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THE EQUILIBRIUM STRUCTURE OF COSMOLOGICAL HALOS: FROM DWARF GALAXIES TO X–RAY CLUSTERS

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RESUMEN

Se describe un nuevo modelo para la estructura de equilibrio post–colapso de objetos virializados que se condensan del fondo cosmológico del universo y se compara con observaciones y simulaciones de halos cosmológicos. El modelo se basa en la suposición que los halos virializados son isotérmicos, lo que lleva a la predicción de una esfera isotérmica única no–singular para la estructura de equilibiro con densidad en el núcleo proporcional a la densidad media del fondo en la época del colapso. Las predicciones de estas esferas isotérmicas están de acuerdo con las observaciones de la estructura interna de halos dominados por materia oscura desde las galaxias enanas hasta los cúmulos emisores de rayos X. Nuestro modelo también reproduce con buena exactitud muchas de las propiedades promedio de halos en simulaciones de MOF, lo que sugiere que es una aproximacion analítica útil para halos con condiciones iniciales realistas. En tanto que las simulaciones de N–cuerpos encuentran perfiles con una cúspide central, nuestro modelo no–singular se ajusta a los halos simulados fuera del pozo de la región más interior. Este modelo puede tambien ser de interés como una descripción de halos en modelos MOF no estándar como los de materia oscura auto–interactuante, los cuales han sido propuestos para eliminar la discrepancia entre los halos con cúspide de las simulaciones MOF y los halos observados con núcleos de densidad uniforme

ABSTRACT

A new model for the postcollapse equilibrium structure of virialized objects which condense out of the cosmological background universe is described and compared with observations and simulations of cosmological halos. The model is based upon the assumption that virialized halos are isothermal, which leads to a prediction of a unique nonsingular isothermal sphere for the equilibrium structure, with a core density which is proportional to the mean background density at the epoch of collapse. These predicted nonsingular isothermal spheres are in good agreement with observations of the internal structure of dark–matter–dominated halos from dwarf galaxies to X–ray clusters. Our model also reproduces many of the average properties of halos in CDM simulations to good accuracy, suggesting that it is a useful analytical approximation for halos which form from realistic initial conditions. While N–body simulations find profiles with a central cusp, our nonsingular model matches the simulated halos outside the innermost region well. This model may also be of interest as a description of halos in nonstandard CDM models like self–interacting dark matter, which have been proposed to eliminate the discrepancy between the cuspy halos of standard CDM simulations and observed halos with uniform–density cores.

Key Words: **COSMOLOGY: THEORY — DARK MATTER — GALAXIES: CLUSTERS: GENERAL — GALAXIES: FORMATION — GALAXIES: HALOS**

1. INTRODUCTION

The question of what equilibrium structure results when a density perturbation collapses out of the expanding background universe and virializes is central to the theory of galaxy formation. The nonlinear outcome of the growth of Gaussian–random–noise cosmological density fluctuations due to gravitational instability in a hierarchical clustering model like CDM is not amenable to direct analytical solution, however. Instead, numerical simulations are required. As a guide to understanding these simulations, as a check on their accuracy, and as a means of extrapolating from simulation results of limited dynamic range, analytical approximations are nevertheless an essential tool. One such tool of great utility has been the solution of the spherical top–hat perturbation problem (cf. Gunn & Gott 1972, Padmanabhan 1993). As used in the Press–Schechter ("PS") approximation (Press & Schechter 1974) and its various refinements, the top–hat model serves to predict well the number density of virialized halos of different mass which form at different epochs in N–body simulations. An analytical model for the internal structure (e.g. mass profile, temperature, velocity dispersion, radius) of these virialized halos would be a further tool of great value for the semi–analytical modelling of galaxy and cluster formation, therefore. Here we shall summarize our attempt along these lines.

Earlier work adopted crude approximations which used the virial theorem to match a collapsing top–hat perturbation either to a uniform sphere or to a singular isothermal sphere, with the same total energy as the top–hat. Our first motivation, therefore, is simply to improve upon this earlier treatment by finding a more realistic outcome for the top–hat problem. As a starting point, we shall adopt the assumption that the final equilibrium is spherical, isotropic, and isothermal, a reasonable first approximation to the N–body and gasdynamic simulation results of the CDM model. As we shall see, the postcollapse analytical solution derived from this assumption quantitatively reproduces many of the detailed properties of the halos found in those simulations, so we are encouraged to believe that our approximation is well justified. Our model is in disagreement, however, with the N–body simulation result that, in their very centers, dark–matter–dominated halos have cuspy profiles (e.g. Navarro, Frenk, & White 1997; "NFW"). By contrast, our model predicts a small, but uniform density core, as required to explain the observed dwarf galaxy rotation curves and cluster mass profiles inferred from gravitational lensing. This discrepancy between the cuspy profiles of the N–body results and the observed dark–matter–dominated halos has led recently to a reexamination of the cold, collisionless nature of CDM, itself, and the suggestion that a variation of the microphysical properties of the dark matter might make it more "collisional", enabling it to relax dynamically inside these halos so as to eliminate the central cusp (e.g. Spergel & Steinhardt 2000). While the details of this suggestion are still uncertain, our model serves to predict its consequences, to the extent that we are able to ignore the details of the relaxation process inside the halo and approximate the final equilibrium as isothermal. In what follows, we shall describe our model and compare its predictions both with CDM simulation results and with observations of dwarf galaxy rotation curves and galaxy clusters.

2. THE TRUNCATED ISOTHERMAL SPHERE MODEL

Our model, as described in Shapiro, Iliev, & Raga (1999) for an Einstein–de Sitter universe and generalized to a low–density universe, either matter–dominated or a flat one with a positive cosmological constant, in Iliev & Shapiro (2001, in preparation), is as follows: An initial top–hat density perturbation collapses and virializes, which leads to a truncated nonsingular isothermal sphere in hydrostatic equilibrium (TIS), a solution of the Lane–Emden equation (appropriately modified in the $\Lambda \neq 0$ case). Although the mass and total energy of the top–hat are conserved through collapse and virialization, and the postcollapse temperature is set by the virial theorem (including the effect of a finite boundary pressure), the solution is not uniquely determined by these requirements alone. In order to find a unique solution, some additional information is required. We adopt the anzatz that the solution selected by nature will be the "minimum energy solution" such that the boundary pressure is that for which the conserved top–hat energy is the minimum possible for an isothermal sphere of fixed mass within a finite truncation radius. As a check, we appeal to the details of the exact, self–similar, spherical, cosmological infall solution of Bertschinger (1985). In this solution, an initial overdensity causes a continuous sequence of spherical shells of cold matter, both pressure–free dark matter and baryonic fluid, centered on the overdensity, to slow their expansion, turn–around and recollapse. The baryonic infall is halted by a strong accretion shock while density caustics form in the collisionless dark matter, instead, due to shell

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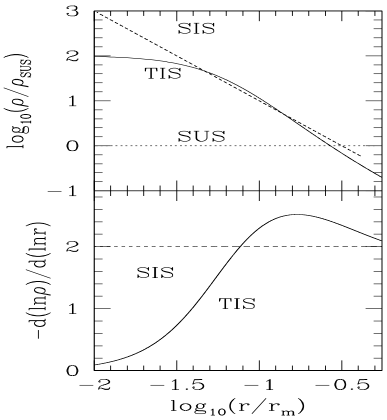

Table 1

	SUS	SIS	TIS ($\Omega = 1$)
$\eta = \frac{r_t}{r_m}$.....	0.5	0.417	0.554
$\dfrac{k_B T_{\rm vir}}{\left(\frac{2}{5}\frac{GMm}{r_{\rm vir}}\right)}$...	1	3	2.16
ρ_0/ρ_t..........	1	∞	514
$\langle\rho\rangle/\rho_t$..........	1	3	3.73
r_t/r_0.............	– NA –	∞	29.4
$\dfrac{\langle\rho\rangle}{\rho_{crit}(t_{\rm coll})}$.....	$18\pi^2$	$18\pi^2\left(\dfrac{6}{5}\right)^3$	130.5

Fig. 1. Density profile of truncated isothermal sphere which forms from the virialization of a top–hat density perturbation in a matter–dominated universe. Radius r is in units of r_m - the top–hat radius at maximum expansion, while density ρ is in terms of the density ρ_{SUS} of the standard uniform sphere approximation for the virialized, post–collapse top–hat. Bottom panel shows logarithmic slope of density profile.

crossing. The postcollapse virialized object we wish to model is then identified with that particular shock– and caustic–bounded sphere in this infall solution for which the mass and total energy are the same as those of our top–hat before collapse and the trajectory of its outermost mass shell was identical to that of the outer boundary of our collapsing top–hat at all times until it encountered the shock. This spherical region of post-shock gas and shell–crossing dark matter in the infall solution is very close to hydrostatic and isothermal and has virtually the same radius as that of the minimum energy solution for the TIS. This confirms our "minimum energy" anzatz and explains the dynamical origin of the boundary pressure implied by that solution as that which results from thermalizing the kinetic energy of infall.

With this "minimum energy" anzatz, we find that a top–hat perturbation collapse leads to a unique, **nonsingular** TIS, which yields a universal, self–similar density profile for the postcollapse equilibrium of cosmic structure. Our solution has a unique length scale and amplitude set by the top–hat mass and collapse epoch, with a density proportional to the background density at that epoch. The density profiles for gas and dark matter are assumed to be the same (no bias). The final virialized halo has a flat density core.

Case I: matter–dominated cases, both flat and low density (see Figure 1). The core size is $r_0 = 0.034 \times$ radius r_t, where r_t is the size of the halo (i.e. truncation radius). The central density is $\rho_0 = 514 \times$ surface density ρ_t. The 1D velocity dispersion σ_V of the dark matter and the gas temperature T are then given by $\sigma_V^2 = k_B T/(\mu m_p) = 4\pi G\rho_0 r_0^2$. [Note: this $r_0 \equiv r_{0,\rm King}/3$, where $r_{0,\rm King}$ is the core radius defined as the "King radius" by Binney & Tremaine (1987, eq. [4-124b])]. Compared to the standard uniform sphere (SUS) and singular isothermal sphere (SIS) approximations (see Table 1), the temperature is $T = 2.16\ T_{\rm SUS} = 0.72\ T_{\rm SIS}$. At intermediate radii, ρ drops faster than r^{-2}.

Case II: flat, $\Lambda \neq 0$ models. The profile varies with epoch of collapse, approaching the universal shape of case I above for early collapse. For example, for $\Omega_0 = 1 - \lambda_0 = 0.3$ and $z_{\rm coll} = (0; 0.5; 1)$, we obtain $r_t/r_0 = (30.04; 29.68; 29.54)$, $\rho_0/\rho_t = (529.9; 520.8; 517.2)$, and $T/T_{\rm SUS} = (2.188; 2.170; 2.163)$, respectively.

3. DWARF GALAXY ROTATION CURVES AND THE $V_{MAX} - R_{MAX}$ CORRELATIONS

The TIS profile matches the observed mass profiles of dark-matter-dominated dwarf galaxies. The observed rotation curves of dwarf galaxies can be fit according to the following density profile with a finite density core (Burkert 1995):

$$\rho(r) = \frac{\rho_{0,Burkert}}{(r/r_c + 1)(r^2/r_c^2 + 1)}. \qquad (1)$$

The TIS profile gives a nearly perfect fit to the Burkert profile, with best–fit parameters $\rho_{0,Burkert}/\rho_{0,TIS} = 1.216$, $r_c/r_{0,TIS} = 3.134$ (see Fig. 2a).

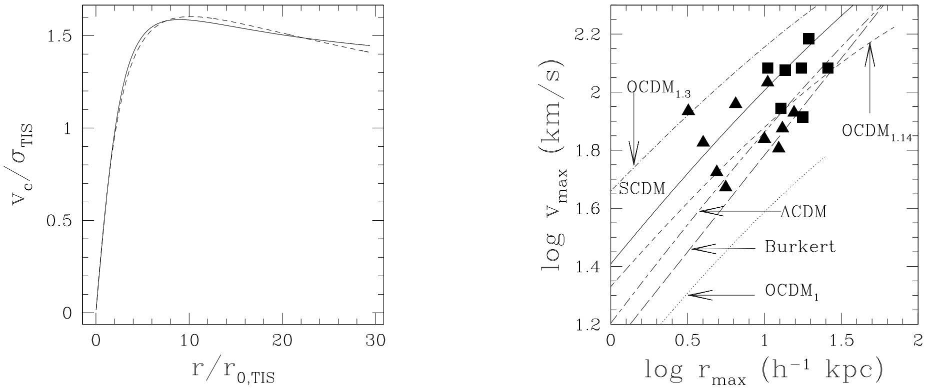

Fig. 2. (a) (left) Rotation Curve Fit. Solid line = Best–fit TIS; Dashed line = Burkert profile, where $\sigma_{TIS}^2 = \langle v^2 \rangle/3 = k_B T/m$. (b) (right) $v_{\max}$–$r_{\max}$ correlations. Observed dwarf galaxies (triangles) and LSB galaxies (squares) from Kravtsov et al. (1998); Burkert: fit to data (Burkert 1995); SCDM: $\Omega_0 = 1$, $\lambda_0 = 0$, $\sigma_{8h^{-1}} = 0.5$ (cluster-normalized); OCDM: $\Omega_0 = 0.3$, $\lambda_0 = 0$ (all COBE–normalized, subscript indicates n-value of tilt); ΛCDM: $\Omega_0 = 0.3$, $\lambda_0 = 0.7$, (COBE–normalized; no tilt); $h = 0.65$ for all.

How well does this best–fit TIS profile predict v_{max}, the maximum rotation velocity, and the radius , $r_{\max}$, at which it occurs in the Burkert profile? We find $r_{max,Burkert}/r_{max,TIS} = 1.13$, $v_{max,Burkert}/v_{max,TIS} = 1.01$ (i.e. excellent agreement).

The TIS halo model explains the observed correlation of v_{max} and r_{max} for dwarf spiral and LSB galaxies, when the TIS halo model is combined with the Press–Schechter model to predict the typical collapse epoch for objects of a given mass (i.e. the mass of the 1-σ fluctuations vs. z_{coll}) (see Figure 2b). Both of the flat, untilted CDM models plotted, cluster–normalized Einstein–de Sitter and COBE–normalized, flat, low–density models ($\Omega_0 = 0.3$ and $\lambda_0 = 0.7$), as well as the slightly tilted ($n = 1.14$) open model ($\Omega_0 = 0.3$) yield a reasonable agreement with the observed $v_{max} - r_{max}$ relation, while the untilted ($n = 1$) and strongly tilted ($n = 1.3$) open models do not agree with the data.

4. GALAXY HALO $M - \sigma_V$ RELATION

Our TIS halo model predicts the velocity dispersion of galactic halos of different mass which form in the CDM model according to N–body simulations. Antonuccio–Delogu, Becciani, & Pagliaro (1999) used an N–body treecode at high resolution (256^3 particles) to simulate galactic halos in regions of a single and of a double cluster. They found that the agreement with the TIS model is quite good, much better than with either of the other two models they considered, namely the singular isothermal sphere and the peak-patch model of Bond & Myers (1996).

5. COMPARISONS WITH GALAXY CLUSTER OBSERVATIONS AND SIMULATIONS

The TIS halo model predicts the internal structure of X–ray clusters found by gas–dynamical/N–body simulations of cluster formation in the CDM model. Our TIS model predictions agree astonishingly well with the mass–temperature and the radius–temperature virial relations and integrated mass profiles derived from numerical simulations by Evrard, Metzler & Navarro (1996; "EMN"). Apparently, these simulation results are not sensitive to the discrepancy between our prediction of a finite density core and the N–body predictions of a density cusp for clusters in CDM. Let X be the average overdensity inside the sphere of radius r, $X \equiv \langle \rho(r) \rangle / \rho_b$. Then the radius–temperature virial relation is defined as $r_X \equiv r_{10}(X)(T/10\,\mathrm{keV})^{1/2}$ Mpc, and the mass–temperature virial relation by $M_X \equiv M_{10}(X)(T/10\,\mathrm{keV})^{1/2}\, h^{-1} 10^{15}\, M_\odot$. A comparison between our predictions of the mass–temperature relation $r_{10}(X)$ and the results of EMN is given in Figure 3a. For the mass-temperature virial relation EMN obtain $M_{10}(500) = 1.11 \pm 0.16$ and $M_{10}(200) = 1.45$, while our TIS

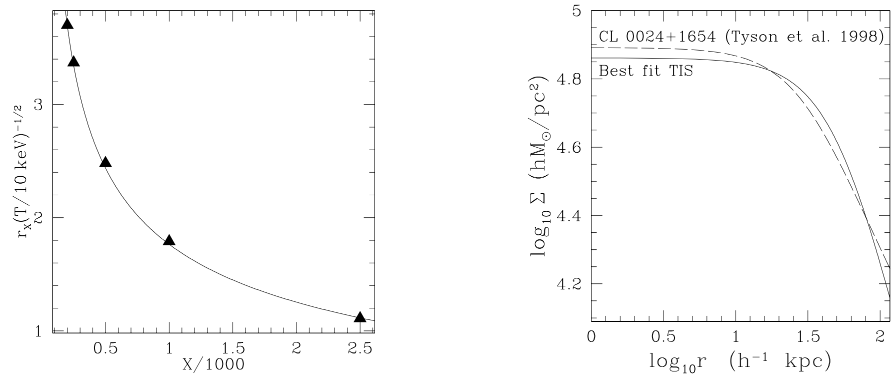

Fig. 3. (a) (left) Cluster radius–temperature virial relation ($z = 0$). (triangles) CDM simulation results as fit by Evrard et al. (1996); (continuous line) TIS prediction. (b) (right) Projected surface density of cluster CL 0024+1654 inferred from lensing measurements, together with the best–fit TIS model.

solution yields $M_{10}(500) = 1.11$ and $M_{10}(200) = 1.55$, respectively.

The TIS model for the internal structure of X-ray clusters predicts gas density profiles $\rho_{\rm gas}(r)$ and X–ray brightness profiles $I(\theta)$ which are well fit by the standard β–profile,

$$\rho_{gas} = \frac{\rho_0}{\left(1 + r^2/r_c^2\right)^{3\beta/2}}, \quad I = \frac{I_0}{\left(1 + \theta^2/\theta_c^2\right)^{3\beta - 1/2}}, \tag{2}$$

with β–values for the TIS β-fit which are quite close to those of simulated clusters in the CDM model but somewhat larger than the conventional observational result that $\beta \approx 2/3$ (see tables below). However, recent X–ray results suggest that the true β-values are larger than 2/3 when measurements at larger radii are used and when central cooling flows are excluded from the fit.

$I(r)$ **(observations)**	β
Jones & Foreman (1999)	0.4-0.8, ave. 0.6
Jones & Foreman (1992)	$\sim 2/3$
Balland & Blanchard (1997)	0.57 (Perseus)
	0.75 (Coma)
Durret et al. (2000)	0.53 (incl. cooling flow)
	0.82 (excl. cooling flow)
Vikhlinin, et al. (1999)	0.7-0.8
(fit by Henry 2000)	
TIS β-fit ($r_c/r_{0,TIS} = 2.639$)	0.904

$\rho_{\rm gas}(r)$ **(simulations)**	β
Metzler & Evrard (1997)	0.826 (DM)
	0.870 (gas)
Eke, Navarro & Frenk (1998)	0.82
Lewis et al. (1999) (adiabatic)	~ 1
Takizawa & Mineshige (1998)	~ 0.9
Navarro, Frenk & White (1995)	0.8
TIS β-fit ($r_c/r_{0,TIS} = 2.416$)	0.846

The TIS halo model can explain the mass profile with a flat density core measured by Tyson, Kochanski & Dell'Antonio (1998) for cluster CL 0024+1654 at $z = 0.39$, using the strong gravitational lensing of background galaxies by the cluster to infer the cluster mass distribution. The TIS model not only provides a good fit to the shape of the projected surface mass density distribution of this cluster within the arcs (see Figure 3b), but when we match the central value as well as the shape, our model predicts the overall mass, and a cluster velocity dispersion in close agreement with the value $\sigma_v = 1200$ km/s measured by Dressler & Gunn (1992). By contrast, the NFW fit which Broadhurst et al. (2000) reports can model the lensing data without a uniform-density core predicts σ_V much larger than observed.

6. SUMMARY

- The TIS profile fits dwarf galaxy rotation curves; combined with the Press-Schechter approximation, it predicts the observed $v_{max} - r_{max}$ relation for dwarf and LSB galaxies.

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- The TIS predicted $M - \sigma_V$ relation agrees with high resolution N–body simulations of galactic halo formation by Antonuccio–Delogu et al. (1999).
- The predicted mass–radius–temperature scaling relations and integrated mass profile of the TIS model match simulation results for clusters in the CDM model in detail. Our solution **derives** the empirical fitting formulae of Evrard et al. (1996), which also agree well with X–ray cluster observations at $z = 0$.
- The TIS X–ray brightness profile matches the β-fit profile with $\beta \approx 0.9$, larger than typically reported by X–ray observers, but very close to the results of gas–dynamical/N–body simulations of X–ray clusters in the CDM model.
- The TIS solution fits the cluster mass profile with uniform–density core derived from strong gravitational lensing data by Tyson et al. (1998) for CL 0024+1654 within the arcs, while accurately predicting the observed σ_V on larger scales, too.

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