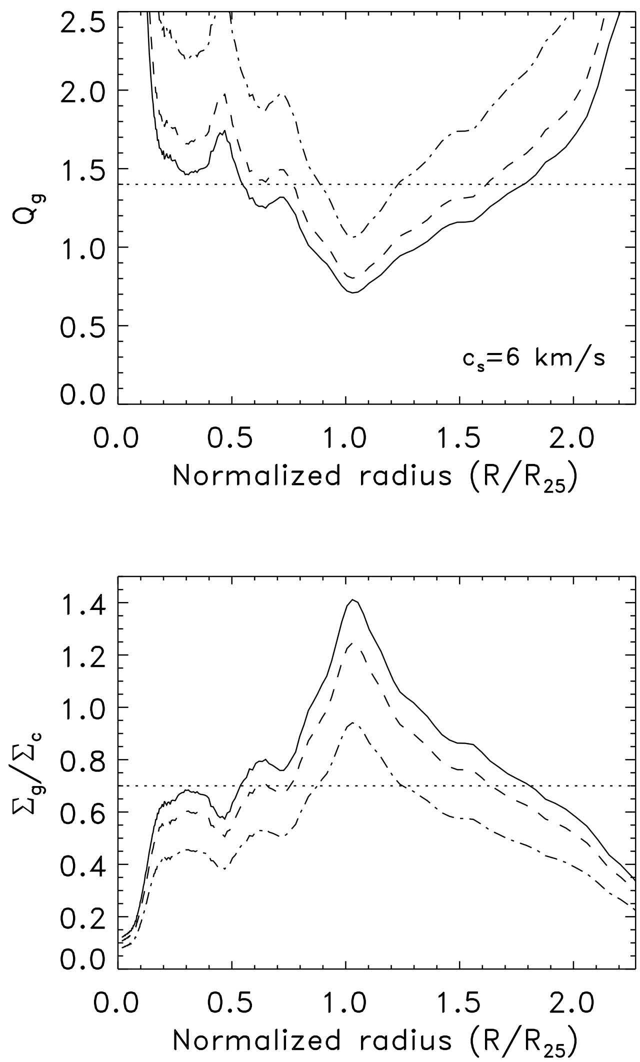

Fig. 4. The Q_g-parameter (top) and Σ_g/Σ_c (bottom) as a function of the galactocentric radius normalized by the optical radius R_{25} for $D = 3.4$ Mpc (solid curve), $D = 3.0$ Mpc (dashed line) and $D = 2.3$ Mpc (dot-dashed line).

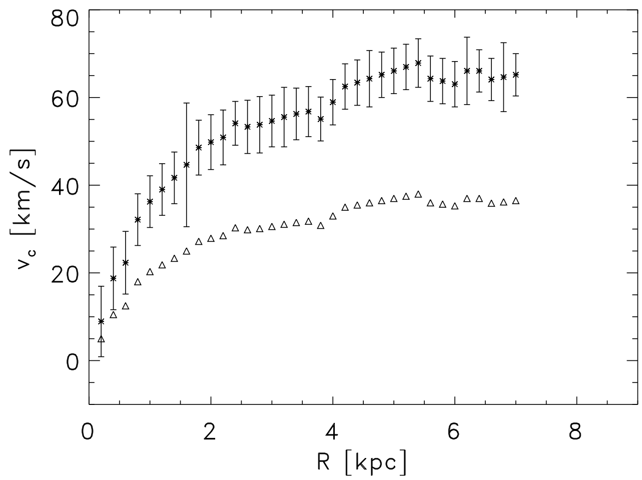

Fig. 5. H I rotation curve of Ho II for two different mean inclination angles: $\langle i \rangle = 49^\circ$ from Oh et al. (2011) (triangles) and for $\langle i \rangle = 27^\circ$ (asterisks with error bars).

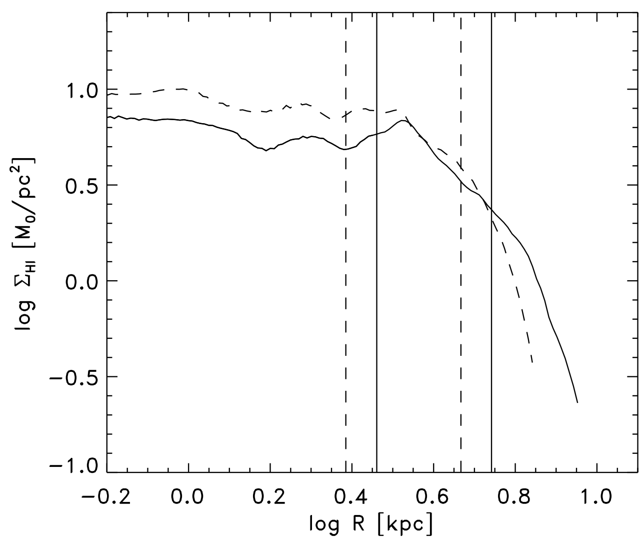

Fig. 6. Radial distribution of the azimuthally averaged H I surface density for a position angle of 175°, for $\langle i \rangle = 49^\circ$ (solid line) and for $\langle i \rangle = 27^\circ$ (dashed line). The vertical lines correspond to the radii R_1 and R_2, which depend slightly on the adopted inclination. The solid line stands for $\langle i \rangle = 49^\circ$ and the dashed lines for $\langle i \rangle = 27^\circ$; the left vertical lines indicate R_1 and the right vertical lines R_2. The projected surface density was taken from THINGS (Walter et al. 2008).

3.5 Mpc (see § 2). If the galaxy distance is smaller than the adopted value, it will result in higher Q_g values at a given angular radius $\alpha = R/D$ because, whereas $\Sigma_g(\alpha)$ is independent of D, the epicyclic frequency varies $\kappa(\alpha) \propto D^{-1}$. Figure 4 shows the Toomre parameter and the ratio between gas surface density and the critical surface density for $D = 3.0$ Mpc. The enhancement of the stability parameter is not significant. In order to have $\Sigma_g/\Sigma_c = 0.7$ at $R_{\mathrm{H}\alpha}$, a value of $D = 2.3$ Mpc is required. As this distance is very unlikely (the distance uncertainty is ∼20%), we conclude that the desired level of stability cannot be achieved by a reasonable adjustment of galaxy's distance alone.

The derived Toomre parameter profile also depends on the adopted inclination angle of the galaxy because both $\Sigma_g(R)$ and $\kappa(R)$ depend on the inclination. In particular, the amplitude of the rotation curve and thereby κ change as $1/\sin i$. We have considered a model in which the inclination angle varies slightly with galactocentric radius from 30° in the inner 2 kpc to 24° in the outer parts, having a mean inclination of Ho II of 27°. In Figures 5 and 6, we compare the rotation curves and the azimuthally-averaged H I surface densities of Ho II for $D = 3.4$ Mpc and two different inclinations; $\langle i \rangle = 49^\circ$ and $\langle i \rangle = 27^\circ$. The resultant $Q_g(R)$ profiles

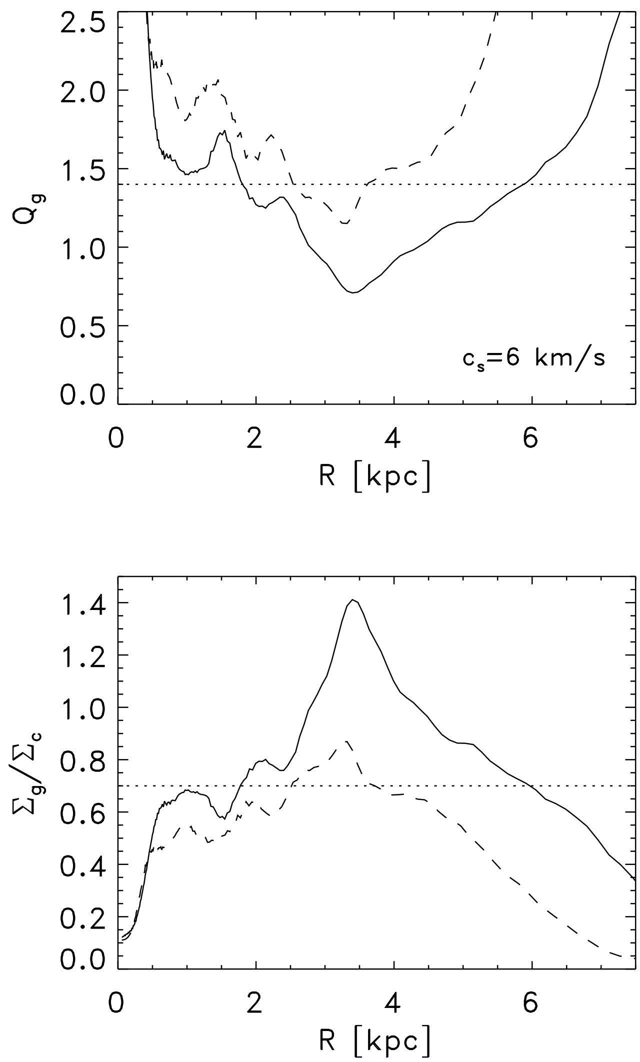

Fig. 7. Radial profile of the gas Toomre parameter (top panel) and Σ_g/Σ_c (bottom panel) in Ho II for $c_s = 6$ km s^{-1}, $D = 3.4$ Mpc and $\langle i \rangle = 27°$ (dashed line). For comparison, we also plot the curves for $\langle i \rangle = 49°$ (solid line). For reference, the horizontal dotted curve corresponds to $Q_g = 1.4$ or, equivalently, $\Sigma_g/\Sigma_c = 0.7$.

are shown in Figure 7, adopting a constant velocity dispersion of $c_s = 6$ km s^{-1}. As expected, the derived stability parameter of the galaxy is higher (more stable) for lower inclination angles.

We see that if the mean inclination angle of Ho II is 27° (and $D = 3.4$ Mpc), then the Σ_g/Σ_c-ratio at $R_{\mathrm{H}\alpha}$ takes a comfortable value of 0.65. This value is similar to those obtained for other dwarf irregular and Sc galaxies (see § 3). For this inclination, Q_g varies between a minimum of 1.2 and a maximum value of 1.6 in the range[4] $R_1 < R < R_2$, and $\langle Q_g \rangle = 1.35$. The radial variation of Q_g between R_1 and R_2 is smaller for $\langle i \rangle = 27°$ than it is when a mean inclination of 49° is adopted (for $\langle i \rangle = 49°$, its minimum value is 0.7 and its maximum value is 1.3). Therefore, a mean inclination of 27° is more consistent with the Quirk hypothesis (1972) that self-regulation leads to a constant value of Q_g. For reference, note that in the sample of 21 galaxies studied in Meurer et al. (2013), log $\langle Q_g \rangle$ has typical rms values of only $\approx$0.05 (see their Figure 4). Our first conclusion is that reasonable values of stability are obtained for a mean inclination $\approx$27°. As a result, Ho II would have a rotation velocity of $\approx$60 km s^{-1} at the outer regions ($R = 4$–7 kpc).

[4]These radii were defined in § 3 as the radii containing 25 and 75 percent of the total neutral hydrogen mass.

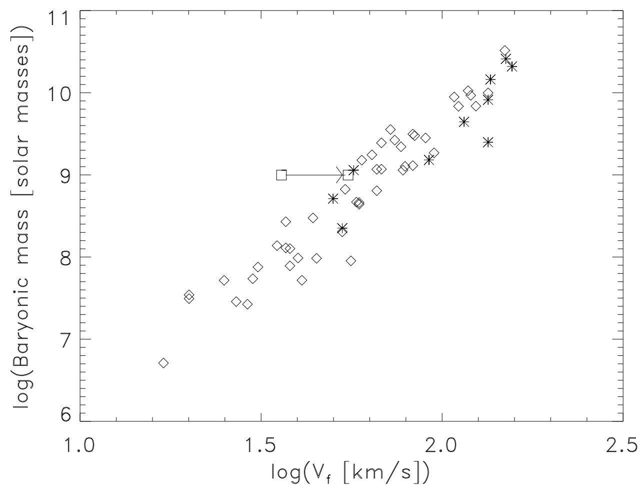

Fig. 8. Baryonic Tully-Fisher relation for the McGaugh (2012) sample of galaxies (diamonds) plus the dwarf irregular galaxies in the Leroy et al. (2008) sample (asterisks). The position of Ho II is indicated with a square for $\langle i \rangle = 49°$ (left square) and for $\langle i \rangle = 27°$ (right square).

An inclination of $\approx$27°, required to place the Ho II disk above the stability threshold, will change the position of Ho II in those diagrams involving the asymptotic circular speed, such as the baryonic Tully-Fisher relationship. Figure 8 shows the baryonic Tully-Fisher relation for the sample of galaxies described in McGaugh (2012), together with the dwarf irregular galaxies described in Leroy et al. (2008). For a mean inclination angle of 49°, Ho II is well outside the baryonic Tully-Fisher relation, whereas a mean inclination of 27° places Ho II within the scatter band.

Zasov & Smirnova (2005) found a tight correlation between the total H I mass of the galaxies and $R_d V_{\mathrm{lmp}}$, where R_d is the radial scalelength of the stellar disk and V_{lmp} is the circular velocity at the last measured point[5]. We wish to check that the advocated mean inclination of 27° is not in conflict with this diagram. Figure 9 displays the total mass gas (neutral gas plus molecular hydrogen) versus $R_d V_{\mathrm{lmp}}$ for the same sample of galaxies as used in Figure 8. We see that, al-

[5]A related relationship is discussed in Lelli et al. (2014).

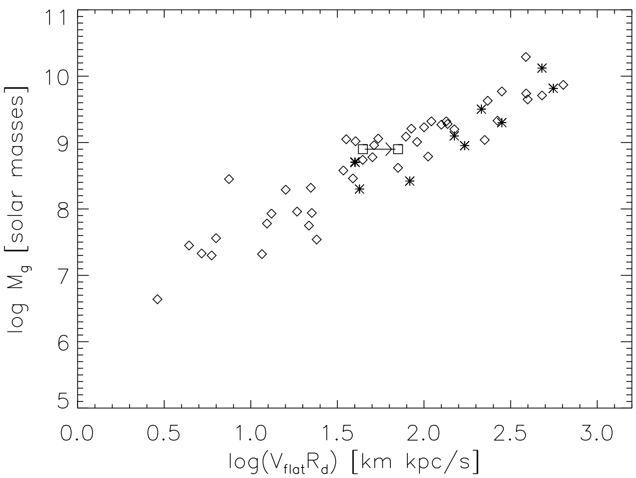

Fig. 9. Total mass of gas versus $V_{\rm flat}R_d$ (where $V_{\rm flat}$ is the asymptotic flat velocity and R_d the exponential radial scale length) for the McGaugh (2012) galaxies (diamonds) and the dwarf irregular galaxies in Leroy et al. (2008) (asterisks). The change in the position of Ho II in this diagram, when the inclination angle varies from $\langle i\rangle = 49°$ to $\langle i\rangle = 27°$, is indicated with squares.

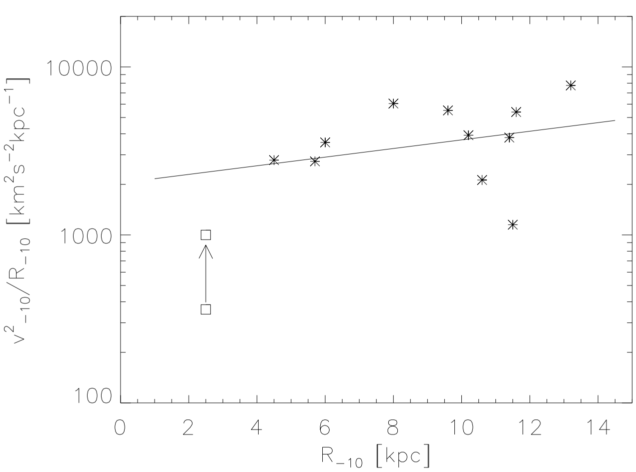

Fig. 10. Radial gravitational acceleration at R_{-10} versus R_{-10} for those galaxies in the Leroy et al. (2008) sample (asterisks) having all the required data and a flattened rotation curve. The position of Ho II is indicated with a square for $\langle i\rangle = 49°$ (lower square) and for $\langle i\rangle = 27°$ (upper square). To guide the eye, a linear fit to the points was drawn (solid line).

though this diagram cannot be used as a diagnostic for the inclination angle, a value $\langle i\rangle = 27°$ keeps Ho II within the scatter in the diagram.

Some authors have investigated a possible correlation between the disk-average star formation efficiency (SFE) and the orbital timescale $\tau_{\rm orb}$, defined as $2\pi R/v_c$ (e.g., Kennicutt 1998). A larger circular velocity implies a shorter orbital period. If such a correlation exists then one could constrain the circular velocity, and thereby the inclination of a galaxy, from observations of the SFE. The question that arises is: Is the SFE of Ho II more consistent with a flat circular velocity of 36 km s^{-1} or with a flat circular velocity of 60 km s^{-1}? To answer this question, we have computed the radial gravitational acceleration v_c^2/R at the radii where SFE= 10^{-10} yr^{-1}, denoted by R_{-10} (this occurs in the outer parts of disks), for the galaxies in the sample of Leroy et al. (2008), excluding those galaxies with a solid-like rotation (i.e. DDO 154, IC 2574, NGC 2976) and the galaxy Ho I because its rotation curve is not flat. Figure 10 shows the radial gravitational acceleration vs R_{-10}. We see that when $\langle i\rangle = 49°$, Ho II is well outside the trend in that diagram, implying that the circular velocity at R_{-10} is too low, as compared to other galaxies. However, an inclination angle of 27° puts Ho II back to normality.

In summary, the stability of the gaseous disk of Ho II, the Tully-Fisher relation, the Zasov & Smirnova (2005) relation, and the present-day star formation efficiency at the outer disk, are all consistent with an H I inclination angle of Ho II (at galactocentric radii $3-5$ kpc) not significantly larger than 27°, which implies that the amplitude of the rotation curve is ~ 60 km s^{-1}.

All the computations above have been derived using $D = 3.4$ Mpc. If we assume a distance of 3.0 Mpc, an inclination angle $\sim 32°$ is needed to have the required level of stability of the gaseous disk. Thus, uncertainties in the distance have a minor role.

5. HO II: IMPLICATIONS FOR MOND

Milgrom (1983) proposed a modification of Newton's gravitation law to explain the dynamics of galaxies without invoking dark matter. Sánchez-Salcedo et al. (2013) provided a sample of gas-rich dwarf galaxies that could be problematic for MOND. One of those galaxies was Ho II. Sánchez-Salcedo et al. (2013) argued that the rotation curve of Ho II is consistent with MOND only if the H I inclination is $\sim 25°$. In the previous sections, we have provided indirect evidence suggesting that, in fact, the inclination of Ho II is possibly close to 25° (see also Gentile et al. 2012). With this new inclination, the amplitude of the rotation curve of Ho II can be matched under MOND.

One of the arguments in favor of an inclination angle of $\approx 25°$ was based on a stability analysis. For significantly higher inclination angles, Ho II would be abnormally unstable, as compared to other galaxies. The level of stability of galaxy, i.e. the Toomre parameter, was studied under Newtonian gravity. However, it depends on the adopted gravitational law. In the following, we will compute the Toomre parameter of the gaseous disk under the MOND framework, $Q_{g,M}$, to see if its level of stability is also reasonable.

The MOND version of Poisson's equation is given by:

$$\nabla \cdot \left[\mu \left(\frac{|\nabla \Phi|}{a_0} \right) \nabla \Phi \right] = 4\pi G \rho, \tag{3}$$

where ρ is the density distribution, Φ the gravitational potential, a_0 is a universal acceleration of the order of 10^{-8} cm s^{-2}, and $\mu(x)$ is some interpolating function with the property that $\mu(x) = x$ for $x \ll 1$ and $\mu(x) = 1$ for $x \gg 1$ (Bekenstein & Milgrom 1984). Milgrom (1989) demonstrated that the Toomre parameter in MOND is:

$$Q_M = \mu^+ (1 + L^+)^{1/2} Q_N, \tag{4}$$

where Q_N is the Newtonian Toomre parameter as defined in equation (1), $L \equiv d \ln \mu / d \ln x$ is the logarithmic derivative of μ, whereas μ^+ and L^+ are the values of μ and L just above the disk.

The two most popular choices for the interpolating function are the "simple" μ-function, suggested by Famaey & Binney (2005),

$$\mu(x) = \frac{x}{1 + x}, \tag{5}$$

and the "standard" μ-function

$$\mu(x) = \frac{x}{\sqrt{1 + x^2}}, \tag{6}$$

proposed by Milgrom (1983). Since we are interested in Ho II, a dwarf galaxy whose dynamics lies in the deep MOND regime (that is, $x = g/a_0 \ll 1$), our results are not sensitive to the exact form of the interpolating function. We will use the simple μ-function with $a_0 = 1.2 \times 10^{-8}$ cm s^{-2}.

In the case of the simple μ-function, the Toomre parameter in MOND differs from the Toomre parameter in Newton by the factor $\mu^+(1+L^+)^{1/2} = \mu^+(2-\mu^+)^{1/2}$. Since this factor is ≤ 1, the Toomre parameter in MOND is lower than it is in the corresponding Newtonian galaxy with a dark halo.

We have computed $Q_{g,M}$ for our model with $\langle i \rangle = 27°$. This inclination is required in order to match the amplitude of the rotation curve with the value predicted by MOND. To derive μ^+ and L^+, we assumed that the disk is very thin so that the vertical Newtonian gravitational field just above and just below the disk has a magnitude $2\pi G(\Sigma_g + \Sigma_\star)$. In order to compute the stellar surface density $\Sigma_\star$, we derived the pixel-to-pixel stellar mass-to-light ratio applying the method proposed by Zibetti, Charlot, & Rix (2009; hereafter ZCR) to the B, V and R images from SINGS (Kennicutt et al. 2003). The azimuthally averaged stellar surface density, using the ZCR stellar mass-to-light ratio ($\Upsilon_{\star,\rm ZCR}$), is shown in Figure 11. For completeness and in order to show its dependence on the adopted inclination, we plot the stellar surface density for two different inclinations (25° and 49°).

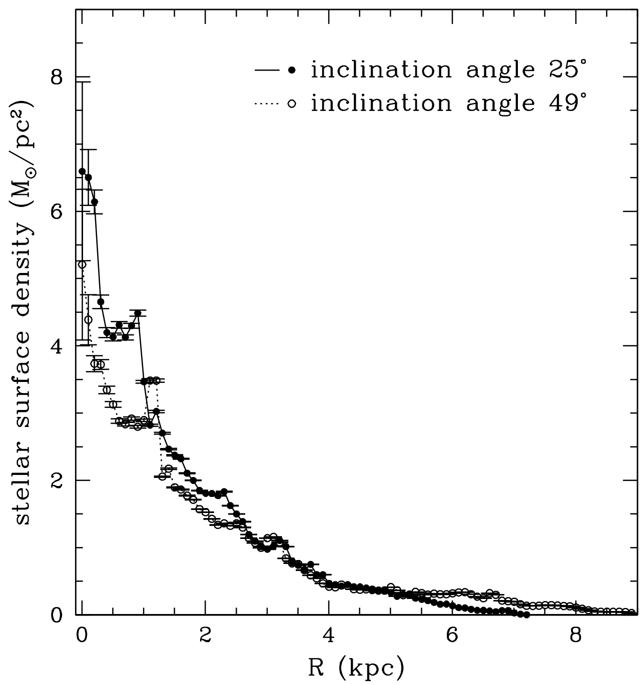

Fig. 11. Azimuthally averaged stellar surface density against radius from the galaxy center for two angle inclinations 25° and 49°, using the ZCR method.

Figure 12 shows $Q_{g,M}$ in Ho II, as a function of radius, using two different choices for c_s: (1) a constant value $c_s = 6$ km s^{-1} (solid line) and (2) the observed H I velocity dispersion from the second-moment map (dashed line). Consider first the Toomre parameter derived using $\Upsilon_{\star,\rm ZCR}$ and $D = 3.4$ Mpc (top panel). What we see in Figure 12 is that, when $c_s = 6$ km s^{-1}, the MOND Toomre parameter is less than 1 at any radius between $R = 1$ and $R = 6$ kpc. We find no correlation between the Toomre parameter, derived assuming that c_s is constant, and the star formation rate (see Figure 2). If the critical surface density is a real threshold for star formation in MOND, then we would expect, following the same reasoning as in § 3, a huge star formation activity throughout the disk of Ho II up to a radius of 6 kpc. However, this is not observed. If, instead of a constant sound speed, we take the observed H I velocity dispersion from the second-moment map, we find no correlation between star formation activity and the Toomre parameter either. More importantly, we find that, adopting the observed velocity dispersion, the gas Toomre parameter $Q_{g,M}$ is larger than 1, as required for stability, within 2.5 kpc, but it is near to 1 in a significant portion of the disk (3 kpc $< R <$ 6 kpc), implying that it is marginally unstable and thereby very responsive to perturbations.

The values of $Q_{g,M}$ are not very sensitive to reasonable changes in the adopted distance D and stellar

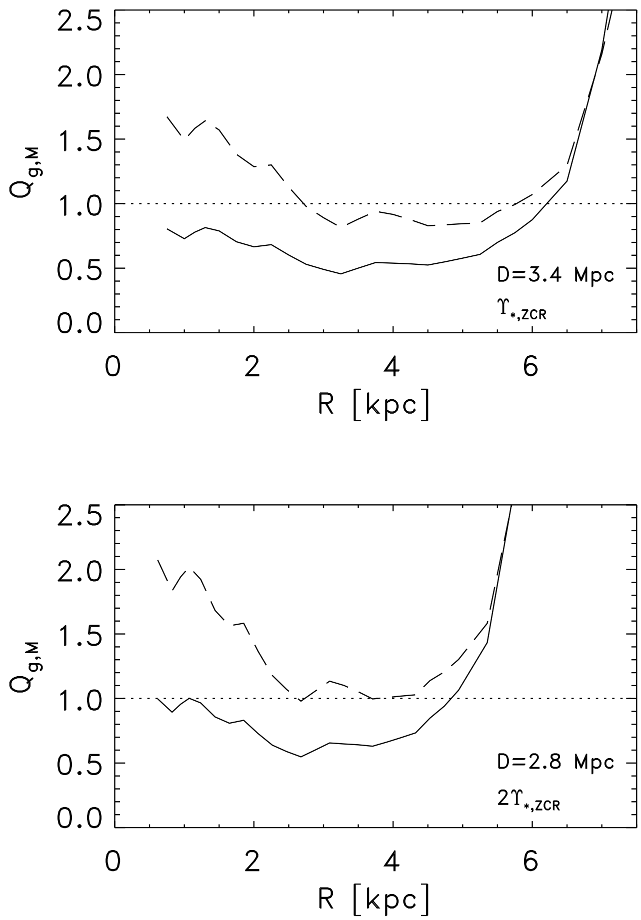

Fig. 12. Radial distribution of the Toomre parameter in MOND using a constant gas velocity dispersion of 6 km s^{-1} (solid line) and using the velocity dispersion from the second-moment map (dashed line). We have used the simple μ-function. In the top panel, we have assumed $D = 3.4$ Mpc and the $\Upsilon_\star$-value derived using the ZCR method. In the bottom panel, we used $D = 2.8$ Mpc and twice the $\Upsilon_{\star,\rm ZCR}$-value. The gas surface density Σ_g was taken as 1.4 times the azimuthally averaged H I surface density. In order to estimate the gas surface density, a mean inclination angle of 27° was used.

mass-to-light ratio (see Figure 12). The effect of adopting a different D can be computed as follows. If the stellar mass-to-light ratio is given, the Newtonian gravitational acceleration $v_{c,N}^2/R$ at a certain angular radius does not depend on the adopted distance. Thus, μ does not depend on D. Therefore, $Q_{g,M}$ depends only on D through κ, which scales as $\propto D^{-1/2}$. This implies that a reduction in the adopted distance from 3.4 Mpc to 2.8 Mpc increases the values of $Q_{g,M}$ by 10%. On the other hand, increasing the stellar mass-to-light ratio leads to a higher $Q_{g,M}$ because the circular velocity and κ will both increase. This effect is however small because the stellar mass is only 20% of the mass in gas, even when a value of $2\Upsilon_{\star,\rm ZCR}$ is assumed. Figure 12 shows $Q_{g,M}$ when the distance is reduced by 20% and the stellar mass-to-light ratio is augmented by a factor of 2. The gas stability parameter in this case is slightly larger than it is for the referecence values but it is still close to unity at the interval 2.5–4.5 kpc in radius.

Sánchez-Salcedo & Hidalgo-Gámez (1999) already noticed that the dwarf irregular galaxies IC 2574 and NGC 1560 have $Q_{g,M} < 1$ along a significant portion of the disk. What stabilizes mboxMONDian gas-rich dwarf galaxies against the formation of grand-design spiral arms and/or widespread star formation up to the radius where $Q_{g,M} \sim 1.4$? We urge MOND simulators to consider the problem of the stability of gas-rich dwarf galaxies as it may provide a key test for MOND.

6. FINAL REMARKS AND CONCLUSIONS

The existence of an intrinsic scatter in the baryonic Tully-Fisher relation may have strong implications in galaxy evolution theories and modified gravity. To establish any scatter, one needs precise determinations of the amplitude of rotation curves and thus good estimates of the inclination of the galaxies, especially for galaxies with low inclinations. It is very difficult to associate a confidence interval to the inclination from tilted-ring analysis. Oh et al. (2011) presented a comprehensive analysis of the mass distributions of seven dwarf galaxies, being especially careful on the effect of inclination on the rotation curves. Regarding Ho II, they claimed that "*all ring parameters are well determined...*" and argued that "*as can be seen from not only the tilted-ring analysis but also the comparison of rotation velocities in the Appendix it is unlikely that Ho II has an inclination (≈ 25°) as low as that inferred from the baryonic Tully-Fisher relation*". If so, Ho II would not satisfy the baryonic Tully-Fisher relation and would be problematic for MOND (Sánchez-Salcedo et al. 2013). Motivated by this result, Gentile et al. (2012) re-analyzed the H I data cube and found that the inclination may be much closer to face-on than previously derived (see also McGaugh 2011). It is clear that the main source of systematic uncertainties in the determination of the amplitude of the rotation curve in Ho II is its inclination.

An amplitude of the rotation curve lower than expected could be caused by an overestimate of its inclination angle in the sky. In this work, we provide some indirect but robust evidence, based on stability arguments, to suggest that the inclination of Ho II is smaller than the value inferred by tilted-ring analysis. We have derived the mean inclination of Ho II in order to place it above the stability threshold at the radius of ∼4 kpc, where the SFE is as low as 2×10^{-11} yr^{-1} (Leroy et al. 2008). We have shown that, under Newtonian gravity, this occurs for a mean inclination of ≈27°. This inclination puts Ho II back into the baryonic Tully-Fisher relation.

One could ask, what is the impact of this new inclination on the position of Ho II in other diagrams? We find that if $\langle i \rangle = 27°$, Ho II lies in the right location in the M_g vs $V_{\rm flat} R_d$ trend. We also find that the radial gravitational acceleration v_c^2/R at the radius where SFE= 10^{-10} yr^{-1} is too small in Ho II as compared to the values in other galaxies for inclination angles of ≈50°, but it is within the scatter when $\langle i \rangle = 27°$.

All the above arguments indicate that Ho II will be set back to normality provided that the mean inclination angle is ≈27°. This inclination is also consistent with the ellipticity of the isocontours of the total H I surface density map, in the outer parts of the galaxy (Gentile et al. 2012).

It is important to note that adopting this inclination, the rotation curve of Ho II is in agreement with MOND (Gentile et al. 2012; Sánchez-Salcedo et al. 2013). However, a similar stability analysis under MOND indicates that the gas Toomre parameter, calculated using the observed velocity dispersion, is close to 1 between $0.8\,R_{25}$ and $1.8\,R_{25}$ in galactocentric radius. To be satisfactory, MOND should explain why gravitational instabilities do not promote a burst of star formation or a grand-design spiral arm with widespread star formation at the outer parts of Ho II, i.e. between $0.8\,R_{25}$ and $1.8\,R_{25}$. Indeed, we were unable to find a possible connection between the degree of large-scale gravitational instability and the locations of star formation. MONDian numerical simulations are needed to understand why gas-rich dwarf galaxies, such as Ho II or IC 2574, appear unperturbed even when the level of self-gravity of their disks is so important.

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