RevMexAA (Serie de Conferencias), **15**, 69–69 (2003)

ON THE MORPHOGENESIS OF PLANETARY NEBULAE

D. Da Rocha[1] and L. Nottale[1]

We generalize the hydrodynamic standard equations by including the effects of the fractality of particle trajectories. We find that the resulting matter density is characterized by a probability distribution of ejection angles that have peaks for some specific values.

In a typical stellar mass ejection process, the interaction between fast and slow winds is described in the standard approach by hydrodynamic (Euler-Newton/continuity) equations. We suggest a generalization of this approach, in which we assume that particle trajectories are subjected to three conditions: (i) fractality of each individual trajectory, (ii) infinity of possible trajectories leading to a fluid-like description $v = v(x(t), t)$, and (iii) the reflection invariance under the transformation $(dt \leftrightarrow -dt)$ is broken as a consequence of non-differentiability, thus leading to a two-valuedness of the velocity vector described by a complex velocity, $\mathcal{V} = (v_+ + v_-)/2 - i(v_+ - v_-)/2$. These three effects can be combined to construct a complex time derivative operator $d'/dt = \partial/\partial t + \mathcal{V}.\nabla - i\mathcal{D}\triangle$ (Nottale 1993, 1996, 1997; Célérier & Nottale 2003). Finally, the real and imaginary parts of the fundamental equation of dynamics (written using this operator: $d'\mathcal{V}/dt = -\nabla\phi/m$), gives back the standard system:

$$m\left(\frac{\partial}{\partial t} + V\cdot\nabla\right)V = -\nabla\left(\phi + Q\right), \qquad (1)$$

$$\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho V) = 0. \qquad (2)$$

That is, the Euler-Newton and continuity equation, but with an additional effective potential energy $Q = -2m\mathcal{D}^2\triangle\sqrt{\rho}/\sqrt{\rho}$.

As a first order approximation, we assume the resultant force to be vanishing, i.e., the ϕ potential energy is constant (Corradi 1999; Dwarkadas, Chevalier, & Blondin 1996; García-Segura, López, & Franco 2001). For stationary solutions we rewrite the $[\rho, V]$ system (equations 1, 2) in terms of a unique $[\mathbf{r}, t]$ complex equation that is written (Nottale 1993; Célérier & Nottale 2003)

$$\mathcal{D}^2 \triangle f + i\mathcal{D}\frac{\partial f}{\partial t} - \frac{\Phi}{2m}f = 0. \qquad (3)$$

[1]LUTH, Observatoire de Paris Meudon, Place Jules Janssen, 92195 Meudon Cedex, France (daniel.darocha, laurent.nottale@obspm.fr).

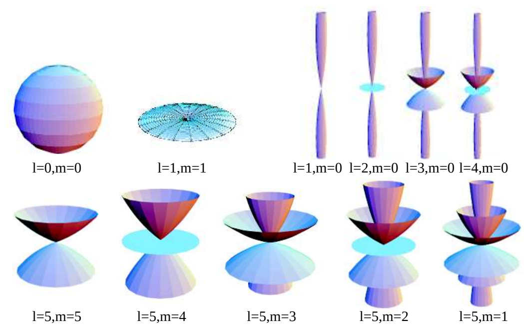

Fig. 1. Solution morphologies.

Solutions like $f(\mathbf{r}, t) = g(\mathbf{r})\exp(-iEt/2m\mathcal{D})$ are allowed by this case, thus we obtain $2m\mathcal{D}^2 \triangle g(\mathbf{r}) - Eg(\mathbf{r}) = 0$, with $E = p^2/2m = 2m\mathcal{D}^2k^2$, the energy of a free particle, and $\rho = |g|^2$. The spherical Laplacian governs the use of the spherical harmonics and the general solution is now constrained by conservation of angular momentum. This solution is written $g(\mathbf{r}) = R(r)\,Y_l^{\hat{m}}(\theta, \phi)$. The specificity of the angular part of the solution, $Y_l^{\hat{m}}(\theta, \phi)$, allows one to present the following results (Da Rocha & Nottale 2003):

1. Numerical values of ejection angles with higher probabilities.

2. Discretization of possible morphologies. All solutions can be classified in three categories: (i) spherical and elliptical shapes, (ii) axial shapes, and (iii) bipolar shapes (see Figure 1).

3. Formation of disc/jet/cone-like structures.

The recent observations of stable discs associated with axial ejections or bipolar shells are consistent with the proposed morphologies.

REFERENCES

Célérier, M. N., & Nottale, L. 2003, Electromagnetic Phenomena, in press (hep-th/0210027)
Corradi, R. L. M. 1999, A&A, 354, 1071
Da Rocha, D., & Nottale, L. 2003, Chaos Solitons & Fractals, in press
Dwarkadas, V., Chevalier, R., & Blondin, J. 1996, ApJ, 457, 773
García-Segura, G., López, J. A., & Franco, J. 2001, ApJ, 560, 928
Nottale, L. 1993, Fractal Space-Time and Microphysics: Toward a Theory of Scale Relativity (London: World Scientific)
———. 1996, Chaos Solitons & Fractals, 7, 877
———. 1997, A&A, 327, 867