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THREE-DIMENSIONAL RADIATIVE TRANSFER

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RESUMEN

Los efectos de la transferencia radiativa (RT) juegan un papel crucial en el historial térmico del medio intergaláctico. Aquí presento los avances recientes en el desarrollo de los métodos numéricos que incluyen RT el la hidrodinámica cosmológica. Estos métodos pueden ser también aplicados a problemas dependientes del tiempo en escalas interestelares y galácticas.

ABSTRACT

Radiative Transfer (RT) effects play a crucial role in the thermal history of the intergalactic medium. Here I discuss recent advances in the development of numerical methods that introduce RT to cosmological hydrodynamics. These methods can also readily be applied to time dependent problems on interstellar and galactic scales.

Key Words: **COSMOLOGY: THEORY — H II REGIONS — HYDRODYNAMICS — INTERGALACTIC MEDIUM**

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The physics of photoionized gases is of crucial importance and many astrophysical applications. As the hydrodynamic modeling of astronomical objects is advancing at a rapid pace we are also forced to consider the complex problem of three-dimensional radiative transfer of ionizing radiation. I report here on recent advances we have made in developing appropriate numerical techniques to achieve this goal. These methods are applicable for a wide range of problems. One particular issue that arises in the formation of the first generation of stars shall serve as motivation.

1. ONE MOTIVATION: STRUCTURE FORMATION AND THE BEGINNING OF THE BRIGHT AGES

The atomic nuclei in the primordial gas (mostly hydrogen and helium) first (re)combined with electrons at a redshift $z \sim 1000$. From the study of absorption spectra of high redshift quasars we know that this gas must have been ionized prior to $z = 5$. Most likely, this reionization process was caused by photoionization from UV radiation produced in protogalactic objects, either by massive stars or by accretion into compact objects. The formation of these first objects in the universe, and their potential impact on subsequent structure formation, is a key issue in physical cosmology. In our standard models of structure formation cosmological objects form via hierarchical build up from smaller pieces. The dynamics is controlled by gravity of the dominant cold dark matter (CDM) component. Baryons will fall into virializing CDM halos in which they may cool and possibly fragment to form stars. The lower limit on the masses of luminous objects that may be formed is determined by (1) the pressure of the primordial gas, which determines whether it can settle in the dark matter halo, and (2) the ability of baryons to cool and collapse to stellar densities.

These issues depend sensitively on the presence of UV photons (see Abel & Haehnelt 1999; Haiman, Abel, & Rees 2000, and references therein). Since the intergalactic medium (IGM) is initially optically thick to $h\nu > 13.6\,\mathrm{eV}$ photons, ionization fronts will be formed around the first sources. Because we believe that the radiation is produced in structures condensed from the IGM by gravitational instability, *the first UV photons will see a clumpy inhomogeneous IGM*. As a consequence the time-varying ionized regions will have complex morphologies.

The above physical processes have prompted us to develop methods for the treatment of RT in three-dimensional cosmological hydrodynamics. In the following we describe the conditions under which the cosmological RT becomes equivalent to classical RT. Then we go on to discuss some possible methods to solve the latter. In this contribution I will only give a brief overview of our work. However, note that Razoumov & Scott (1999) offer a different approach, and Gnedin (2000) chose to use dramatic simplifications constructed such as to mimic the expected effects of radiative transfer in order to study the reionization of intergalactic hydrogen.

2. COSMOLOGICAL RADIATIVE TRANSFER

The equation of cosmological radiative transfer in comoving coordinates (cosmological, not fluid) is:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \frac{\hat{n}\cdot\nabla I_\nu}{\bar{a}} - \frac{H(t)}{c}(\nu\frac{\partial I_\nu}{\partial \nu} - 3I_\nu) = \eta_\nu - \chi_\nu I_\nu \ , \tag{1}$$

where $I_\nu \equiv I(t, \vec{x}, \vec{\Omega}, \nu)$ is the monochromatic specific intensity of the radiation field, $\hat{n}$ is a unit vector along the direction of propagation of the ray; $H(t) \equiv \dot{a}/a$ is the (time-dependent) Hubble constant, and $\bar{a} \equiv \frac{1+z_{em}}{1+z}$ is the ratio of cosmic scale factors between photon emission at frequency ν and the present time t. The remaining variables have their traditional meanings (e.g., Mihalas 1978). Equation (1) can be recognized as the standard equation of radiative transfer with two modifications: the denominator $\bar{a}$ in the second term, which accounts for the changes in path length along the ray due to cosmic expansion, and the third term, which accounts for cosmological redshift and dilution.

One could, in principle, attempt to solve equation (1) directly for the intensity at every point given the emissivity η and absorption coefficient χ. However, the high dimensionality of the problem (three positions, two angles, one frequency and time = 7D!) not to mention the high spatial and angular resolution needed in cosmological simulations would make this approach impractical for dynamic computations. Therefore, we proceed through a sequence of well-motivated approximations which reduce the complexity to a tractable level.

2.1. *Local Quasi-Static Approximation*

We begin by eliminating the cosmological terms and factors. This can be understood on simple physical grounds. Before the universe is reionized, it is opaque to H and He Lyman continuum photons. Consequently, ionizing sources are local to scales of interest, and not at cosmological distances. In particular, this means that the multiplicative term $H(t)/c$, which is simply the reciprocal of the Hubble horizon at the time t, will ensure the cosmological term to be small as long as the opacity is much smaller than the horizon scale. If this is not the case, and we have a simulation box size much smaller than the mean free path, then we are at the limit where the cosmological terms will modify the boundary values but still not be important as the photons transverse the box. Therefore, setting $\bar{a} \equiv 1$, equation (1) reduces to its standard, non-cosmological form:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{n}\cdot\nabla I_\nu = \eta_\nu - \chi_\nu I_\nu \ , \tag{2}$$

where now ν is the instantaneous, comoving frequency.

Thinking of the special case of a point source that switches on instantaneously, one realizes that initially the ionization front propagates at the speed of light. It slows down as the "incoming flux" of neutrals grows with the increasing ionization surface. This ensures that the light crossing time (1/(c times I-front distance)) eventually becomes shorter than the timescales of change of the emissivities and absorption coefficient on the right hand side of equation (2). At this point an explicit integration can take long time steps, making the term $1/c\,\partial I_\nu/\partial t$ negligible. After this moment, it will suffice to solve the static classic equation of radiative transfer,

$$\hat{n}\cdot\nabla I_\nu = \eta_\nu - \chi_\nu I_\nu \ . \tag{3}$$

3. METHODS

3.1. *Brute Force: Ray Tracing*

Abel, Norman, & Madau (1999) give a method that integrates this quasi–static approximation along rays casted from point–sources. That method has the particular advantage that it will ensure photon conservation

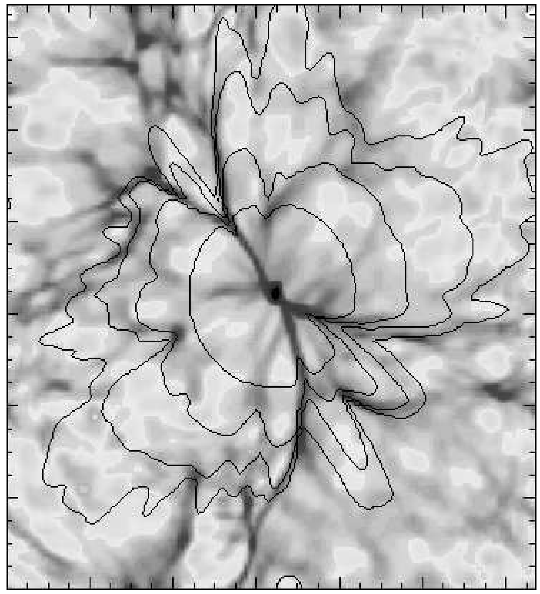

Fig. 1. Propagation of an R-type I-front in a 128^3 cosmological density field produced by a mini-quasar with $\dot{N} = 5 \times 10^{53}$ s^{-1}. The solid contours give the position of the I-front at 0.15, 0.25, 0.38, and 0.57 Myr after the quasar has switched on. The underlying grey-scale image indicates the initial H I density field.

independent of resolution by exploiting the known analytic solution of radiative transfer for a homogeneous slab. Consider the simple case of pure absorption: across a computational cell, where we assume the density of absorbing material to be constant, the outgoing photon number flux is simply $e^{-\tau}$ times the incoming one. Hence, the number of absorbed photons must be $(1-e^{-\tau})$ times the incoming flux. So, one can compute the number of photoionizations per second by adding all these $(1-e^{-\tau})$ terms for the rays that pass through this cell. Now, by definition, the number of photoionizations is equal to the number of photons absorbed. As a consequence, this method always propagates ionization fronts at the correct speed *independent* of resolution. This is a highly desirable feature of any method of multi-dimensional radiative transfer. This control of accuracy comes at high computational cost. In this method, the $1/r^2$ drop in the photon flux in an optically thin region around the source is captured by the simple fact that many more rays traverse through cells near the source than cells further away. Obviously, a large amount of computational time is wasted in computing the flux of such an optically thin cell, where it would simply be given by $I(0)/r^2$. This can be solved as discussed below. Nevertheless, this method can be used for a variety of realistic cases. This can be seen from Figures 1 and 2. Both of them employed the ray-tracing of Abel, Norman, & Madau (1999), and are shown as illustrations of the practicality of this method.

3.2. *Ionization Front Tracking*

Let us point out a simple way of solving a specific problem. If one is interested in the propagation of an R-type ionization front in a static medium it suffices to integrate the jump condition

$$n_{HI}(r)\frac{dR}{dt} = \frac{F(0)}{4\pi R^2} - \int_0^R \alpha(T(r)) n_{HII}(r)\, n_e(r) dr \ , \tag{4}$$

where R, α, and $F(0)$ denote the radius of the I-front, the recombination-rate coefficient, and the ionizing photon rate, respectively. Dividing by n_{HI} we can integrate equation (4) explicitly along rays[1] and find the time at which the ionization fronts arrives at a given cell. Storing the arrival time in a 3D array allows one to

[1] Where one uses the raytracing technique of Abel et al. (1999).

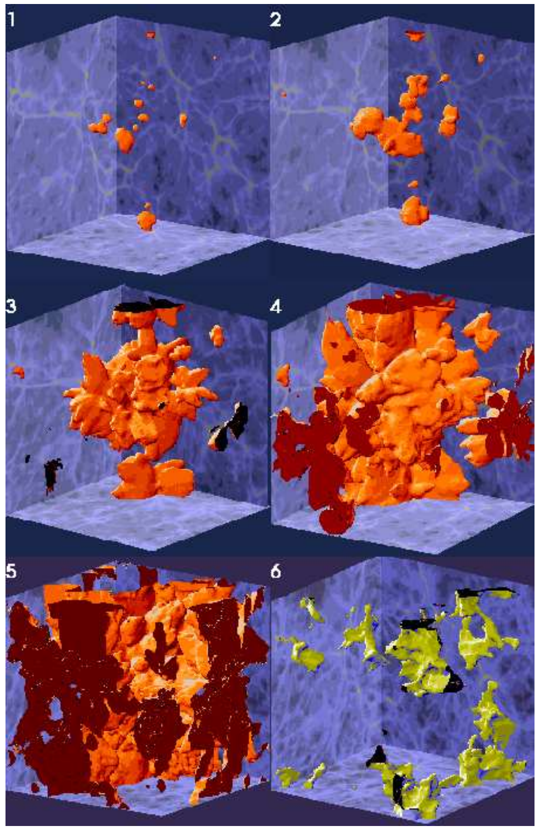

Fig. 2. Visualization of a time series of a 3D radiative transfer simulation. Here, we assume 30 sources with an ionizing luminosity proportional to their halo mass that switch on simultaneously at redshift 7. The density distribution is taken from a standard cold dark matter Lyman alpha forest simulation in a periodic box of 2.4 comoving Mpc carried out by Bryan et al. (1999). Hence, the box linear size corresponds to 300 kpc. Six different panels are shown. Each slice at the cube edges illustrates the log of the hydrogen density. The iso-surfaces visualize the boundaries of ionized regions. The last panel, number six, illustrates the remaining neutral islands that have not been ionized yet.

investigate the time dependent morphology of the ionization front by simply taking iso-surfaces on this array of arrival times. Such data is also interesting for computing how many ionizing photons are used to ionize a given volume in a static case, etc. To get the full time evolution of the ionization front of one source on a 128^3 numerical grid requires a one minute computation (wall clock) on a workstation.

3.3. *Computer Graphics*

In many fields, e.g., biomedical imaging, interactive volume rendering of 3-dimensional data is highly desirable. A lot of effort went into designing fast algorithms that yield optical depths from a light source to the observer's eye. Not every such method will be suitable for application in astrophysics. In particular, one needs to worry about exact photon (energy) conservation. However, imagine one has a method that gives the optical depth to a source at every point in the computational volume. For the case of pure attenuation one then also knows the photon number flux (photons per second per area) everywhere from

$$\vec{F}(\vec{r}) = \frac{F_{source}(0)}{|\vec{r}|^2}\, e^{-\tau(\vec{r})} \frac{\vec{r}}{|\vec{r}|}, \tag{5}$$

where $\vec{r}$ denotes the vector from the source to the point of interest. Now from the obvious "discontinuity equation"

$$\nabla \cdot \vec{F}(\vec{r}) - \dot{n_{HI}} = 0 \ , \tag{6}$$

one can ensure that the number of photoionizations $\dot{n_{HI}}$ is computed self-consistently, independently of resolution. Such a method has been implemented and tested by Abel & Welling (2000), and found to give speeds in excess of a factor hundred as compared to Abel et al. (1999) in the limit of large ionized regions.

3.4. *Moment Methods*

The methods presented above focus on the correct implementation of radiative transfer for point sources. However, ideally we also want to be able to treat regions of diffuse emission as those arising due to, e.g., bremsstrahlung and recombination radiation. In Norman, Paschos, & Abel (1998) we have outlined a possible approach to treat point sources and diffuse radiation by means of a variable Eddington tensor formalism. Although we have improved significantly on some ingredients of this method (e.g., deriving an analytic expression for the Eddington tensors in the pre-overlap stage) we have not succeeded as yet in constructing a stable implementation.

4. CONCLUDING REMARKS

For the applications to numerical cosmology some of the methods of 3D radiative transfer discussed above will have to be combined. I-front tracking is useful to initialize the environment of new sources. The methods drawn from Computer Graphics can be used to compute accurate boundary conditions for the moment methods that are the most promising in the limit of many sources. A number of interesting problems still need to be solved before cosmological radiation hydrodynamics can become a standard tool for the study of the formation and evolution of structure in the universe. However, the existing techniques should be employed for the study of interstellar problems in which only a few sources are of interest. Planetary nebulae and H II regions are ideal candidates for such three-dimensional radiation magnetohydrodynamic modeling.

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