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SPH WITH RADIATIVE TRANSFER: METHOD AND APPLICATIONS

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RESUMEN

Se describe un método explícito para el transporte de radiación en SPH. El método usa la aproximación de difusión y los cálculos hidrodinámicos se realizan con un código SPH tridimensional. Varios estados de energía así como la disociación del hidrógeno molecular son considerados en el cálculo de la energía para llegar a una determinación más realista de la temperatura. Se han simulado colapsos no isotérmicos de nubes centralmente condensadas y se han comparado los resultados con aquellos obtenidos con el método de diferencias finitas de Myhill & Boss (1993). Los resultados obtenidos con estos dos métodos tan completamente diferentes, están en buena concordancia. También calculamos el colapso de una nube cilíndrica en rotación para investigar la formación y evolución de discos. Aqui se presentan algunos de estos resultados.

ABSTRACT

An implicit method for radiative transfer in SPH is described. The diffusion approximation is used in this method, and the hydrodynamic calculations are performed by a fully three–dimensional SPH code. Various energy states and the dissociation of hydrogen molecules are considered in the energy calculation for a more realistic temperature determination. We have performed non–isothermal collapse simulations of a centrally condensed cloud, and have compared with the results of finite difference calculations performed by Myhill & Boss (1993). The results produced by the two completely different numerical methods agree well with each other. We also perform a rotating cylindrical cloud collapse to investigate the formation and evolution of disks, and present some results.

Key Words: **METHODS: NUMERICAL — STARS: FORMATION**

1. INTRODUCTION

It is essential to follow the exact thermal evolution of a collapsing cloud to understand the early stages of star formation, because some important dynamical processes, for example fragmentation, are closely related to the thermal evolution. Therefore, many previous studies have tried to unveil the thermal evolution of a collapsing cloud. These previous studies can be divided into two streams. One is to solve the radiative transfer equation directly in a one-dimensional hydrodynamic code (e.g., Yorke 1979, 1980; Masunaga, Miyama, & Inutsuka 1998; Masunaga & Inutsuka 2000). This approach can track the exact thermal evolution of a collapsing cloud, however, some important physics, for example rotation, cannot be included in one–dimensional calculations. Furthermore, it is impossible to observe fragmentation in one–dimensional calculations. The other way is to use a multi–dimensional hydrodynamic code with an approximated radiative transfer process (Boss 1984; Myhill & Boss 1993). Larson (1969) used the diffusion approximation in his one–dimensional simulations, and the Eddington approximation was used by Winkler & Newman (1980a), Winkler & Newman (1980b), Boss (1984) and Myhill & Boss (1993). This approach is less exact than the first approach, but the detailed dynamical evolution can be seen clearly.

The multi-dimensional calculations with radiative transfer published so far have been done mostly with codes using the finite difference method. Although Smoothed Particle Hydrodynamics (hereafter SPH) codes are now used quite commonly in calculations of self–gravitating clouds, so far only two attempts to include radiative transfer in an SPH code have been published (Lucy 1977; Brookshaw 1994). There are two major difficulties with radiative transfer in SPH. One is the treatment of the double derivative which appears in the diffusion term of the energy equation, and the other problem is the large difference between the radiative and dynamical time scales. Viau (2001) developed an effective implicit scheme for radiative transfer in a fully three–dimensional SPH code that solves these two problems.

We have performed a non–isothermal cloud collapse using the test initial conditions of Myhill &

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Boss (1993) with the implicit code (Viau 2001), and compared our results with those of Myhill & Boss (1993). The comparison should be useful because the two methods are completely different from each other. We also performed a rotating cylindrical cloud collapse to observe the formation and evolution of disks.

The implicit scheme which combines the diffusion approximation and SPH is described in section 2. The non–isothermal cloud collapse simulations to test the implicit code and the comparison with Myhill & Boss (1993) are given in section 3. In section 4, we present the collapse of a rotating cylindrical cloud. The summary is in section 5.

2. NUMERICAL METHOD

2.1. *SPH and the diffusion approximation*

SPH (Lucy 1977; Gingold & Monaghan 1977) is a grid–free and fully Lagrangian method, and therefore has been widely used in gravitational collapse simulations. We have used a common form given by

$$\rho = \sum_j m_j W_{ij}, \tag{1}$$

$$\frac{\Delta \mathbf{v}}{\Delta t} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}\right) \nabla_i W_{ij} - \nabla^2 \Phi_i, \tag{2}$$

$$\frac{\Delta u}{\Delta t} = \sum_j \frac{m_j}{2} \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}\right) (\mathbf{v}_i - \mathbf{v}_j) \cdot \nabla_i W_{ij}, \tag{3}$$

where W is the kernel function, and all variables have their usual meaning. Here Π_{ij} is the artificial viscosity (see Monaghan (1992) for details). Our SPH code uses a tree algorithm to find neighbours and calculate self–gravity, and individual–time–step algorithm to reduce the computational time.

The diffusion approximation has been adopted in our three-dimensional SPH code for the treatment of radiative transfer, because it is very hard to trace the exact behavior of individual photons in the multi–dimensional hydrodynamic code. The energy equation with the diffusion term is given by

$$\frac{Du}{Dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} - \frac{1}{\rho} \nabla \cdot \mathbf{F}, \tag{4}$$

where $\mathbf{F}$ is the radiative flux, and is given by

$$\mathbf{F} = -\frac{4acT^3}{3\kappa_R \rho} \nabla T, \tag{5}$$

where a is $\frac{4\sigma}{c}$, c is the speed of light and σ is the Stephan–Boltzmann constant. Here κ_R is the Rosseland mean opacity. For κ_R, we have used the model developed by Yorke (1979) with the recent values for the extinction factor provided by Preibisch, Ossenkopf, Yorke, & Henning (1993) in low temperature regions ($T < 316\,\mathrm{K}$). In high temperature regions ($708 < T < 12500$) we have used the model of Alexander & Ferguson (1994).

If a parameter, $Q(\equiv -16\sigma T^3/3\kappa_R \rho)$ is defined, equation (4) may be rewritten as

$$\frac{Du}{Dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} - \frac{1}{\rho} \nabla \cdot (Q \nabla T). \tag{6}$$

Equation (6) contains a double derivative, and this ∇^2 operation is very sensitive to the disorder in the particle distribution, so it may cause numerical noise in the simulation. In order to avoid this ∇^2 operator, we have used a formulation suggested by Brookshaw (1994). Finally, the energy equation becomes

$$\frac{\Delta u}{\Delta t} = \sum_j \frac{m_j}{2} \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}\right) (\mathbf{v}_i - \mathbf{v}_j) \cdot \nabla_i W_{ij} + \sum_j \frac{m_j}{\rho_i \rho_j} \frac{(Q_i + Q_j)(T_i - T_j)}{|\mathbf{r}_i - \mathbf{r}_j|^2} (\mathbf{r}_i - \mathbf{r}_j) \cdot \nabla_i W_{ij}. \tag{7}$$

2.2. *An implicit scheme for radiative transfer*

The radiation time scale is much shorter than the dynamical time scale in a collapsing cloud, therefore, an implicit scheme has been developed (Viau 2001). We describe the implicit scheme here.

First of all, define a function $\mathcal{F}$ from equation (4),

$$\mathcal{F} = u_i^* - u_i^0 + \delta t \left(\frac{P}{\rho} \nabla \cdot \mathbf{v}\right) + \delta t \left(\frac{1}{\rho} \nabla \cdot \mathbf{F}\right), \tag{8}$$

where u_i^0 is the former step value, and u_i^* is the updated value by the iteration. After that define a range in temperature characterized by, say, T_L and T_R, and find u_L and u_R from the corresponding boundary temperatures. Using the bisection method or the Van Wijingaarden–Dekker–Brent method (e.g. Press, Teukolsky, Vettering, & Flannery 1992), find u_i^* and repeat the whole procedure until $\mathcal{F}(u_i^*) < tol$. Here tol is a tolerance factor related to the resolution of the simulation, and set to 10^{-5} in our simulations.

The specific internal energy, u, is determined from

$$u = u(H) + u(H_2) + u(H_{2diss}) + u(He) + u(M), \tag{9}$$

where $u(H)$, $u(H_2)$, $u(He)$ and $u(M)$ are the specific internal energies of atomic hydrogen, molecular hydrogen, helium and metals, respectively. Here

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$u(H_{2diss})$ is the dissociation energy of hydrogen molecules. With this implicit scheme for relating the flux to the temperature, we can integrate equation (7) on the dynamical time scale.

3. TEST FOR NON–ISOTHERMAL COLLAPSE

In order to confirm the validity of the implicit radiative transfer code, we have performed test simulations for a centrally condensed cloud. This test was proposed by Myhill & Boss (1993), and the comparison should be meaningful because the method of Myhill & Boss (1993) and ours are based on two completely different philosophies but deal with exactly the same subject.

3.1. *Initial conditions*

The same initial conditions of Myhill & Boss (1993) have been used to compare the results directly. The initial cloud has a mass of $1.087\mathrm{M}_\odot$ and a radius of 1.1×10^{16}cm. Solid–body rotation has been imposed around the z–axis, and the angular velocity is $8.2 \times 10^{-12}\mathrm{s}^{-1}$. The cloud is initially spherical but centrally condensed, and its density profile is given by

$$\rho = \frac{\rho_i}{\sqrt{x^2 + 4y^2 + 4z^2}}, \tag{10}$$

where $\rho_i = 4.28 \times 10^{-16}\mathrm{g/cm}^3$.

We have performed three tests with 50000 particles for each one. The only difference between each test is in the calculation of the specific internal energy. Tests 1 and 2 use the energy equation of state of an ideal gas for the derivation of the energy. 5/3 and 7/5 are used as the specific heat ratio in Tests 1 and 2, respectively. The mean molecular weight is fixed at 2.385. Test 3 uses the more realistic energy calculation with radiative transfer explained in the previous section. Note that the mean molecular weight is not constant in Test 3 due to the dissociation of hydrogen molecules.

3.2. *Results and comparison*

The central part of the cloud collapses very quickly while the outer part is still isothermal because the cloud is already centrally condensed. Figure 1 shows the evolution of the central part of the collapsing cloud. In the adiabatic regime, the slope of Test 1 (dots) is steeper than that of Test 2 (long–dash). In Test 3 (solid line), the evolution of the central core follows the slope $\gamma = 5/3$ initially, but it becomes closer to $\gamma = 7/5$ when the density becomes greater than $\sim 10^{-12}\mathrm{g/cm}^3$. This change in the slope is due to the energy of hydrogen molecules. The rotational energy of hydrogen molecules is not

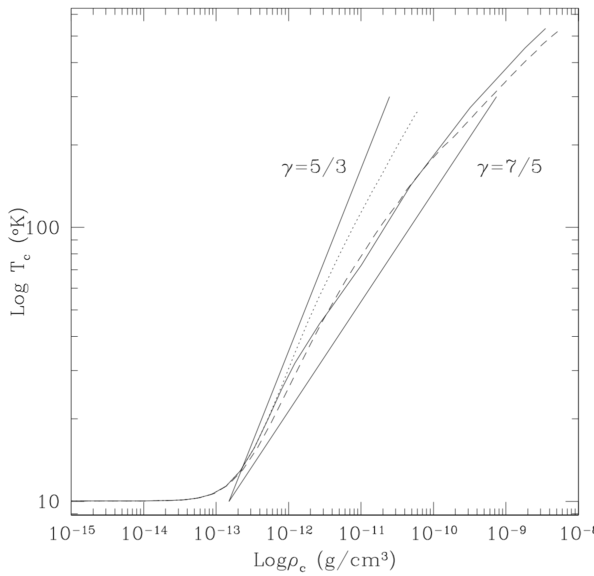

Fig. 1. Evolution of the density and temperature of the cloud center. Dotted, long–dashed and solid lines are the result of Tests 1, 2 and 3, respectively. The effective γ value for Test 3 is nearly 5/3 in the range $10^{-13}g/cm^3 < \rho_c < 10^{-12}\mathrm{g/cm}^3$, and then changes to 7/5. See text for details.

important at very low temperatures, so the contribution of the translational energy is dominant. However, as the temperature increases, the rotational energy becomes more important, so the effective γ value becomes closer to 7/5.

We have compared our results with those of Myhill & Boss (1993) in Table 1. The central temperatures of SPH are somewhat lower than those of Myhill & Boss (1993). We may presume a few reasons for this difference. First of all, the treatment of the radiative transfer is different. We have used the diffusion approximation in our simulations, while Myhill & Boss (1993) used the Eddington approximation. It is not easy to predict the resultant difference due to the different treatments. Secondly, there is a difference in the energy calculation of hydrogen molecules. According to the $u(H_2)$ calculation of Boss (1984) (See Appendix B of Boss (1984)), the transition temperature from $\gamma = 5/3$ to 7/5 is 100 K. However, the transition temperature is variable in our calculation, and $\simeq 40$ K in Test 3. Therefore, the temperature increase should be slower in our simulation.

4. CYLINDRICAL CLOUD COLLAPSE: DISK FORMATION AND EVOLUTION

We have performed a rotating cylindrical cloud collapse to observe the formation and evolution of

TABLE 1

COMPARISON THE RESULTS[a]

ρ_c	$2.2 \times 10^{-12} g/cm^3$	$1.7 \times 10^{-12} g/cm^3$
Test 1	49.9 K	42.6 K
Test 2	39.2 K	34.3 K
Test 3	41.0 K	36.8 K
CC	67.0 K	…
SC	…	56.0 K

[a]Here CC and SC mean Cartesian and Spherical codes, respectively, from Myhill & Boss (1993).

disks. This calculation is similar to those of Bonnell, Arcoragi, Martel, & Bastien (1992), except that the code includes the radiative transfer described here. The length and mass of the initial cloud is 0.23pc and $2M_\odot$. The ratio between the length and diameter is 2, and initial Jeans number (ratio of gravitational to thermal energies) is 1.07. The cloud rotates around an axis perpendicular to the axis of symmetry with an angular velocity of $10^{-14}s^{-1}$. 8000 particles have been used in the simulation.

As the cloud collapses, two condensations form on the axis of symmetry and move at the speed of sound towards the equatorial plane. Because of rotation, a disk forms around each condensation. Using a black body approximation for each particle in a disk (Nelson, Benz, & Ruzmaikina 2000), we computed the spectral energy distribution in Figure 2. While the overall shape is similar to that of Young Stellar Objects, the total luminosity is too low by many orders of magnitude, presumably because there is no radiation from the central object to heat the disk at this early stage in our calculation.

5. SUMMARY

We have presented a fully three–dimensional implicit radiative transfer method within a hydrodynamic SPH code. It uses the diffusion approximation and includes various thermodynamics (e.g. dissociation of hydrogen molecules).

We have performed a non–isothermal cloud collapse to check the validity of our code, and the results are comparable to those of Myhill & Boss (1993). We also presented a rotating cylindrical cloud collapse to observe the disk formation and evolution.

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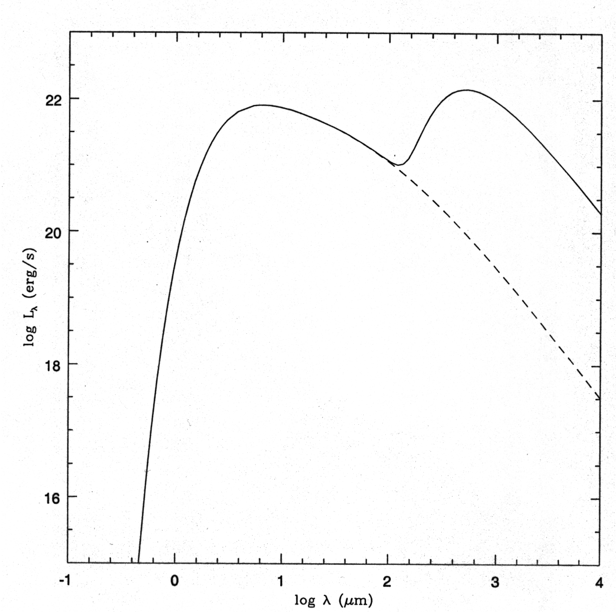

Fig. 2. Spectral energy distribution for the disk which formed in a collapsing cylindrical cloud. The secondary peak at longer wavelengths is obtained when the envelope surrounding the disk is included.

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