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SPH AND RADIATIVE COOLING

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I have investigated the effects of including a more realistic cooling law in SPH simulations of accretion discs. In order to improve the efficiency of the simulations, I have used Strang operator splitting, and I have developed a method of timestep control which guarantees the accuracy and stability of the simulations.

Strang splitting (Strang, 1968) is used to numerically solve a differential equation where the differential operator is split into two or more parts. This scheme is appropriate for any such equation, but is applied most effectively in cases where one part of the equation has a different timescale to the rest, or is somehow more difficult to solve or less stable.

Using Strang splitting means that the simulation is no longer constrained to move with the smallest timestep. Those physical processes which move more slowly can be separated from those in the fast lane. Each process can also be solved with the most appropriate numerical scheme, meaning that the accuracy and stability is improved. On the whole, using Strang splitting can result in dramatic improvements in speed, accuracy and efficiency.

When using operator splitting, we are no longer able to base the timestep size on an appropriate physical condition. Ideally, one would hope that the larger of the two natural timesteps was sufficiently small to ensure stability of the entire scheme, however, this may not be guaranteed. Instead, a method of determining an appropriate timestep is required, hopefully one which easily adapts to changing simulation conditions.

One simple solution is to generate some estimate of the error involved, then demand that this error be beneath a preset limit. In this way, not only is stability guaranteed, but an upper limit is also set on the overall error.

I have run several simulations of thin accretion discs around an isolated, one solar mass object. The first of these was an isothermal simulation, the rest used more realistic cooling laws.

Figure 1 compares the stepsize required by the Courant condition with the stepsize required to resolve the gravitational motion of the innermost particles. Clearly, if the simulation were to proceed at

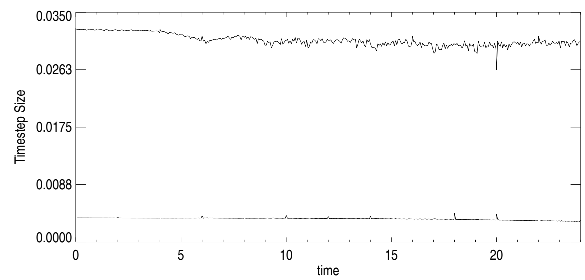

Fig. 1. The upper line is the timestep taken by the Strang Splitting routine, the lower the step that would be taken if Strang Splitting were not used.

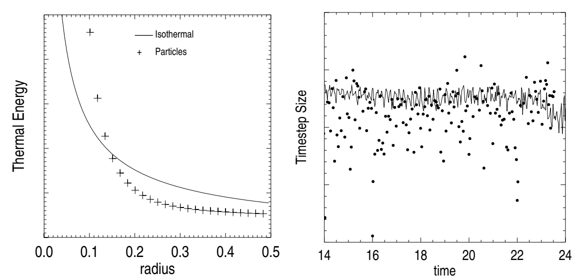

Fig. 2. Left: The solid line is the energy of an isothermal disc, the points are the energy of the cooled disc. Right: the solid line is the Courant condition stepsize, the points are the size required to maintain accuracy and stability.

this lower stepsize, it would take around ten times as long. Strang Splitting provides a way of avoiding problems of this nature.

Figure 2 shows on the left the temperature structure resulting from using an optically thin cooling law. On the right is a comparison of stepsizes from the physical conditions and the numerical stability. In general, the numerical stability stepsize is the smaller of the two. This means that if the physical stepsize were taken the simulation would be unstable or the accuracy would be less than the required limit.

Further details, explanations and evidence of the improvements gained using these methods may be obtained by contacting the author.

REFERENCES

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