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THE EXO-PLANETARY SYSTEM OF 55 CANCRI AND THE TITIUS-BODE LAW

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RESUMEN

El reciente descubrimiento de un quinto planeta ligado a 55 Cancri (Fischer et al. 2007) nos ha motivado a investigar si este sistema exo-planetario se ajusta a alguna forma de la ley de Titius-Bode (TB). Encontramos que una simple relación TB exponencial reproduce muy bien los cinco semiejes mayores observados siempre y cuando se asigne el número 6 al planeta con el semieje más grande. Esta forma de contar deja un vacío en la posición $n = 5$, una situación curiosamente reminiscente a la ley TB en nuestro propio sistema planetario, antes del descubrimiento de Ceres. La aplicación de una ley TB exponencial a 55 Cancri nos permite predecir la existencia de un planeta con $a \approx 2.0$ UA y con un periodo de $P \approx 1130$ días localizado en la gran brecha entre $a = 0.781$ UA ($P = 260$ días) y $a = 5.77$ UA ($P = 5218$ días) correspondientes a los dos más grandes periodos observados. Con menos certeza, también predecimos un séptimo planeta en $a \approx 15$ UA con $P \approx 62$ años.

ABSTRACT

The recent discovery of a fifth planet bound to 55 Cancri (Fischer et. al 2007) motivated us to investigate if this exo-planetary system fits some form of the Titius-Bode (TB) law. We found that a simple exponential TB relation reproduces very well the five observed major semi-axis, provided we assign the orbital $n = 6$ to the largest a. This way of counting leaves empty the position $n = 5$, a situation curiously reminiscent of TB law in our planetary system, before the discovery of Ceres. The application of an exponential TB relation to 55 Cancri allows us to predict the existence of a planet at $a \approx 2.0$ AU with a period of $P \approx 1130$ days located within the large gap between $a = 0.781$ AU ($P = 260$ days) and $a = 5.77$ AU ($P = 5218$ days). With less certainty, we also predict a seventh planet at $a \approx 15$ AU, with $P \approx 62$ years.

Key Words: planetary systems — planets and satellites: general — stars: individual (55 Cancri)

1. INTRODUCTION

The two hundred year old saga of the Titius-Bode law is well known (see Nieto 1972 for a well documented review). Since the discovery by Bode, that the Titius relation "predicted" the major semi-axis of Uranus, a frantic search for "the lost planet" at position $n = 5$ was initiated at various european observatories. The discovery of Ceres by Piazzi on the night of January 1st, 1801, with the major semi-axis predicted by TB, and the fact that Thebe, with the Galilean satellites of Jupiter, as well as the more massive satellites of Saturn also follow a TB relation, initiated a debate about the meaning of TB which is still alive nowadays. Is the TB law a matter of chance (Lynch 2003; Dubrulle & Graner 1994; Neslušan 2004)? Is it consequence of the early physical conditions in the protoplanetary disk (Graner & Dubrulle 1994)? Is it a reflection of a process of dynamical relaxation in a system of planets subject to their mutual gravitational perturbation (Hayes & Tremaine 1998; Hills 1970; Ovenden 1975)?

TABLE 1

PARAMETERS OF 55 CANCRI

Apparent visual magnitude	5.96
Hipparcos parallax	79.8 ± 0.84 mas
Distance	12.5 ± 0.13 parsecs
Absolute visual magnitude	5.47
Effective Temperature	5234 ± 30K
Rotation velocity $v \sin i$	2.4 ± 0.5 km s^{-1}
Luminosity	0.6 $L_\odot$
Spectral type	G8V/K0V
Mass	0.94 ± 0.05 $M_\odot$

Because of the previous considerations and in view of the growing number of multiple exo-planetary systems, we decided to check whether the 55 Cancri system, for which a fifth planet has been recently announced (Fischer et al. 2007), follows the TB law.

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TABLE 2

OBSERVED PROPERTIES OF THE 55 CANCRI EXO-PLANETARY SYSTEM*

n	1	2	3	4	5
Year of discovery	2004	1996	2002 (1)	2007	2002 (2)
Observed semiaxis major (AU)	0.038 $\pm 1.0 \times 10^{-6}$	0.115 $\pm 1.1 \times 10^{-6}$	0.24 $\pm 4.5 \times 10^{-5}$	0.781 ± 0.007	5.77 ± 0.11
Period (days)	2.817 $\pm 1 \times 10^{-4}$	14.651 $\pm 7 \times 10^{-4}$	44.344 $\pm 7 \times 10^{-3}$	260 ± 1.1	5218 ± 230
$M \sin i$ (Jovian masses)	0.034 ± 0.0036	0.824 ± 0.007	0.169 ± 0.008	0.144 ± 0.04	3.835 ± 0.08

*Taken from Fischer et al. 2007.

The traditional TB relation is essentially a geometric progression in the number n, the running number of a planet according to its distance to the central star. This geometric relation can be represented by an exponential in n.

We tried to represent our planetary system by an exponential in n and found a good fit. Having verified that an exponential fit for the Solar System was a good approximation we tried to represent the 55 Cancri system also by an exponential TB. The exponential fit to the first 5 major semi-axes taken from Table 2 do not seem to support a TB relation for the planets in 55 Cancri; however when we take the largest major semi-axis to be $n = 6$ then the corresponding fit is very good. The vacancy left at $n = 5$ leads us to propose the existence of a new planet with a major semi-axis $a \approx 2$ AU.

2. THE 55 CANCRI SYSTEM

The star 55 Cancri (55 Cnc = HD 75732 = HR3522 = HIP 43587 = GJ 324A) is a well observed nearby star, with a common proper motion companion at 1550 AU of separation; in Table 1 we list some of its parameters taken from the paper by Fischer et al. (2007). In Table 2 the observed parameters for its planetary system are listed, including the year of discovery of each planet. Note in this table the enormous spacing between planets $n = 4$ and $n = 5$, whose major semi-axes are, respectively, less than 1 AU and more than 5 AU, and whose periods are 260 days and 5218 days.

3. THE TITIUS-BODE LAW

The equation

$$a = 0.4 + 0.3 \times 2^n \qquad (a \text{ in AU}) \tag{1}$$

represents the classical Titius-Bode law. Note the peculiar ordering system: Mercury corresponds to $n = -\infty$, Venus to $n = 0\ldots$

In equation (2)

$$a = 0.1912 e^{0.5594n} \tag{2}$$

we present our best exponential fit to the Solar System excluding Mercury and Pluto, but including Uranus and Neptune. Our exponential TB fit to the Solar System excludes Mercury because in the traditional TB relation not only it is given an orbital number $n = -\infty$, devoid of any physical meaning, but also the value of the constant (0.4) is arbitrarily chosen to give the approximately correct values of a for Mercury and the Earth. At the other end of our planetary system we exclude Pluto not only because of its pathological orbit, but also because we do not know if it is an object from the Kuiper belt, captured into the region of the outer planets, or a satellite ejected from Neptune; in any case its present orbit has followed a dynamical evolution different from that of the rest of the planets.

In Figure 1 we plot equation (2), as well as the observed values of the major semi-axis for all the planets. Note that although equation (2) does not include the value of a for Mercury, it predicts orbitals close to the observed ones up to Neptune, $n = 9$. The traditional TB relation gave a good fit up to Uranus, but a very poor one for Neptune. Our exponential TB relation gives a poor representation for Uranus, but a good one for Neptune.

4. THE TITIUS-BODE LAW AND THE 55 CANCRI EXO-PLANETARY SYSTEM

The success of our exponential TB fit to the Solar System led us to try an exponential fit to the major semi-axis of the five planets as listed in Table 2; our

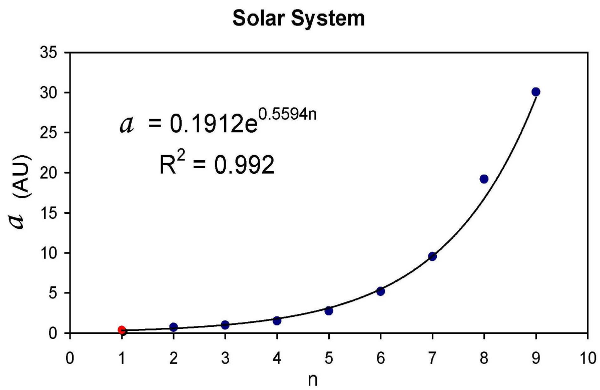

Fig. 1. An exponential TB fit to the Solar System. This fit includes major semi-axis from Venus ($n = 2$) up to Neptune ($n = 9$). Note that the extrapolation of equation to $n = 1$, gives $a(1) = 0.335$ AU, close to the observed value $a = 0.387$.

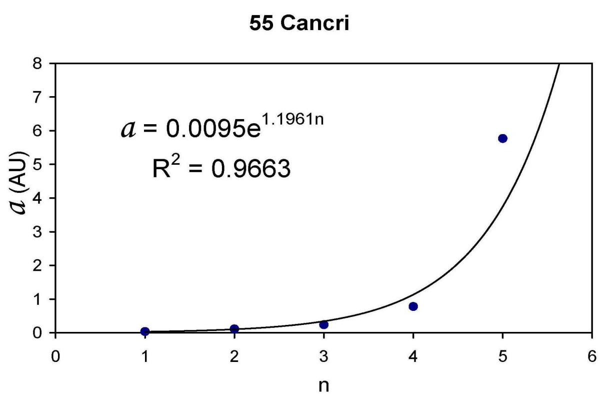

Fig. 2. Exponential TB fit to the 5 major semi-axis of the 55 Cancri system, taken from Table 2.

first fit is shown in Figure 2 which, as can be seen, does not offer a very convincing case for the extension of TB law to 55 Cancri, in fact planets $n = 4$ and $n = 5$ deviate significantly from the exponential fit; fortunately we were not discouraged by this result so we tried another TB exponential fit only to the first four planets, which have very small error in their major semi-axis, Figure 3 shows this second fit. In this case we find a remarkably good TB representation of the first 4 planets with a correlation coefficient of R^2=0.99. However, the fifth planet lies very far from this 4-planet TB exponential, but very close to the extrapolation when we assign to it $n = 6$, as can be seen in Figure 3. This convinced us that the 55 Cancri system may follow TB law, but if this is the case then a third fit to the five planet is in order but now taking the largest a as $n = 6$. Figure 4 shows this fit; it is given by

$$a = 0.0142e^{0.9975n} \quad (R^2 = 0.997) \tag{3}$$

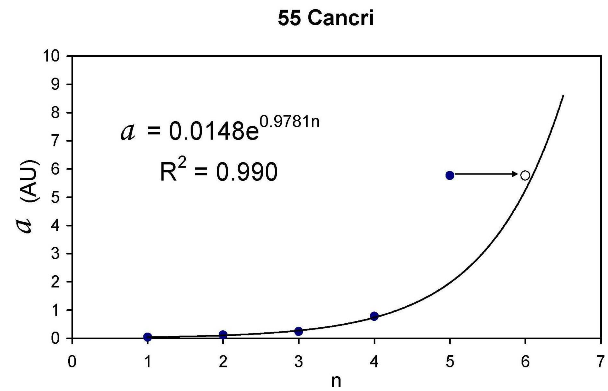

Fig. 3. An exponential TB fit to the four closest planets in 55 Cancri System. Note that the planet $n = 5$ ($a =$ 5.77 AU) deviates considerably from the 4 planet TB relation, however the extrapolation of this relation to $n = 6$ corresponds to a major semiaxis $a = 5.24$ AU, close to the observed one, $a = 5.77$ AU

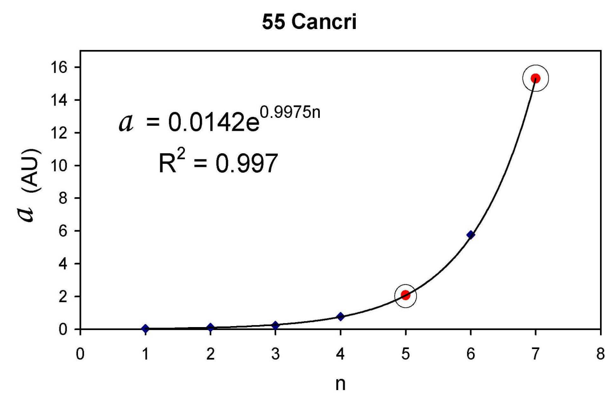

Fig. 4. The TB exponential fit to the 5 observed planets of 55 Cancri, where we count the farthest observed one as $n = 6$. Planets $n = 5$ and $n = 7$, predicted by TB, are shown as open circles.

Equation (3) is a remarkable confirmation that the 55 Cancri system follows a TB law if we are willing to accept that $n = 5$ corresponds to a planet not yet discovered, a situation curiously similar to the history of TB in the Solar System before the discovery of Ceres. In view of the success of TB law (Figure 4) and following the analogy with the Solar System we predict the existence of a sixth planet (asteroidal belt?) $n = 5$ (at $a \approx 2$ AU), and with less certainty we extrapolate equation (3) to predict yet another planet at $n = 7$ (at $a \approx 15$ AU)

In Table 3 we list the observed elements of the 55 Cancri system, the TB fit and the corresponding

TABLE 3

THE TITIUS-BODE FIT TO THE 55 CANCRI SYSTEM AND THE TWO NEW PLANETS

n	1	2	3	4	5	6	7
Observed semiaxis major (AU)	0.038 $\pm 1 \times 10^{-6}$	0.115 $\pm 1.1 \times 10^{-6}$	0.24 $\pm 4.5 \times 10^{-5}$	0.781 ± 0.007		5.77 ± 0.11	
Titius-Bode (AU)	0.039	0.104	0.283	0.768	**2.08**	5.643	**15.3**
Error %*	2.63	9.57	17.92	1.72		2.19	
Period (days)	2.817 $\pm 1 \times 10^{-4}$	14.651 $\pm 7 \times 10^{-4}$	44.344 $\pm 7 \times 10^{-3}$	260 ± 1.1	**1130**	5218 ± 230	**22530**

*(| a obs $-a$ (TB) | / a obs) $\times$ 100.

percentage errors, as well as the two new planets predicted by the Titius-Bode law.

5. CONCLUSIONS

In the present paper we show that the exponential Titius-Bode law holds for the 4 closest planets of 55 Cancri and that its extrapolation fits well the major semi-axis of the fifth planet, provided it is assumed that it occupies the sixth orbital.

The Titius-Bode law is valid for the exo-planetary system 55 Cancri, and may be valid for other exo-planetary systems as well.

Having found another planetary system where the Titius-Bode law is valid makes it rather unlikely that it is due to chance.

The Titius-Bode law allows us to predict two new planets for the 55 Cancri system:

$$a \approx 2.0 \text{ AU} \qquad P \approx 3.1 \text{ years}$$
$$a \approx 15.0 \text{ AU} \qquad P \approx 62 \text{ years}$$

The existence of two hot Jupiter-like planets ($n = 1, 2$) in this system opens the problem of how to understand the persistence of the Titius-Bode law against the phenomenon of planet migration.

The validity of TB for the 55 Cancri exo-planetary system does not yet help to understand the physics behind it. However, it may help to discover new planets by paying special attention to periodic signals in the radial velocities at values close to the predicted periods.

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REFERENCES

Dubrulle, B., & Graner, F. 1994, A&A, 282, 269
Fischer, D. A., et al. 2008, ApJ, 675, 790
Graner, F., & Dubrulle, B. 1994, A&A, 282, 262
Hayes, W., & Tremaine, S. 1998, Icarus, 135, 549
Hills, J. G. 1970, Nature, 225, 840
Lynch, P. 2003, MNRAS, 341, 1174
Neslušan, L. 2004, MNRAS, 351, 133
Nieto M. M. 1972, in The Titius-Bode Law of Planetary Distances: Its History and Theory (Oxford: Pergamon Press)
Ovenden, M. W. 1975, Vistas in Astronomy, 18, 473