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AXISYMMETRIC ROTATOR MODELS AS PULSARS: ABJECT FAILURE

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RESUMEN

El bien conocido modelo de Goldreich & Julian (1969) postuló que aún una estrella de neutrones en rotación con un momento de campo magnético dipolar alineado sería un pulsar activo. Aunque se sospechó desde 1980 que este modelo no podría funcionar como se propuso, hemos visto el resurgimiento de modelos que no perciben los problemas básicos subyacentes. A menos que esos modelos identifiquen explícitamente el vicio de la falla de los modelos GJ, no se les debe dedicar tanto esfuerzo teórico.

ABSTRACT

The well–known Goldreich & Julian (1969) model postulated that even a rotating neutron star with an aligned magnetic dipole moment would be an active pulsar. Although it was suspected as long ago as 1980 that this model could not work as posed, we have now seen a resurgence of models that do not realize the basic underlying problems. Unless such models can explicitly identify where the failure of the GJ model is vitiated, they may not be a useful place to devote so much theoretical effort.

Key Words: **MAGNETOHYDRODYAMICS — PLASMAS — PULSARS: GENERAL — STARS: NEUTRON**

1. INTRODUCTION

Although Goldreich & Julian (1969) are usually credited with calculating the "Goldreich and Julian charge density" around rotating conductors having an aligned magnetic dipole moment, this was apparently first estimated for stars in general by Davis, (1947; 1948). Later Hones & Bergeson (1965: HB) extended this treatment for the more general case of a non–aligned dipole moment, here applied to planetary magnetospheres. However, except for the parameter space (explicit values of magnetic moment, stellar radius, and rotation rate), these estimates are as valid for planets as for pulsars. Indeed, the application to, say, the Earth's magnetosphere is even more plausible than to pulsars, since the magnetosphere is known to be abundantly full of plasma to provide the (negligible) space–charge separation about the Earth: the Hones and Bergeson (HB) charge density. Accordingly, HB imposed the so–called exact MHD condition which assumes that the magnetic field lines are perfect conductors along these field lines and perfect insulators across them. Consequently, the magnetic field lines become electrical equipotentials, with the value of each potential being determined at the stellar/planetary surface. Given this potential, one can take the Laplacian to obtain the required charge separation. Since the magnetic field lines then effectively extend the same electric fields as inside the star, the magnetosphere should then rigidly corotate with the star. (In the case of the Earth's magnetosphere, the interaction with the external solar wind is the dominatant perturbation, which largely overwhelms the effects of the Earth's rotation.)

Goldreich and Julian proposed exactly the same electrodynamics, simplified to the aligned case, but to the case of neutron stars. Neutron stars, unlike planets, are not expected to maintain quasi–neutral plasma in their magnetospheres owing to the intense gravitational field. Nevertheless, they too assumed ideal MHD, which then required the neutron star to be surrounded by space charge (we try to make the distinction between charge–separation, which can always be obtained at any given point in space given sufficient *local* conduction charges, and space–charge, which can only be obtained by transport of charge from some source *elsewhere*). Indeed, GJ explicitly proposed that this space–charge comes from the neutron star surface (pulled off by the huge electric fields that would otherwise appear, as we will detail). They then appealed to the supposed rigid corotation of the plasma with the star to claim that there must be a "wind" beyond the distance at which the corotational velocity would exceed the speed of light. And thirdly, they assumed that the lost plasma would "have" to be replaced from the stellar surface.

2. HISTORICAL IMPACT

The impact on the radio pulsar field would be difficult to overstate.

2.1. *The Cartoon Model*

First, it elevated the magnetic polar caps to a position of preeminence: they were where the replacement particles "had" to be accelerated from. Consequently, popular textbooks now feature a sphere (neutron star) surrounded by dipole magnetic field lines with wiggly lines coming out of the magnetic poles (the radio emission). These dipoles are always *inclined* to give the "light–house" sweeping the beam to produce radio *pulses* as seen by the fixed observer, as shown in Figure 1. The completely plausible assumption that axisymmetry must be broken to get pulses is here muddled together with the completely unexamined assumption that tilting the axis "shouldn't make much difference" (i.e., if the aligned system is active, so should be the inclined system).

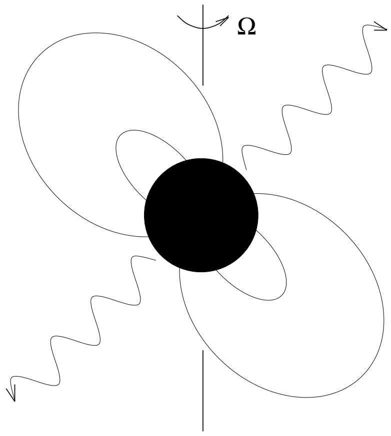

Fig. 1. Tilting the aligned rotator to get pulses. Wiggly lines represent the supposed location of radio emission at the magnetic polar caps. Thin lines represent closed magnetic field lines near the neutron star (black sphere).

2.2. *The Hollow Cone Model*

Additionally, this general picture has been taken up by phenomenologists who interpret details of pulse shapes (which may vary from a single spike to as many as five distinct "components"). On this basis the components become concentric circles of emission surrounding the polar caps. To account for some data, it is necessary to suppose that each such annulus requires additional variable levels of emission depending on the azimuth about the polar cap. It is difficult for models of such flexibility to fail, and they may paint a quite misleading picture of what is actually happening.

2.3. *Acceleration from the Magnetic Polar Caps*

From a theoretical point of view, the largest impact was to funnel theoretical effort into how particles might be accelerated up from the magnetic polar caps and how, in such a scenario, the coherent radio emission might be generated. Only a handful tried to "flesh out" the details of the global magnetospheric physics inspired by the GJ model.

3. NON–NEUTRAL PLASMAS

Ironically, the space–charge at any location surrounding the neutron star in this scenario would have to be of just one or the other charge. It would not be the quasi–neutral plasma familiar to plasma physics texts of the time (or even now!). It was about 5 years later that laboratory experiments began to be conducted with such plasmas (Malmberg & deGrassie 1977), although the basic device had been known decades earlier: the Penning trap. In a Penning trap, particles of *one* sign are trapped in a quadrupole electric field (approximated by two end caps at one voltage and a ring electrode of the opposite voltage). If a charged particle at the center is repelled by the end caps, it will simply move to the ring electrodes. To prevent this, a uniform magnetic field is applied. Now the particle will $\mathbf{E} \times \mathbf{B}$ drift around the symmetry axis. The particle trajectory can be rather complicated, and consequently the question of what would happen if one put a lot of particles in the trap was dismissed by describing the likely configuration as a "cloud" of particles. But in fact it is easy to see (Michel 1991a, § 4.2) that a *cold* system of trapped particles, all of the same charge, could form a *uniform sphere* at the center, which would be in rigid $\mathbf{E}\times\mathbf{B}$ rotation. By definition, the charge density would have to drop from the finite value inside the sphere to zero outside, and therefore there would be an abrupt discontinuity at the surface. Such a discontinuity is only found for non–neutral plasmas; ambipolar diffusion allows quasi–neutral particles to follow magnetic field lines (if this were to happen in the Penning trap, the "trap" would not even trap a single particle!).

4. ROTATING MAGNETIZED NEUTRON STAR AS A PARTICLE TRAP

To make this section as accessible as possible, we will minimize the details but show how they follow as a natural consequence of the physics.

4.1. *The GJ solution*

The GJ potential about the neutron star is given by

$$\Phi = \Phi_0 \frac{a}{r} \sin^2 \theta, \tag{1}$$

where Φ_0 depends on the spin rate, etc., and the exact value is only relevant insofar as whether it is huge compared to the surface work function (pulsar models) or tiny (rotating magnets in the laboratory). Here a is the stellar radius, and in fact this functional form describes dipolar magnetic field lines since that is what $\Phi =$ constant must give in ideal MHD. The Laplacian gives the (quadrupolar) HB charge density

$$\rho = \rho_0 \frac{1}{2}(3\cos^2\theta - 1), \tag{2}$$

with ρ_0 being what is often stated as "the" charge density. Since θ is the angle from the pole, we see that the charges are of one and the same sign over each polar cap, and opposite in the equatorial regions. This distribution is shown schematically in Figure 2.

As advertised, we show negative particles over each pole (a common assumption if particles from the polar caps are to be the ones radiating radio waves), and positive particles in the equatorial zone. We omit depicting the dipolar magnetic field lines. For simplicity we model the magnetic field as being dipolar all the way to the center of the star. More general cases are straight–forward, but unnecessarily complicate this discussion.

Note, that there is a *large* positive charge symbol at the center of the star. This is a huge intrinsic charge that is *required* by the model. If one averages eq. 1 over a spherical surface, it averages to a non–zero $1/r$, which is a monopole moment, the central charge. It is easy to forget this charge, **but it has to be there**. And it is crucial to understanding how the system "works". The dashed circle could be taken as the surface of the star. As such, we see that the GJ distribution does double duty. It ensures that the field lines are equipotentials *inside* the star. This is essential, otherwise currents would flow inside the conducting star to create exactly the required charge distribution. They also perform the less obvious service of requiring the same distribution (now as space–charge) in what otherwise would have been the insulating vacuum!

It is convenient also to imagine the dashed circle as being at an arbitrary position, because it emphasizes how the electric forces are assembled to give equipotential magnetic field lines. Inside the dashed circle, we

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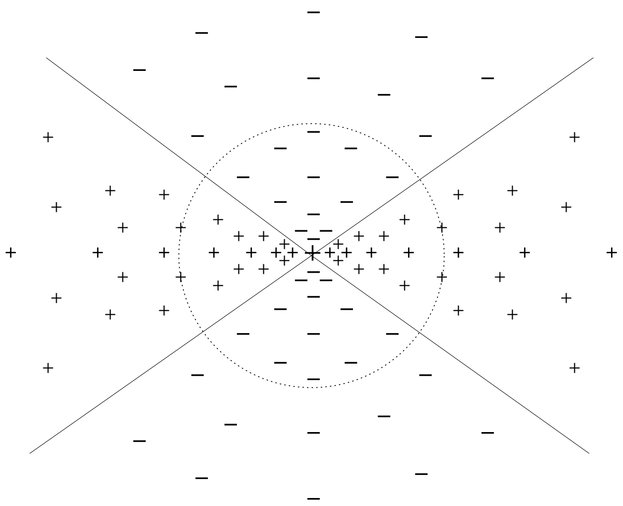

Fig. 2. The Hones & Bergeson charge distribution for the aligned case adopted by Goldreich and Julian. This quadrupolar distribution is one sign over the poles (here negative) and the other in the equatorial belt, both with densities dropping off as $1/r^3$ with radial distance. Please notice the central charge (larger plus sign).

have the central charge and the quadrupolar charge distribution. These would give at and beyond the circle a monopole $(1/r)$ potential and external quadrupole $(1/r^3)$ potential. The charges outside the circle provide an internal quadrupole (r^2) potential. The resultant fields all conspire to give $\mathbf{E} \cdot \mathbf{B} = 0$ on the dashed circle. If we did not have all three terms:

1. the central charge $(1/r)$,
2. the external quadrupole moment from charges inside $(1/r^3)$, and
3. the internal quadrupole moment from charges outside (r^2),

in exactly the correct proportions, we would not have $\mathbf{E} \cdot \mathbf{B} = 0$ *anywhere*! This analysis might seem a convoluted way of simply describing the potential in eq. 1, but it will greatly simplify the discussion of other charge distributions that, unlike the GJ model itself, actually satisfy the GJ assumption that the particles all originate from the surface. Note that the quadrupole moment strengths depend on what radius (R) the dashed line is at, so the external $(1/r^3)$ quadrupole potential does not actually drop faster than the monopole as R is increased because more space–charge is also encompassed by increasing R. In the same way, the internal monopole (r^2) does not increase faster than the monopole because less space–charge is encompassed. They all scale together, regardless of distance.

As a final aside, note that if we plotted isodensity contours outside the star, the space–charge would be found in nested polar domes and equatorial tori about the star, a topology that we will repeatedly see.

5. THE LABORATORY LIMIT

For pulsars, the work function is expected to be negligible compared to Φ_0, so the simple boundary condition is that zero (i.e., negligible compared to the internal charge–separation and surrounding space-charge) surface charge will be on the stellar surface. In the opposite laboratory limit, it is difficult to spin a spherical magnet of sufficient size and field strength to even approach the work function and we have the case where *all* of what would have been the space charge is confined to the surface. This leads to the charge configuration schematically shown in Figure 3, where the quadrupolar space–charge has now been confined to be a quadrupole surface charge.

It should be clear that insofar as the conducting star is concerned, the internal volume remains $\mathbf{E} \cdot \mathbf{B} = 0$ provided that that the quadrupolar surface charge is just such as to provide the correct internal quadrupole (the central charge and internal quadrupolar charge–separation being unchanged). Outside the star we have complete "violation" of ideal MHD: there is no plasma! But there can be no doubt that Figure 3 is indeed the correct result. One might quibble that this system has a net positive charge since the quadrupole distributions

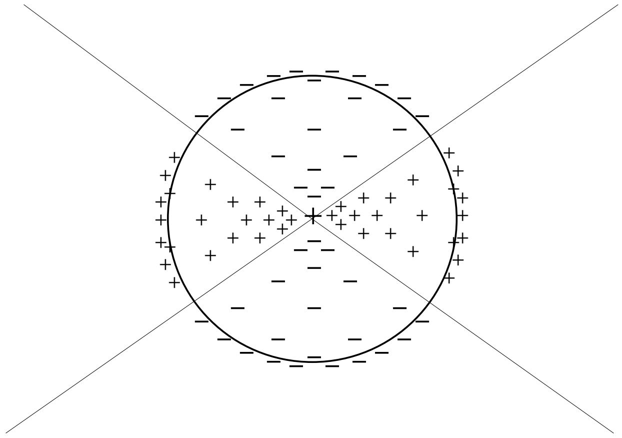

Fig. 3. The surface charge distribution for an aligned rotating conducting sphere in the laboratory, with a magnetic dipole at the center. The interior is exactly the same as for Figure 2 and the surface charge has to produce exactly the same internal quadrupole moment as the extended space–charge in the GJ "solution".

are charge neutral. But all one has to do is add an additional uniform surface charge to neutralize the central charge; this produces no internal electric fields so ideal MHD continues to hold in the interior, while ideal MHD is still violated outside. Indeed, that is the source of the electric fields that the GJ model invokes to pull the surface charge off the star in the first place! But we can consider this surface–charge solution to be an initial condition prior to the charges being pulled off.

6. THE ACTUAL RESULT

The above two limiting charge configurations (GJ and laboratory limit) are simple and each can be written out in explicit analytic form. Were it not for the tunnel vision determined to produce a pulsar model, the result of releasing space–charge from the surface of the laboratory solution would have intuitively led to the case shown in Figure 4.

This has to be the result starting from Figure 3 because the positive particles in the equatorial zone have to simply follow the hugely strong magnetic field lines as they depart the star. Consequently, they can only move a short distance from the star. It should additionally be clear that if the positive charges cannot escape, loss of negative particles must be limited; we cannot endlessly charge the system positive. In fact, the negative charges cannot escape the system anyway; their self–repulsion (evidenced by the $\mathbf{E} \cdot \mathbf{B} \neq 0$ outside the star in the Laboratory case above) indeed acts to carry them off the stellar surface, but the central positive charge will dominate at large distances (unlike the GJ case, where the quadrupolar charge distribution extends to "infinity" and cannot be gotten past). Consequently, there is a trapping point along the polar magnetic field line where charges can accumulate. If we were to just slightly exceed the work function, only a few charges would be trapped over the magnetic polar caps, and these charges would repel the remaining ones on the surface and help the work function maintain them. For an actual pulsar, since the work function is generally assumed to approach zero, all of the particles will leave the surface and completely fill the trapping region from the surface outwards. However, they do not move out to arbitrarily large distances since again at some distance one will be outside of the quadrupolar dome and torus space–charge and the $(1/r^3)$ fall off will always allow the attractive central charge to win out. So the trapped domes will only extend to this distance. However, unlike the first two solutions, there is no analytic solution for this free boundary problem (i.e., where exactly the surfaces of discontinuity will be): this case can only be treated using numerical simulations, as we have done in (Smith, Michel & Thacker 2000).

If we return to the electrostatics involved, we see that for the stellar interior to satisfy ideal MHD, the external charges have to provide exactly the same internal quadrupole moment as did the external charges in the GJ solution or the surface charges in the Lab model. Not only that, but the distribution must not produce any other multipoles! This might seem to be an impossible condition, but it can be shown explicitly to hold.

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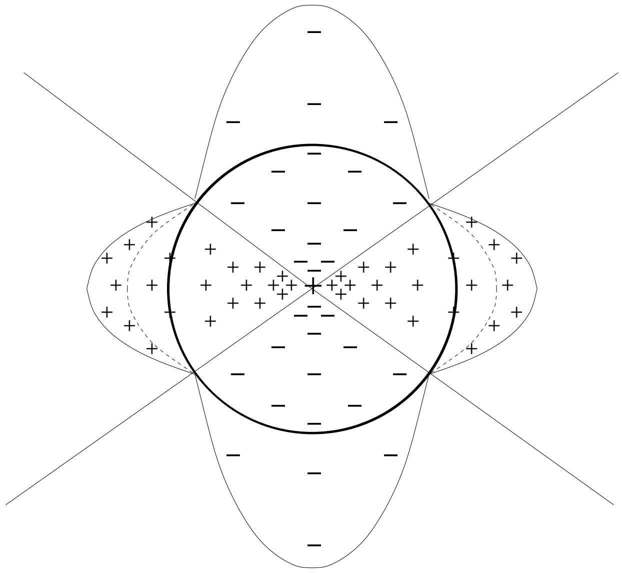

Fig. 4. The space-charge distribution for particles released from an aligned rotating conducting sphere, with a magnetic dipole at the center. The interior is again exactly the same as for Figure 2 and the finitely extended space–charge here has to produce exactly the same internal quadrupole moment as the infinitely extended space–charge in GJ.

However, the requirement that the same internal quadrupole be obtained produced an interesting complication to the distribution in Figure 4. First, the (polar) domes are clearly threaded by magnetic field lines from the star. As such, they must be (from ideal MHD) equipotentials and they must then have exactly the same quadrupolar space–charge density as for the GJ case (i.e., eq. 2). The only difference is that their extent is terminated by the discontinuity to the vacuum and they are literally formed into domes. Second, the equatorial torus looks like it too would just be the same quadrupolar space–charge, but terminated by the discontinuity surface. But taking the GJ space–charge, which extends to infinity, and simply truncating it to form domes and a torus, would *have* to reduce the quadrupole moment, and we would no longer have a solution that gave ideal MHD inside the star. And we cannot simply scale up ρ_0 because that is determined by the *stellar* surface potential. Or putting it another way, the charge–separation density just below the surface has to equal the space–charge density just above the surface.

6.1. *System Charge*

We can have a range of system charges and therefore a continuous set of solutions with domes and tori depending on what the overall system charge is. If the system charge is reduced towards zero, it follows that the domes are less confined by the monopole moment and can extend to higher altitudes over the magnetic poles. In the limit of zero system charge, the domes would extend to “infinity.” But loss of the particles would lead to an increase in the system charge, and the dome would shrink to below the loss region.

7. RELEVANCE TO PULSAR RESEARCH

Twenty years ago, the configuration in Figure 4 was shown by Michel (1980) and five years later it was numerically demonstrated (Krause-Polstorff & Michel 1985a, b). Recently, we have decided to extend this analysis to the inclined case as well, as had already been pioneered by Thielheim & Wolfsteller (1994). In so doing, we have repeated the 1985 numerical calculations (Smith, Michel & Thacker 2000) and performed a number of additional simulations. We now calculate the multipole moments formed by the external space–charge and numerically confirm that only the quadrupole is non–zero and that it has exactly the correct magnitude.

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Let us recall the GJ assumptions:
1. space–charge in the magnetosphere is from particles pulled from the surface,
2. the charges follow magnetic field lines,
3. ideal MHD everywhere (even vacuum!),
4. space–charge beyond the "light–cylinder" is lost as a wind,
5. the lost space–charge "has" to be replaced from the surface.

The present work makes no quibble with the first two assumptions, but the third is simply inconsistent with the first two. It is impossible to fill the entire magnetosphere from the surface without adding in some other (unknown) piece of physics. Once this chain of assumptions is broken, the remaining ones fail totally. The fourth becomes irrelevant because there will now be no plasma to extend to the light–cylinder (thousands of neutron radii away for typical pulsars). The fifth we have been able to simulate by simply creating a GJ charge distribution and then truncating it as if it had somehow been depleted. Hopefully the reader can by now appreciate that truncating the GJ solution simply removes the internal quadrupole moment provided by those particles. Suddenly the entire system is no longer in force balance, but rather than emit particles it collapses into the domes and torus configuration! These results are reported in Smith, Michel & Thacker (2000).

8. DISCUSSION

When discussing these results with other researchers, it becomes clear that some hold the view that the KPM simulations involve an "alternative" or "controversial" model. Evidently this is due to a gradual back–sliding into thinking that the GJ model is still a viable one. But ours is not an "alternative" model, it is exactly the model GJ proposed but correctly solved by not imposing ideal MHD everywhere at the start. The resultant domes and torus model satisfies ideal MHD *wherever there is space–charge*. Ideal MHD is inapplicable to the vacuum. Ideal MHD is meaningless for the Penning trap. And the supposed "controversy" is non–existent. These solutions have been examined by others (Zachariades 1993, Neukirch 1993, Thielheim & Wolfsteller 1994) and confirmed (see also Rylov 1976). No one has published any argument claiming to find fault with these results. Yet recent publications have dealt with (1) acceleration from the polar caps, and (2) aligned rotators with accelerating gaps (dimensions chosen arbitrarily). Contopoulos, Kazanas & Fendt 1999, have numerically solved the full GJ model (GJ only discuss the near magnetosphere and speculate on what happens at light–cylinder distances), but drop the assumption that particles are available only from the surface. Instead they assume (effectively) that particles can appear out of the vacuum wherever needed to maintain ideal MHD. In effect they succeeded in completing the solutions attempted by Pelizzari (1976) and reported in (Michel 1991a). What physics this formal solution might correspond to is quite unclear (i.e., where does the space–charge come from).

9. OUTLOOK

Since radio pulsars do function and presumably are rotating magnetized neutron stars, it is a fair question to ask what the simplest viable model might be. Clearly the simplest possible model is the aligned rotator. But what can be done to break the conundrum that the space–charge is trapped close to the star in domes and a torus. If the system could be neutralized, the domes would extend to "infinity", but this in itself is little help since losing charge from the domes would just re–charge the system. The tight confinement of the torus is the real problem in obtaining a theoretically active system. The torus is not in rigid corotation, however, so it is entirely possible that the plasma viscosity would cause it to diffuse outwards and produce a distant thin extension. However, this hardly seems a likely mechanism for a dynamic system (requiring huge currents to close) unless finally having both the domes and torus extend to "infinity" can trigger some new form of activity. It is difficult to estimate even the tiny current that might flow since at large distances the particles should see only the dipole magnetic field ($B \approx 1/r^3$) and monopole electric field ($E \approx 1/r^2$) in which the $\mathbf{E} \times \mathbf{B}/B^2$ drift velocity scales as $E/B \approx r$ and viscous diffusion would asymptotically vanish.

If some source of ionization could fill in the domes and torus to fill up the magnetosphere, we might be back in business. But the most promising mechanism would seem to be some sort of pair production mechanism (either magnetic pair conversion of a gamma ray in the vacuum or pair production as a radiative correction to Compton scattering). But neither of these seem plausible in the remote regions at the light cylinder distances for typical pulsars. Certainly they are plausible mechanisms close to the neutron star and should act to fill in between the domes and torus there (Smith, Michel & Thacker 2000). Indeed, even the first step of initiation of magnetic pair production can not take place until one is within a few tens of neutron star radii (for the strongly magnetized pulsars having periods of the order of a second: (Michel 1991b).

Unless one can identify a mechanism to provide copious ionization at light–cylinder distances, it is difficult to see how an aligned rotator model can work. Phenomenological models can always be constructed with arbitrary features to force activity. Certainly if you could run a wire to the magnetic pole and a wire to the equatorial zone and short the two out, spectacular things would happen.

9.1. *A simple extension*

If not an aligned rotator, perhaps an inclined rotator. After all, one needs the inclination to get rotationally paced pulses in the first place. Once one has an inclined rotator, one realizes that there is another piece of tunnel vision inherited from the old GJ model, namely that centrifugal forces drive a plasma wind from the system, hence the relevance of a "light–cylinder" in the first place. But an inclined system must general electromagnetic waves of huge amplitudes beyond the "wave zone", which is of course at exactly the same distance away (see Michel & Li 1999). To imagine that these waves, which drive away both signs of charge, play no role in the pulsar physics is certainly an unwise practice. Only for a perfectly aligned rotator will large amplitude electromagnetic waves be suppressed. But even in the case of the inclined rotator, the huge distances to the wave zone may still pose difficulties. It seems imperative that some research be directed to this much more promising model.

REFERENCES

Contopoulos, I., Kazanas, D. & Fendt, C. 1999, ApJ, 511, 351
Davis, L., Jr. 1947, Phys. Rev., 72, 632
Davis, L., Jr. 1948, Phys. Rev., 73, 536
Goldreich, P. & Julian, W. H. 1969, ApJ, 157, 869 (GJ)
Hones, E. W., Jr. & Bergeson, J. E. 1965, J. Geophys. Res., 70, 4951 (HB)
Krause-Polstorff, J. & Michel, F. C. 1985a, MNRAS, 213, 43p
Krause-Polstorff, J. & Michel, F. C. 1985b, A& A, 144, 72 (KPM)
Malmberg, J. H. & deGrassie, J. S. 1977, Phys. Rev. Lett., 35, 577
Michel, F. C. 1980, Ap&SS, 72, 175
Michel, F. C. 1991a, Theory of Neutron Star Magnetospheres, (Chiacgo: U. Chicago Press)
Michel, F. C. 1991b, ApJ, 383, 808
Michel, F. C. & Li, H. 1999, Phys. Rep., 318, 227
Neukirch, T. 1993, A&A, 274, 319
Pelizzari, M. 1976, PhD thesis, Rice University
Rylov, Yu. A. 1976, Sov. Astron. — AJ, 20, 23
Smith, I. A., Michel, F. C. & Thacker, P.D. 2000, MNRAS, submitted
Thielheim, K. O. & Wolfsteller, H. 1994, ApJ, 431, 718
Zachariades, H. A. 1993, A&A, 268, 705

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