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THE STELLAR MASS AND ANGULAR MOMENTUM PROBLEM IN STAR FORMATION

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RESUMEN

En esta contribución describo algunas ideas viejas y nuevas que explican la relación log-normal de ambas: la función inicial de masa (IMF) y la distribución de separación del semi-eje mayor de estrellas binarias de baja masa. Resalto los artículos clásicos de Peter Bodenheimer, encaminados hacia la solución de ambos problemas, basados en la fragmentación rotacional jerárquica de una nube de gas colapsante. Describo también modificaciones más recientes y que por ende involucran a la fragmentación bajo una turbulencia supersónica.

ABSTRACT

In this contribution I describe some old and new ideas explaining the log-normal shapes of both the Stellar Initial Mass Function (IMF) and the semi-major axis separation distribution of low-mass binary systems. I highlight the classical papers of Peter Bodenheimer towards the solution of both problems, based on the rotational hierarchical fragmentation of a collapsing gas cloud. Modern modifications thereof involving supersonic turbulent fragmentation are also described.

Key Words: **STARS: BINARIES: GENERAL — STARS: FORMATION — STARS: LUMINOSITY, MASS FUNCTION — STELLAR DYNAMICS**

1. PERSONAL INTRODUCTION

I first met Peter Bodenheimer at the Max-Planck-Institut für Physik und Astrophysik in Munich/Freimann in 1978, in the second year of my PhD. Peter had a huge influence on my research in star formation and my first models of the stellar Initial Mass Function (IMF), the topic of my thesis. In particular, Peter's 1978 ApJ paper on the evolution/fragmentation of rotating interstellar clouds left a lasting impact. In this paper, Peter described a scenario of hierarchical fragmentation of a collapsing massive gas cloud into successively smaller pieces based on the transfer of spin angular momentum into orbital motion at each fragmentation stage ("ring cascade"). The initial conditions in the cloud led to final fragments which in many cases had the masses and angular momenta appropriate to observed main-sequence binary and multiple systems. Thus, in this classic paper Peter tried to kill two birds with one stone, solving both the mass and angular momentum problem in star formation together!

When Peter gave me a preprint of his paper and discussed it with me, I quickly realized how to generalize his fragmentation scheme into a theoretical model of the IMF by allowing his fixed branching ratios of mass and angular momentum (10 %) to take on more random values (5 – 40 %) at each step; see Fig. 1 & 2. This simple change then led to a Monte Carlo model of the origin of the observed log-normal IMF (Miller and Scalo 1979) which at the same time had the potential of explaining the frequency distribution of orbital angular momenta or semi-major axes of the resulting binary systems (Abt and Levy 1976). Indeed, the explicit formulation of that idea became one of the pillars of my PhD thesis submitted in 1981. The corresponding paper "A statistical theory of the log-normal IMF" was finally published in Monthly Notices a few years later (Zinnecker 1984).

2. IMF AND CENTRAL LIMIT THEOREM

A similar Monte Carlo model of the log-normal IMF was also proposed by Elmegreen and Mathieu (1983) without resorting to a specific physical picture of fragmentation process, but refining Larson's (1973) original crude probabilistic (analytical) scheme of hierarchical fragmentation. Both, the Zinnecker (1981, 1984) and the Elmegreen and Mathieu (1983) Monte Carlo simulations of hierarchical fragmentation and the IMF had in common the application of the central limit theorem where the fragment mass function always converged to a log-normal shape after only 4 or 5 fragmentation events, regardless of the functional form for the distribution of the stochastic variables.

3. STAR FORMATION AS A RANDOM MULTIPLICATIVE PROCESS

A random hierarchical process for star formation is but a special case of a random multiplicative pro-

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cess which in itself is more general. The latter would not only include a temporal sequence of fragmentation steps but would also incorporate simultaneous influences which represent the initial conditions for the onset of the fragmentation and condensations process. Thus, another set of random variables in the multiplicative process which may affect the formation of a particular stellar mass is the mixture of initial conditions in the cloud, such as gas density and gas temperature, specific angular momentum, strength of turbulence, mass-to-magnetic flux ratio as well as the geometry of the incipient condensation. Although some of the initial conditions may not be statistically independent, we may assume approximate factorization. In the simplest case, the stellar mass would be given by the thermal Jeans mass, which factorizes exactly into functions of gas density and gas temperature. Indeed, for the observed density range of 10^3 to 10^6 cm^{-3} and temperature range of 10 to 50 K in molecular clouds, Jeans masses of 0.1 to 100 solar masses are obtained, i.e. exactly the observed range of stellar masses. Probably, however, things are not that simple, and the mixture of quasi-independent initial conditions (which enter the random multiplicative process) may consist of several (of order 5) components, including the ones listed above. If so, the central limit theorem again applies, and the resulting stellar mass distribution will be Gaussian in the log of the mass, i.e. log-normal (Zinnecker 1985, Les Houches).

The story of the log-normal IMF continues: 10 years later, Adams and Fatuzzo (1996) developped yet another log-normal IMF model, due to the central limit theorem, based on a theory of stellar masses self-regulated by bipolar outflows (the random variables being the mass accretion rate and the accretion timescale). Finally, mention must be made of the recent simulations of turbulent fragmentation and the prediction of a log-normal IMF by Padoan and Nordlund (2002). According to their theory, multiple shocks due to supersonic turbulent flows generate a log-normal probability distribution for the gas density and also for the Jeans mass (assuming a constant isothermal gas temperature). Whether bipolar outflows or turbulent flows can solve the angular momentum problem in star formation remains to be seen (Edwards 1994). Of course, none of these theories addresses the question of how to solve the the third problem of star formation, the magnetic flux problem (only magnetically supercritical cores can collapse and form stars); cf. Appenzeller 1982, Shu et al. 2004.

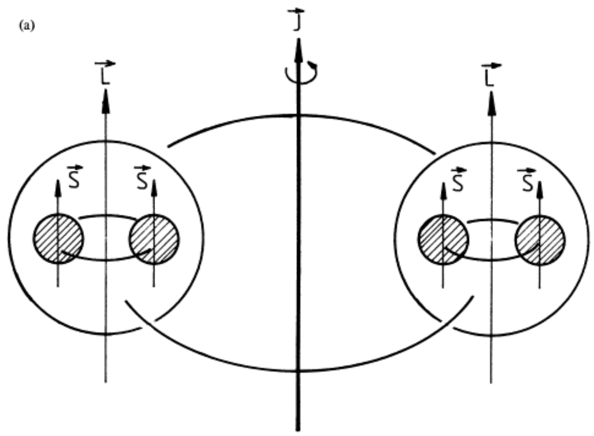

Fig. 1. Schematic illustration of Bodenheimer's (1978) ring fragmentation cascade (successive conversion of spin to orbital angular momentum).

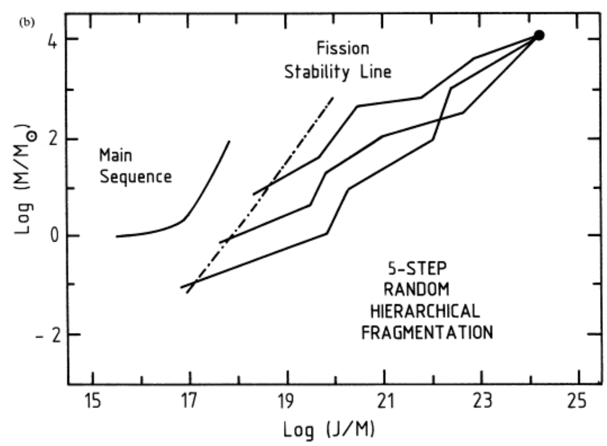

Fig. 2. Random walk in mass vs. specific angular momentum space (from Zinnecker 1984, based on Bodenheimer 1978, his Fig. 2).

4. ANGULAR MOMENTUM PROBLEM

In the mid-1990's, Peter in an article for ARA&A reviewed the angular momentum problem in star formation (Bodenheimer 1995). In Table 1 of this review, characteristic values of angular momenta per unit mass (specific A.M.) of observed molecular clouds (10^{23} cm^2 s^{-1}, scale 1 pc) and dense core (10^{21} cm^2 s^{-1}, scale 0.1 pc) were compared with those of rapidly spinning T Tauri stars ($\sim 10^{18}$ cm^2 s^{-1}) and the orbital motion of typical binary systems ($\sim 10^{20}$ cm^2 s^{-1}). Obviously, an efficient mechanism to reduce the specific A.M. during the molecular cloud fragmentation process is needed. If specific A.M. were strictly conserved during cloud collapse, only very wide visual binary systems with correspondingly large orbital A.M. could form (e.g.

Mouschovias 1977).

One such mechanism is the successive step-by-step conversion of spin to orbital A.M. in hierarchical fragmentation (as discussed above). Another is "magnetic braking" (for a review, see Mouschovias 1991). Magnetic fields threading molecular clouds can effectively exert a torque on the rotating neutral gas, as long as the magnetic field is "frozen" and the magnetic flux is conserved during contraction. However, at high enough gas density flux freezing will break down, due to an insufficient degree of ionization in the cloud. Then magnetic braking ceases, leaving behind dense pre-stellar cloud cores whose specific spin A.M. still exceeds the orbital A.M. of most binary stars. Indeed, in a careful study, Simon et al. (1995, their Fig. 8) compared the distributions of specific A.M. of rotating dense ammonia cores in Taurus with a representative sample of pre-Main Sequence binary systems in Taurus, noting that there is some overlap (the spin A.M. of the most slowly rotating cores matches the orbital A.M. of the widest binary systems). However, it must also be noted that the average observed spin A.M. of dense cores is about a factor of 10 higher than the median orbital A.M. of the T Tauri binaries (see the review of Mathieu 1994). Something remains to be explained here! This was the motivation for a recent paper in which we tried to relate the separation distribution of the binary population to the properties of star-forming cores (Sterzik, Durisen, and Zinnecker 2003).

5. THE SEMI-MAJOR AXIS DISTRIBUTION OF LOW-MASS BINARY SYSTEMS

To some extent, the problem was already realized by Burkert & Bodenheimer (2001) and Kroupa & Burkert (2001). The former paper suggested that the total spin A.M. of dense cores can be overestimated, if the turbulent nature of the velocities inside the cores are misinterpreted as systematic rotational motion. The latter paper took a different approach, investigating whether it is possible to broaden an initially narrow period or semi-major axis distribution by stellar dynamical interactions in a very young and compact cluster, but concluded it does not work under any circumstances. On the other hand, Larson (1997) had pointed out that it may work if the role of dynamical friction and gas drag are taken into account.

Our own paper, cited above, proposes yet another model for the observed broad log-normal period or semi-major axis distribution of binary and multiple G and K-tpye main sequence stars (Duquennoy & Mayor 1991, Eggenberger et al. 2003). This model is based on the picture of gravo-turbulent fragmentation (MacLow & Klessen 2004) where gravitionally supercritical cores are swept together in convergent supersonic turbulent flows. This means that the typical outcome of supersonic turbulent compression is a prompt condensation containing several thermal Jeans masses (rather than only one or two), thus leading to small N clusters (N = 3,4,5, ...) as envisaged in the work of Clarke, Pringle, and collaborators. We note that the number of thermal Jeans masses is given by $N = 1/(2\alpha_o)$ where $\alpha_o = E_{thermal}/E_{grav}$ is the initial ratio of thermal to gravitational energy at the onset of collapse. Now, starting from initial conditions with a number of Jeans masses ($\alpha_o \sim 0.1$) implies a rapid nearly pressure-free collapse (free-fall time of the order of 1000 years), stopped only by rapid rotation (centrifugal barrier) at high gas densities (of the order of 100,000 times the prompt initial gas density, but still in the isothermal regime). As Tohline (2002) has reminded us, in such a situation density perturbations can grow on a quasi-stationary background and several objects are likely to form, with separations of the order of the centrifugal radius $R_c = \beta_o R_o$, where R_o is the initial radius of the collapsing configuration and $\beta_o = Peter = E_{rot}/E_{grav}$ is the ratio of rotational to gravitational energy of the initial turbulent fragment (typically $\beta_o = 0.02$, corresponding to a centrifugal shrink factor of 50 and a mean density increase of $50^3 = 125,000$).

While R_c is still a factor of 10 too large compared to the peak (40 AU) of the log-normal binary separation distribution, chaotic N-body interactions will lead to a final binary system or hierarchical triple with typical binary component separations decreasing by the required factor of 10 w.r.t. the initial size of the small N system, as numerical simulations confirm. This is due to dynamical hardening connected to the ejection of the odd member of the initial multiple system (cf. Reipurth & Clarke 2001). We then postulate that the occurrence of a wide range of binary separations results from the chaotic gravitational interactions in young multiple systems (see Fig. 3). The widest binaries form from N = 2 initial conditions, as there is no way to harden these binaries. N = 1 conditions may occur too (rarely), but most single stars should be the ejecta from a few-body system at birth.

How to make very close (spectroscopic) binaries? This is a long-standing unsolved problem (e.g. Mathieu 1994). Fission of rapidly rotating protostars is unlikely (Tohline & Durisen 2001), but disk fragmentation of the second collapsing embryonic core trig-

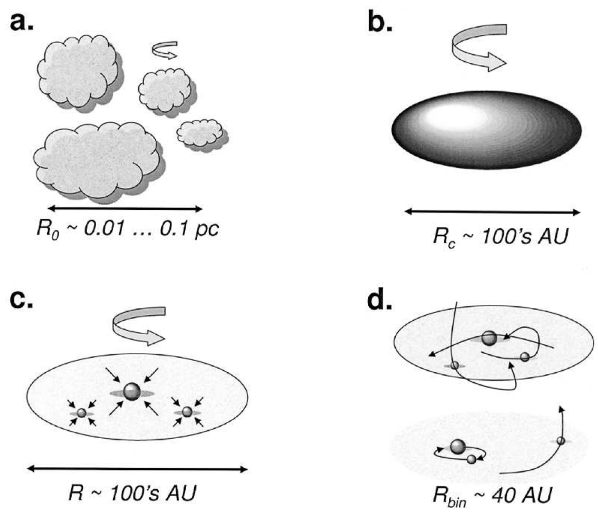

Fig. 3. Evolutionary stages from dense molecular cloud cores produced by prompt turbulent fragmentation towards final binary and multiple stellar systems. Typical system scales are indicated (from Sterzik et al. 2003).

gered by molecular hydrogen dissociation may work (Bonnell 2001). In our model (Sterzik et al. 2003), the key process is the Kozai mechanism. Kozai (1962) found that the gravitational interaction of a third body in polar hierarchical orbit around a normal wide binary drives the eccentricity of that binary towards unity, which causes tidal dissipation of orbital energy and orbital A.M. at peri-astron, thus a secular circularization of the orbit and a dramatic reduction of the orbital period (Kiseleva et al. 1998).

Let me finish this discussion about binary star formation and component separations by returning to the central limit theorem. It is possible that the stochastic distribution of the initial collapse parameters α_o and β_o (first introduced by Black & Bodenheimer 1976 and Larson 1978) are wide enough to account for the fragmentation into a wide variety of binary and multiple systems; the broad log-normal semi-major axis distribution may thus be yet another manifestation of the powerful central limit theorem.

I thank Peter Bodenheimer for his inspiration and friendship over the years and wish him all the best for the future.

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