RevMexAA (Serie de Conferencias), **15**, 223–225 (2003)

ORBITAL DYNAMICS ANALYSIS OF BINARY SYSTEMS IN MASS-LOSS SCENARIOS

M. Andrade, and J. A. Docobo

Universidade de Santiago de Compostela, Spain

RESUMEN

Analizamos la dinámica de sistemas binarios con pérdida de masa dependiente del tiempo y efecto periastro—esto es, un supuesto aumento de la pérdida de masa durante el paso por el periastro—mediante técnicas analíticas y numéricas. Estudiamos así la variación de los elementos orbitales en función de tres pequeños parámetros, los cuales describen la pérdida de masa dependiente del tiempo (α_1, α_2) y por efecto periastro (β). Se demuestra que este último produce variaciones seculares en algunos elementos orbitales: excentricidad, semieje mayor y período. Finalmente, sugerimos que ciertos comportamientos anómalos observados en los elementos orbitales de algunas binarias cerradas pueden ser explicados teniendo en cuenta el efecto periastro. Con la intención de mostrar esto incluimos también algunas aplicaciones prácticas.

ABSTRACT

We analyze the dynamics of binary systems with time-dependent mass loss and periastron effect—i.e., a supposed enhanced mass loss during periastron passages—by means of analytical and numerical techniques. In this way, we study the variation of orbital elements in terms of three small parameters, which describe the time-dependent mass loss (α_1, α_2) and the mass loss by periastron effect (β). It is demonstrated that the latter gives rise to secular variations in some orbital elements: eccentricity, semi-major axis and period. Lastly, we suggest that some of the anomalous behavior observed in some close binaries can be explained by taking into account the periastron effect. As a demonstration some practical applications are also included.

Key Words: **BINARIES: GENERAL — STARS: MASS LOSS**

1. INTRODUCTION

From the moment of its formation, every star undergoes a continuous time-dependent mass loss. In the framework of celestial mechanics this problem, known as Gylden-Meščerskij problem, has been exhaustively addressed by Hadjidemetriou (1963,1966), Prieto & Docobo (1997a,b), Docobo, Blanco, & Abelleira (1999), Andrade & Docobo (2001), among others.

In a binary system, in addition to time-dependent mass loss, there can be another loss due to the gravitational interaction between the two components. Therefore, we could suppose that, close to periastron, there is an appreciable enhancement of mass loss. This phenomenon will be called the *periastron effect*, and it will be more noticeable the greater the eccentricity and the smaller the minimum distance between the two stars.

Such an effect could possibly explain the relatively high eccentricity found in some tidally strong interacting binary systems. In fact, the periastron effect could counteract the tidal circularization predicted by Zahn's theory (1977, 1989).

2. MASS-LOSS LAWS WITH PERIASTRON EFFECT

Of the whole set of laws that take into account periastron effect by means of its dependence on distance, only some of them give rise to new behavior in the evolution of the orbital elements, such as secular variations of eccentricity.

In this work we propose to study the following time- and distance-dependent mass-loss law

$$\dot{\mu}(t; r, p_\theta) = -\alpha_1 \mu_1^n(t) - \alpha_2 \mu_2^q(t) - \beta \frac{p_\theta}{r^2}. \qquad (1)$$

Here, the first two terms represent time-dependent mass loss for the first and the second component, respectively. Each of them is the Eddington-Jeans law, where μ_1 and μ_2 are uniformly decreasing, continuous and differentiable functions of time, α_1 and α_2 are two small parameters close to zero, and n and q are two real numbers between 0.4 and 4.4. Lastly, the third term introduces the periastron effect, where r is the distance between the two components, p_θ is the total angular momentum and β is another small parameter close to zero.

3. ANALYTICAL INTEGRATION

3.1. *Hamiltonian Formulation*

The Hamiltonian corresponding to this problem of time- and distance-dependent mass loss is expressed in polar-nodal variables in the following way

$$H(r,\theta;p_{\mathrm{r}},p_\theta;t)=\frac{1}{2}\left(p_{\mathrm{r}}^2+\frac{p_\theta^2}{r^2}\right)-\frac{\mu(t)}{r}+\beta\frac{\theta}{r}. \quad (2)$$

This can be considered as the Hamiltonian function of a perturbed Gylden-Meščerskij problem, where such a perturbation is introduced by the last term.

3.2. *Application of a Triparametric Perturbation Method*

Given that mass loss depends on three small parameters α_1, α_2 (related to time) and β (related to distance), it is reasonable to consider an expansion of the Hamiltonian up to a certain order of perturbation. This problem can be analytically integrated using a triparametric method for non-autonomous systems (Andrade 2002), which is based on the application of a Lie transformation to the initial canonical system with the aim of obtaining a new, more easily integrable Hamiltonian. With the introduction of the dilogarithm function of complex argument (Osácar & Palacián 1994), we are in a position to obtain the generating function in closed form. However, it is not easy to evaluate such functions and, at present, we are still dealing with this calculation.

4. NUMERICAL INTEGRATION

As an alternative to the analytical approach, we can analyze the behavior of the system by means of a numerical integration.

4.1. *Application of a Runge-Kutta Method*

From the Hamiltonian given by equation (2) we can obtain the canonical equations of motion. These equations, together with another two for time-dependent mass loss, form a system of six differential equations

$$\begin{aligned}
\frac{dp_\theta}{dt}&=-\frac{\partial H}{\partial\theta}=-\frac{\beta}{r},\\
\frac{dp_{\mathrm{r}}}{dt}&=-\frac{\partial H}{\partial r}=\frac{p_\theta^2}{r^3}-\frac{\mu(t)}{r^2}+\beta\frac{\theta}{r^2},\\
\frac{d\theta}{dt}&=\frac{\partial H}{\partial p_\theta}=\frac{p_\theta}{r^2},\\
\frac{dr}{dt}&=\frac{\partial H}{\partial p_{\mathrm{r}}}=p_{\mathrm{r}},\\
\frac{d\mu_1}{dt}&=-\alpha_1\mu_1^n,\quad \frac{d\mu_2}{dt}=-\alpha_2\mu_2^q,
\end{aligned} \quad (3)$$

where $\mu=\mu_1+\mu_2$. The integration of this system, by use of a fourth order Runge-Kutta method, allows us to calculate the time- and distance-dependent coordinates, momenta and orbital elements.

TABLE 1

INITIAL CONDITIONS[a]

P_0	T_0	f_0	ω_0	a_0	e_0
2π	0.0	0.0	0.0	1.0	0.2
μ_{10}	μ_{20}	n	q		
0.7	0.3	2	2		

[a]The orbital elements are presented in the first row, while in the second one the initial masses and superscripts are given.

TABLE 2

VALUES OF PARAMETERS

Cases	A	B	C	D
α_1	10^{-6}	0	10^{-5}	10^{-7}
α_2	10^{-7}	0	10^{-6}	10^{-7}
β	0	10^{-7}	10^{-7}	10^{-7}

4.2. *Plots*

In order to show the behaviour of the variables with time, a set of initial conditions (orbital elements) and fixed values for the superscripts that appear in equations (3) are chosen. These are shown in Table 1.

We now describe four cases that broadly represent the phenomenology of this problem (see Table 2), however, only Case C will be plotted.

4.2.1. *Case A: Purely Time-Dependent Mass Loss*

This is known as the Gylden-Meščerskij problem, when the system is losing mass with time. Therefore, the β parameter, which is related to distance-dependent mass loss, will be zero.

As is well known, the angular momentum of the system is a constant of motion. With respect to the orbital elements, the periastron and eccentricity arguments exhibit a periodic evolution, while the semi-major axis and period show a secular increase.

4.2.2. *Case B: Mass Loss by Pure Periastron Effect*

Let us consider an extreme case where there is a strongly enhanced mass loss close to periastron without time-dependence, that is, a *pure periastron*

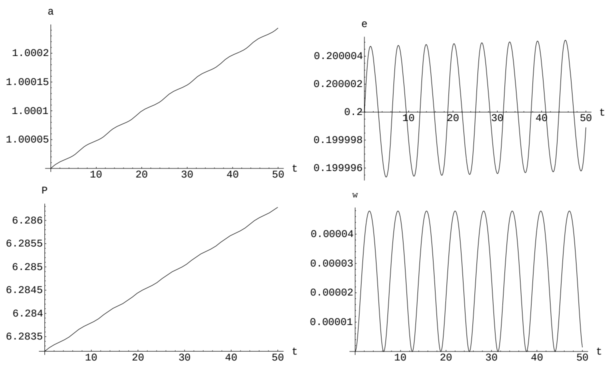

Fig. 1. Orbital elements for Case C.

effect. Now, the α parameters, which have to take into account time-dependent mass loss, will be zero.

The first consequence is that angular momentum of the system is no longer a constant of motion: in contrast it shows a secular decrease. This will be true for any case when β is larger than zero.

The periastron effect gives rise to a periodic regression of periastron argument. However, the most noteworthy variations happen in eccentricity, which now shows a strong secular increase, and semi-major axis and period, which in contrast with the previous case, show a strong secular decrease.

4.2.3. *Case C: Time-Dependent Mass Loss plus Periastron Effect*

Now we shall suppose that the β parameter is slightly smaller than the α parameters. In this way, we are considering time-dependent mass loss plus periastron effect as a perturbation.

As expected, time-dependent mass loss is dominant, and therefore the mass, periastron argument, semi-major axis and period show behavior like the first case. However, it is very remarkable that the eccentricity again shows a smooth secular variation (see Figure 1), which will be more important as the β parameter increases. That is, depending on how we fit the three parameters, the perturbation will be stronger or weaker, giving rise to larger or smaller secular increases. Certainly, this case is the most interesting one.

4.2.4. *Case D: Time-Dependent Mass Loss and Periastron Effect*

In this last case, we will suppose that the β parameter is of the same order as the α parameters.

Now, almost all the orbital elements have a behaviour similar to the second case and thus the eccentricity shows a secular variation. However, although the semi-major axis shows a secular decrease, the period doesn't show the same behaviour. Instead, it exhibits an unexpected time-dependent oscillation.

5. CONCLUSIONS

With this small parameter perturbation model the time-dependence of the whole set of orbital elements could be calculated over long timescales and even for high eccentricities. Moreover, it would be useful for describing certain anomalies found in some binary systems (Kennedy 2000; Soker 2000).

We propose to carry out an exhaustive examination of other mass-loss laws with periastron effect by means of analytical methods of integration. Later, successive perturbations would be added, such as those due to mass exchange between the two stars, the influence of accretion disks and the existence of electromagnetic fields.

This work is a part of the Investigation Project AYA 2001-3073 supported by the Ministerio de Ciencia y Tecnología (Spain).

REFERENCES

Andrade, M. 2002, in Métodos de Dinámica Orbital y Rotacional: IV Jornadas de Mecánica Celeste, eds. S. Ferrer, T. López, & A. Vigueras (Murcia: Prensas Universitarias), 113 (in Spanish)

Andrade, M., & Docobo, J. A. 2001, in Highlights of Spanish Astrophysics II: 4th Scientific Meeting of the Spanish Astronomical Society (SEA), eds. J. Zamorano, J. Gorgas, & J. Gallego (Dordrecht: Kluwer), 273

Docobo, J. A., Blanco, J., & Abelleira, P. 1999, in Monografías de la Academia de Ciencias de Zaragoza, 14, II Jornadas de Mecánica Celeste, eds. A. Elipe, & V. Lanchares (Zaragoza: Academia de Ciencias de Zaragoza), 33 (in Spanish)

Hadjidemetriou, J. D. 1963, Icarus, 2, 440

__________. 1966, Icarus, 5, 34

Kennedy, H. D. 2000, RASNZ, 38, 201

Osácar, C., & Palacián, J. 1994, Celes. Mech. & Dyn. Astron., 60, 207

Prieto, C., & Docobo, J. A. 1997a, A&A, 318, 657

__________. 1997b, Celes. Mech. & Dyn. Astron., 68, 53

Soker, N. 2000, A&A, 357, 557

Zahn, J.-P. 1977, A&A, 57, 383

__________. 1989, A&A, 220, 112

M. Andrade and J. A. Docobo: Astronomical Observatory R. M. Aller, Universidade de Santiago de Compostela, P.O. Box 197, Santiago de Compostela, Spain (oandrade,oadoco@usc.es).