RevMexAA (Serie de Conferencias), **22**, 91–94 (2004)

GLOBAL EVOLUTION OF SOLIDS IN PROTOPLANETARY DISKS: A SIMPLE MODEL

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RESUMEN

Discutimos varios aspectos de la evolución de sólidos en discos protoplanetarios. Mientras que cuerpos pequeños en el disco crecen en tamaño, se desacoplan del gas y resultan en una distribución significativa de sólidos dentro del disco. Mostramos que la evolución de los sólidos debe de ser considerada por modelos de formación de planetas via el mecanismo de acrecentamiento del núcleo - captura de gas.

ABSTRACT

Various aspects of the evolution of solids in protoplanetary disks are discussed. While small bodies present in the disk grow in size, they decouple from the gas, resulting in a significant redistribution of solids within the disk. It is shown that the evolution of solids must be taken into account in models of planet formation via the core accretion - gas capture mechanism

Key Words: **ACCRETION, ACCRETION DISKS — STARS: PLANETARY SYSTEMS: FORMATION — STARS: PLANETARY SYSTEMS: PROTOPLANETARY DISKS**

1. INTRODUCTION

It has long been recognized that solid particles in protoplanetary disks evolve differently from the gas (see e.g. Weidenschilling & Cuzzi 1993 and references therein). The differences between the global evolution of the gaseous and solid components of the protoplanetary disk have been calculated by Stepinski & Valageas (1996, 1997). The most important distinction between the evolution of gas and solids is that the gas maintains its form, while the solids do not. Initially the solids are all in the form of fine dust, but, given enough time, they convert into roughly 1 – 10 km sized bodies (plenetesimals) on essentially Keplerian orbits. It is generally thought that this transition is achieved via the buildup of progressively more massive particles by the process of coagulation. Once planetesimals are formed, their further aggregation leads to planets. In general, the mass distribution of the solids evolves due to gas-solid coupling, coagulation, sedimentation, and evaporation/condensation. Stepinski & Valageas (1996, 1997) have shown that the solids decouple over time from the gaseous component, resulting in significant departures from a constant solids-to-gas mass ratio in the disk. Thus, in limited regions of an evolved disk, the surface density of solids can be considerably higher than it was at the beginning of the evolution.

In the present communication we briefly describe the results of numerical simulations in which the global evolution of solids is modeled with the help of a code based on ideas of Stepinski & Valageas (1996). We developed a computationally efficient method that tracks the evolution of solids from an early stage, when they are in the form of small dust grains, to the stage when most solids are in the form of planetesimals. We begin with the presentation of the numerical method, followed by the discussion of the basic properties of disk models. We identify the radial drift of solids as the principal factor influencing the properties of the final planetesimal swarm, and we show that when the drift is accounted for, the snowline does not have to coincide with the water condensation radius. Finally, we demonstrate that the drift may promote the formation of giant planet cores.

2. METHODS OF CALCULATION

The protoplanetary disk is modelled as a two-component fluid consisting of gas and solids. The evolution of the gas component is described by an analytic solution to the viscous diffusion equation, which gives the surface density of the gas as a function of radius r and time t (Stepinski 1998). The viscosity is given by the usual α model. The temperature of the gas is calculated in the thin-disk approximation, assuming vertical thermal balance, according to equations (2) through (6) in Stepinski (1998). Relative velocities at which the coagulation of grains occurs are calculated from the turbulent disk model

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of Stepinski & Valageas (1997). To account for the sedimentation of solids toward the midplane of the disk, the vertical thickness of the solid particle distribution at each radius is calculated and is evolved in time.

The solids are in the form of grains (dust particles) of various sizes, characterized by their sublimation temperature, T_{evap}, and their bulk density, ϱ. The model of their evolution includes the gas drag effect, sedimentation, coagulation and evaporation.The main assumptions used in the calculation are (1) at each radius the particles are all assumed to have the same size (which, of course, varies in time), (2) all collisions between particles lead to coagulation, (3) in disk regions with temperature exceeding T_{evap} all water is in the form of vapour and evolves at the same radial velocity as the gas component, (4) when at a given radius the disk temperature falls below T_{evap}, all of the local vapour condenses immediately into grains with sizes a_{min} (the minimum size of a grain allowed by the model), (5) the systematic radial velocity of grains is entirely determined by the effects of gas drag, and (6) the evolution of solids does not affect the evolution of the density or the temperature of the gas.

With the above approximations, the evolution of solids is governed by a set of two equations. The first is the standard continuity equation for Σ_s, the surface density of solids. The second can be viewed as the continuity equation for the size-weighted surface density of solids $\Sigma_a(r) \equiv a(r)\Sigma_s(r)$, where $a(r)$ is the radius of particles at a distance r from the star. The set is solved numerically on a moving grid whose outer boundary follows the motion of the outer edge of the dust disk. Further details of the code are given in Kornet et al. (2001).

3. A GRID OF MODELS

The formalism described in §2 is so efficient that it allows for a broad search through plausible initial conditions. The initial conditions are parametrized by the quantities m_0 (the mass of the disk in $M_\odot$), and j_0 (the total angular momentum of the disk in units of 10^{52} g cm^2 s^{-1}). Once m_0 and j_0 are specified, the analytic solution of Stepinski (1998) gives the gas surface density Σ_g as a function of radius at $t = 0$. At the beginning of the evolution the ratio Σ_s/Σ_g and the particle radius a are independent of r (values of 0.01 and $a = a_{min} = 10^{-3}$ cm are adopted, respectively). We checked that the results are not sensitive to the adopted value of a_{min} (see Kornet et al. 2001). The sampled region of the parameter space was confined by the minimum

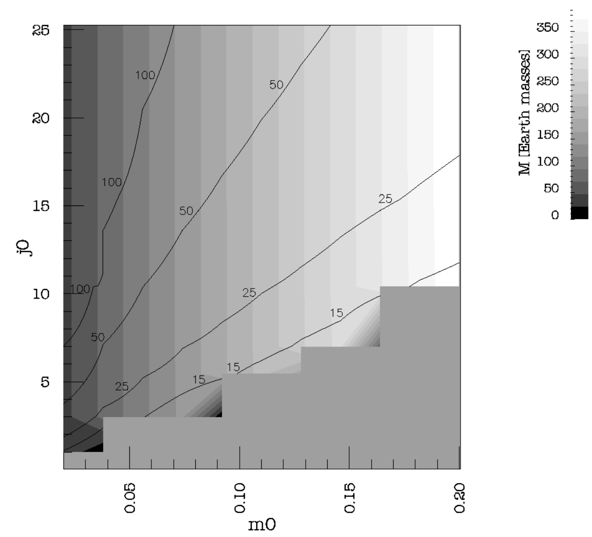

Fig. 1. Final mass and outer radius of the solid disk, as functions of the initial disk mass m_0 (in solar masses) and angular momentum j_0 (in units of 10^{52} g cm^2 s^{-1}). The contours give the outer radius in AU, and the grey scale gives the mass in $M_\oplus$. The grey region at the lower right indicates disks in which the solid component has completely accreted onto the star. The viscosity parameter α is equal to 0.01.

and maximum values of j_0 and m_0 as indicated by observations (Fig. 1). We also varied the viscosity parameter α, which characterized the turbulence in the gaseous component, and influenced the disk evolution rate (four values ranging from 10^{-4} to 10^{-1} were used). Finally, we varied the composition of solids, which in our approach was directly related to their bulk density and evaporation temperature (three species: water ice, low-temperature silicates, and high-temperature silicates were considered).

Our most important finding is that the diversity of planetary systems emerges naturally from the process of the evolution of solids in protoplanetary disks of diverse initial configurations. In other words, if protoplanetary disks form in a variety of masses and sizes, this variation alone leads to the formation of extremely different planetary systems. In particular, we have found that for certain initial configurations of protoplanetary disks, the planetesimal swarm, and thus the planetary system, does not form at all. These configurations are characterized, in general, by small specific angular momentum (small initial sizes relative to their initial masses). In these disks solids migrate into the evaporation zone where they are destroyed and accreted onto the star before they have time to grow and stop their inward motion.

The size of the no-swarm domain of initial conditions depends on the species of solids and on the timescale of disk evolution (related to the viscosity parameter α). Configurations with large values of specific angular momentum (large initial sizes with respect to their masses) tend to redistribute solids during their evolution without losing them to the star. The residual swarms have relatively large masses. Finally, configurations with intermediate values of specific angular momentum lose a fraction of solids during their evolutions, producing less massive swarms. Statistical analysis of astronomical observations may, in principle, establish relative frequencies of different types of initial conditions. Coupled to our model, it may predict relative frequencies of different sorts of potential planetary systems.

4. AN ALTERNATIVE LOOK AT THE SNOWLINE

In the present literature, the snowline is defined as the location where the temperature of the disk is equal to the sublimation temperature of water-ice. The surface density of solids increases rapidly beyond the snowline because water-ice, the most abundant species of solids, becomes available, and its contribution dominates the value of Σ_s. In the core accretion - gas capture scenario of giant planet formation high values of Σ_s are necessary to produce solid cores on time scales consistent with the presence of a gaseous nebula. Thus, at least in such a scenario, the importance of the snowline derives from the notion that it marks the inner edge of the giant planet formation zone.

However, the work by Stepinski & Valageas (1997), recently confirmed by more detailed calculations of Weidenschilling (2003), indicates that due to coagulation and sedimentation the solid particles grow on a time scale that is short in comparison to the lifetime of the disk. As they grow, they drift toward the star, and, having attained $\sim$ 1 km sizes they settle onto fixed Keplerian orbits. The ability of the solids to stop migrating inwards allows for a situation in which the location of the snowline, R_{sl}, does not coincide with the sublimation radius, R_{evap}. We find that the final difference between R_{sl} and R_{evap} may be truly substantial (Fig. 2), and we show that a zone of high surface density of solids, in which giant planets may form, results from the global redistribution of solids rather than from the thermal structure of the disk.

The final location of the snowline, R_{sl}^f, depends on the initial parameters of the disk and on the value of α. Specifically, R_{sl}^f is equal to the value of the evaporation radius at the moment when solids at

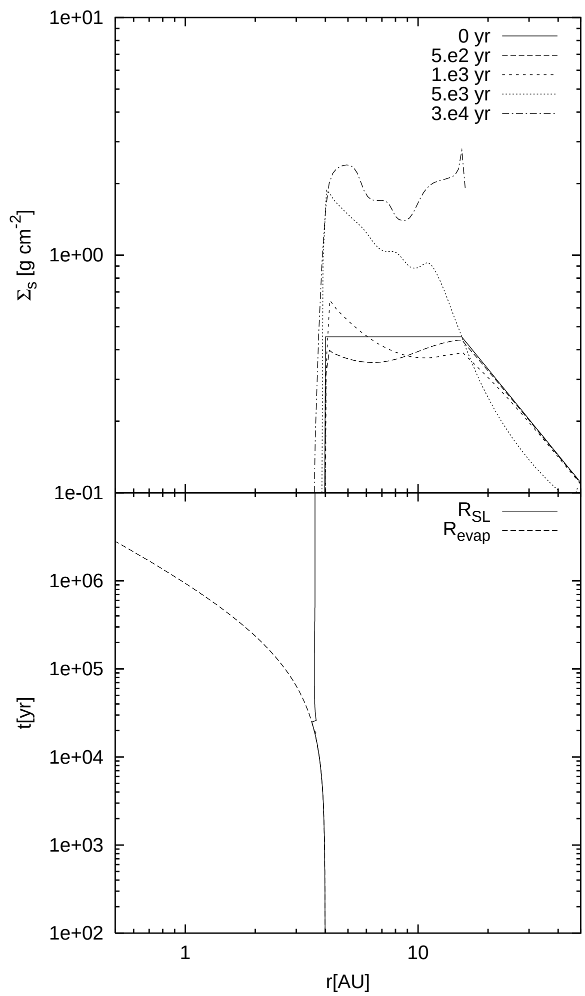

Fig. 2. The evolution of solids in an $\alpha = 0.1$ disk with $m_0 = 0.02$ and $j_0 = 1$. Top: surface density of solids at the times indicated in the frame (solids that have sublimated but not yet accreted onto the star are not taken into account). Bottom: locations of the sublimation limit (dashed) and snowline (solid) as a function of time. The snowline is defined as the inner edge of the region where ice grains are present.

R_{evap} are so large that they settle into Keplerian orbits. Thus, it is determined by a complicated interplay between coagulation, sedimentation and radial drift of the solids, and the local properties of the gas. Beyond R_{sl}^f the disk is enriched in solids due to the drift of solids from still larger radii. The relatively high surface density of solid material is achieved naturally, as a result of solids decoupling from the gas.

5. THE FORMATION OF PLANETS IN THE 47 UMA SYSTEM

The formation of the inner giant planet in the 47 UMa system has been studied by Bodenheimer et al. (2000) under the assumption that it formed *in situ* by the core accretion – gas capture process. They found that *in situ* formation is possible, but the surface density of the protoplanetary disk must be about 20 times higher than that of the minimum mass solar nebula. Such a disk is likely to be gravitationally unstable at larger radii, most probably leading to the formation of a much more massive planet at a distance of about 10 AU. Also, the required $\dot{M}$ was much higher than the values of $\sim 10^{-8}$ $M_\odot$ yr^{-1} in typical observed disks around young stars.

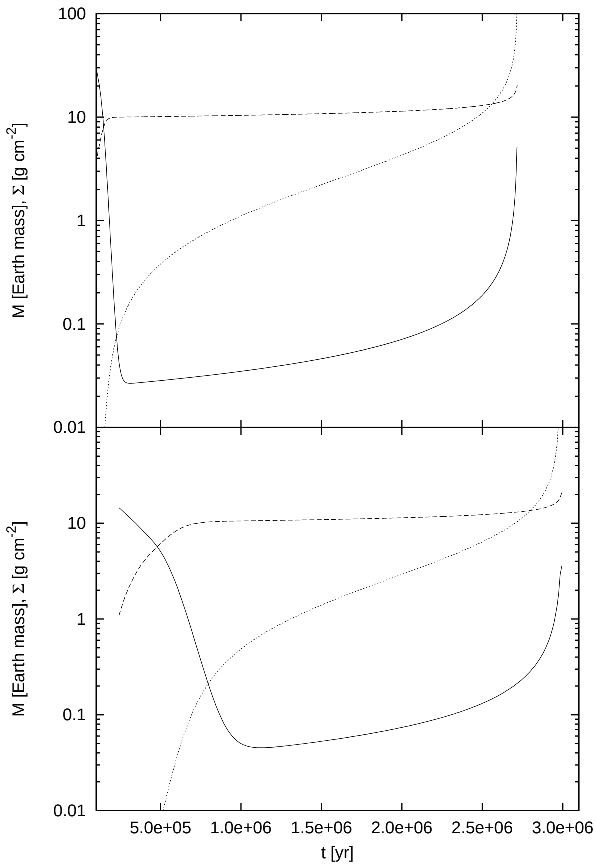

Fig. 3. Formation phase of the inner (top) and outer (bottom) planet in 47 UMa. Dashed, dotted, and solid lines indicate, respectively, core mass in $M_\oplus$, envelope mass in $M_\oplus$, and solid surface density in g cm^{-2} remaining in the feeding zone of the planet, all as a function of time in years (counted from the beginning of the disk evolution).

The basic assumption of their disk model is that the ratio of gas to solids does not change in space or time, which, as we indicated above, in most cases is not true. Here we consider alternate disk models, in which the evolution of gas and solids is not necessarily coupled (see also Kornet et al. 2002). These models start with a uniform, solar ratio of solids to gas, and evolve for 10^7 yr, following the buildup of initially small particles up to the size of a few km. Also, we take into account the outer planet, described by Fischer et al (2002).

Our results indicate that (1) The formation of planets in the 47 UMa system can proceed via the standard core accretion – gas capture process in a disk with reasonable parameters (mass 0.16 $M_\odot$ and outer initial radius 40 AU). (2) At their present distances from the star both planets can form in only about 3 Myr (Fig. 3). (3) The solid cores of both planets may be relatively small, 21 and 16 $M_\oplus$ for the inner and outer planet, respectively (for the inner planet, Bodenheimer et al. 2000, find a large core of 69 $M_\oplus$, which seems far too high if the structure of 47 UMa planets is similar to the structure of Jupiter). (4) As a result of the radial drift of solids with respect to the gas, the surface density of solids at 2–4 AU can be considerably higher than in the minimum mass solar nebula. This is the main factor enabling the formation of planets in 47 UMa.

M.R. and K.K. benefited from the European Commission RTN grant No. HPRN-CT-2002-00308. The work of P.B. is supported in part by a NASA grant NAG5-9661.

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