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# GRAVITATIONAL INSTABILITIES IN DISKS: FROM POLYTROPES TO PROTOPLANETS?

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RESUMEN

Las inestabilidades gravitacionales (GIs) ocurren probablemente en discos que circundan a objetos estelares jóvenes, durante su temprana fase en la que presentan una envolvente. Aquí, reseñamos el saber acerca de la no-lineariedad de las GIs en la formación de planetas y la evolución del disco. Los investigadores están de acuerdo en que, bajo un enfriamiento lo suficientemente rápido, los discos se fragmentan en condensaciones densas o estructuras arqueadas, pero no hay un consenso universal sobre si un enfriamiento lo suficientemente rápido como para producir fragmentación ocurre en realidad, y si ocurre, que todos los fragmentos que se formen evolucionen en proto-planetas ligados.

ABSTRACT

Gravitational instabilities (GI's) probably occur in disks around young stellar objects during their early embedded phase. This paper reviews what is known about the nonlinear consequences of GI's for planet formation and disk evolution. All researchers agree that, for sufficiently fast cooling, disks fragment into dense clumps or arclike structures, but there is no universal agreement about whether fast enough cooling to cause fragmentation ever occurs and, if it does, whether any clumps that form will become bound protoplanets.

*Key Words:* **HYDRODYNAMICS — INSTABILITIES — STARS: PLANETARY SYSTEMS: FORMATION — STARS: PLANETARY SYSTEMS: PROTOPLANETARY DISKS**

## 1. INTRODUCTION

Interstellar cloud cores have specific angular momentum comparable to that of planetary orbits (Bodenheimer et al. 1994), and so young stars that form from collapsed clouds are commonly surrounded by gas disks of Solar System size (Calvet et al. 2000). In some cases, especially for the youngest systems, these disks can be comparable in mass to the central star (Osorio et al. 2003). As reviewed by Durisen et al. (2003), numerical simulations show that circumstellar gas disks become susceptible to instabilities driven by self-gravity when the Toomre (1964) stability parameter $Q = c_s\kappa/\pi G\Sigma$ becomes less than about 1.5 to 1.7. Here $c_s$ is the sound speed, $\kappa$ the epicyclic frequency, and $\Sigma$ the surface mass density. Fits of thermal structures to observed embedded disks (Osorio et al. 2003) yield $Q$'s lower than the gravitational instability (GI) limit.

In seminal papers, Boss (1997, 1998) revived the classic idea (Kuiper 1951, Cameron 1978) that gas giant planets might be formed directly, all at once, by GI's in disks. A debate now rages between advocates of gas giant formation by disk instability and advocates of the "standard" core-accretion theory (Wuchterl et al. 2000) . In this paper, I briefly summarize what is known about GI's in disks, where serious disagreements remain, and how our understanding is likely to progress.

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## 2. PETER BODENHEIMER'S CONTRIBUTIONS

Peter's direct and indirect contributions to the study of disk instabilities span several decades. I have space here to acknowledge only a few.

**Rapidly Rotating Polytropes.** Numerical studies of GI's require equilibrium starting models. Peter pioneered use of the self-consistent-field (SCF) method to create axisymmetric models of rapidly rotating stars (Bodenheimer & Ostriker 1973). My group uses a grid-based variant of SCF (Hachisu 1986) to create 2D initial equilibrium models of star/disk systems (Pickett et al. 1997, 2003). An example is shown in Figure 1.

**Disk Formation in Cloud Collapse.** In the 90's, Peter coauthored several papers on disk formation during cloud collapse (e.g., Yorke et al. 1993, Yorke & Bodenheimer 1999) which demonstrate that GI's occur in this early phase of star formation.

**Angular Momentum Transport in Disks.** When GI's grow in disks, they produce strong trailing spiral arms. Gravitational torques along these arms transport angular momentum outward and

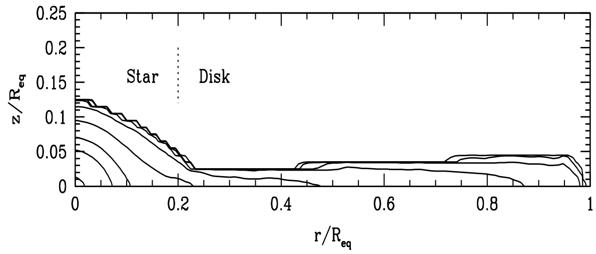


Fig. 1. An example of a star/disk equilibrium model created by the SCF method. Shown are meridional plane density contours with the rotation axis along the vertical axis and the equatorial plane on the horizontal axis. The fat central region is a slowly rotating "star" which makes a smooth transition to a thin, nearly rotationally supported disk. The disk in this particular model has $Q = 1.35$. (Figure courtesy of M. K. Pickett)

mass inward. Mass transport mechanisms in disks have been a major focus of Peter's research, including not only GI's (Laughlin & Bodenheimer 1994), but also turbulent viscosity, convection, and baroclinic instabilities.

**Gas Giant Planet Formation.** Peter was among the first to model the evolution of a Jupiter formed by GI's (Bodenheimer 1974), but, more recently, he is better known for his role in collaborations that have produced the most detailed 1D core-accretion models currently available (e.g., Pollack et al. 1996, Bodenheimer et al. 2003).

## 3. SECURE RESULTS

**Heating and Cooling Determine GI Amplitude.** Numerical hydrodynamics simulations have confirmed the suggestion by Goldreich & Lynden-Bell (1965) that the nonlinear behavior of GI's is determined by the balance of heating and cooling in the disk (Pickett et al. 1998, 2000a,b, Nelson et al. 2000, Gammie 2001, Boss 2001, 2002, Johnson & Gammie 2003, Pickett et al. 2003, Rice et al. 2003). The amplitude of the nonaxisymmetric structure increases inversely with the cooling time (Mejía 2004). The preponderance of evidence suggests that GI's are intrinsically 3D and global. The vertically uppermost layers of the disk are heated disproportionately, the disk surface becomes severely distorted, and the complex spiral structure tends to be dominated in the nonlinear regime by low-order modes of considerable radial extent (see Figure 2).

**Disks "Fragment" for Strong Cooling.** All researchers previously mentioned, plus Mayer et al. (2002, 2003), agree that, for strong enough cooling, the dense spiral structure seen in Figure 2 breaks

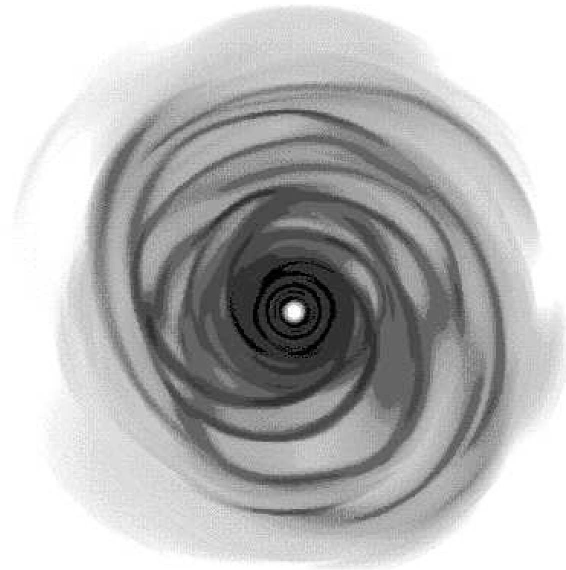

Fig. 2. An equatorial plane logarithmic grey scale of the density in a disk with 0.13 $M_\odot$ orbiting a 1 $M_\odot$ star. The initial inner and outer radii of the disk are 2.3 AU and 40 AU. This snapshot shows the disk after 18 outer rotations. The sustained cooling time in the disk is set to one outer rotation period everywhere. The disk image above is a full 170 AU on a side due to disk expansion driven by the GI's. (Figure courtesy of A. C. Mejía, animation at http://westworld.astro.indiana.edu/)

up into dense clumps or arclets. For local, thin-disk calculations, Gammie (2001) finds that such fragmentation occurs when $t_{\rm cool}\Omega < 3$, where $\Omega$ is the angular rotation speed of the disk and $t_{\rm cool}$ is the cooling time. Numerical simulations in 3D with both Smoothed Particle Hydrodynamics (SPH) (Rice et al. 2003) and grid-based codes (Mejía 2004) confirm this result. Researchers further agree that assuming the disk gas to behave isothermally is equivalent to very strong cooling and causes fragmentation when $Q < 1.4$ to 1.5. An example of isothermal disk fragmentation is shown in Figure 3. I emphasize that "fragmentation" here means only that the disk shatters into very dense localized structure. It does not imply that the clumps and arclets are permanent bound objects.

**GI's Produce Steady "Gravitoturbulence".** GI's can be initiated by growth of a single dominant unstable mode, but, after many rotations, as shown in Figure 2, a disk with moderate cooling settles into a complex, steady-state, nonlinear, self-gravitating turbulence with a uniform unstable $Q$-value sustained by the balance of heating and cooling (Tomley et al. 1991, Gammie 2001, Pickett et al.

Fig. 3. An equatorial plane logarithmic grey scale of mass density for the $Q = 1.35$ star/disk model of Figure 1 after several outer rotations at high azimuthal resolution. The star is kept hydrodynamically inactive in this simulation. (Figure courtesy of M. K. Pickett and J. Rosheck)

2003, Mejía 2004). Fourier analysis of simulations by my own group reveals pervasive nonlinear coupling of a large number of modes with a broad range in number of arms. In this quasi-steady state, substantial accretion rates are measured which appear to be associated with low-order global modes. For the disk in Figure 2, an underlying two-armed mode dominates mass inflow over the 10 to 30 AU region at a strongly fluctuating rate which averages about $2\text{x}10^{-6}\ M_\odot/\text{yr}$. Angular momentum from inside 30 AU is transferred to the the outer disk, which expands. Other authors report similar rates of mass inflow for similar parameters.

## 4. DISAGREEMENTS

**When is a Clump a Protoplanet?** This is the crux of the problem for planet formation. Clumps form in all strongly cooled, low-$Q$ disks, but do they become long-lived, gravitationally bound protoplanets? Some researchers would answer this question with a resounding,"Yes!" In the most dramatic example to date, for isothermal disks with $Q < 1.4$, Mayer et al. (2002, 2003), using a 3D SPH code with a large number of particles, report the formation of permanent bound clumps of multiple Jupiter masses which they can follow for very many orbits. In fact, the end states of their simulations resemble gas giant planetary systems. One long-lived dense clump is reported for a grid-based codes by Boss (2000) in one of his isothermal simulations. The clump shown in his Figure 1 survives at least two orbits before falling through the inner boundary of his grid. In this calculation, to mimic adaptive mesh refinement, Boss increases his fixed-grid resolution by hand as the clump grows. As far as I know, he leaves his resolution fixed in most other published simulations and does not follow other high density "clumps" for many orbits.

My own research group, on the other hand, never sees *permanent* clumps, even though we agree that strongly cooled disks fragment. We always find that, even though clumps may satisfy simplistic boundedness tests based on self-gravity and instantaneous internal energy content, the clumps exist in a dynamic environment where tidal stresses, velocity shears, and interactions with other clumps and arclets cause them to form and vanish usually in a fraction of an orbit. Our published simulations to date have somewhat limited resolution (Pickett et al. 2003), but, in more recent isothermal simulations with improved resolution, as in Figure 3, we still do not see *permanent* clumps form, even though the disk is strongly fragmented into very dense pieces.

This ongoing controversy may hinge on questions of numerical methodology. There are numerical instabilities that can simulate permanent clump formation in both SPH and grid-based codes (see Nelson 2003 for a discussion). There are also differences in implementation of physics and in grid shape, centering, and other numerical features, like artificial viscosity. Preliminary results from our own hydrodynamics code suggest that, even without heating, artificial viscosity can have a severe effect on the behavior of GI's and clumps. Planet formation is a problem of such importance that we cannot accept any one numerical result at face value. We must require that all competent researchers be able to obtain essentially the same results for the same conditions. So far, this is not the case.

**How Fast do Disks Cool?** Although it remains important to determine the true outcome of the academic isothermal case, it is more important to determine whether permanent bound clumps can form in disks with more realistic physics. Given the critical importance of cooling for fragmentation, radiative cooling must be computed accurately using real opacities. So far, only a few groups have addressed this problem, with mixed outcomes. Nelson et al. (2000) and Johnson & Gammie (2003) treat radiative cooling by approximate techniques in thin disk calculations and have somewhat conflicting answers about clump formation. Boss (2001, 2002) and my own group (Mejía et al. 2003, Mejía 2004) both use flux limited radiative diffusion in 3D but obtain opposite results. Boss finds rapid cooling under all conditions and attributes it to convection. Mejía finds slow cooling and no convection. The main differences are in the handling of boundary conditions at the surface of the optically thick disk. This discrepancy sounds another serious cautionary note. Not only do researchers not agree on whether clumps become bound, long-lived objects, but they also do not yet agree whether cooling is fast enough to allow clumps to form in the first place.

## 5. WHAT LIES AHEAD

**Desiderata.** Definitive conclusions about clump longevity will require that calculations have exquisite resolution and use impeccable numerical techniques. All results must be viewed skeptically. None should be widely accepted until confirmed by more than one research group using different methodologies.

**Final Speculations.** Opacity effects, irradiation, and boundaries will play critical roles in regulating whether GI's cause or assist planet formation. Edges could be particularly important, because permanent clump formation seems easier in situations where there are no violent radial motions. Johnson & Gammie (2003) have already demonstrated a significant sensitivity of fragmentation to an opacity gap, and Durisen et al. (2004) suggest that core accretion may be accelerated in dense rings which form in their simulations near disk boundaries between GI active and inactive regions. Such rings can be seen in the center of the disk in Figure 2. It may turn out that GI's and core accretion are complementary rather than competitive processes.

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