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DYNAMICS OF THERMAL FRONTS

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RESUMEN

El modelo de fuente puntual de Zel'dovich et al. (1985) para flujos adelante y atrás de un frente de combustión se generaliza a frentes térmicos (de calor y fríos) arbitrarios que se propagan en plasmas no-homogéneos. En particular, se analizan soluciones auto-similares del tipo potencias.

ABSTRACT

The point source model proposed by Zel'dovich et al. (1985) for analyzing the flows ahead and behind a spherical flame front is generalized to study arbitrary thermal (heat and cooling) fronts propagating in nonhomogeneous plasmas. In particular, self-similar power-like solutions are analyzed.

Key Words: plasmas

1. INTRODUCTION

At advanced nonlinear stages of thermal instability in optically thin plasmas, a two-phases medium develops with an interface zone, where, in addition to strong temperature gradient (where the heat conduction goes into play), interchange of mass between the cold and hot phases occurs. Mass exchange between these two phases through evaporation of cold clouds and condensation of hot intercloud gas can play a key role in the global dynamics of the interstellar medium (McKee & Begelman 1990), in star formation (Ibáñez & Parravano 1983; Ibáñez & Bessega 2000), in thermonuclear flame-like fronts at late stage of evolution of Type Ia supernovae explosions (Bell et al. 2004), in dynamics of cooling flows (Churazov & Inogamov 2004), in tokamak plasmas (Tokar 2002). Therefore, the understanding of the dynamics of thermal fronts is an important problem in astrophysical as well as in laboratory plasmas. In the present work the point source model proposed by Zel'dovich et al. (1985) for analyzing the flows ahead and behind a spherical flame front is generalized to study arbitrary thermal fronts propagating in nonhomogeneous plasmas.

2. SELF-SIMILAR EQUATIONS

In spherical symmetry self-similarity demands that the physical variables can be written in the form

$$p = \rho_0(t)\dot{R}^2(t)f(\xi) \quad \rho = \rho_0(t)g(\xi) \quad u = \dot{R}(t)v(\xi)\,, \tag{1}$$

where $\dot{R}(t) = dR(t)/dt$ and $\xi = r/R(t)$. As usually, if one chooses $R(t), \rho_0(t)$ as the basic scales, then $\dot{R}(t)$ and $\rho_0(t)\dot{R}^2(t)$ can be taken as velocity and pressure scale, respectively. Therefore, the mass motion and energy equations reduce to

$$\frac{R}{\rho_0\dot{R}}\frac{d\rho_0}{dt} + \frac{dv}{d\xi} + (v-\xi)\frac{d}{d\xi}\ln g + 2\frac{v}{\xi} = 0\,, \tag{2}$$

$$\frac{R}{\dot{R}^2}\frac{d\dot{R}}{dt} + (v-\xi)\frac{dv}{d\xi} + \frac{1}{g}\frac{df}{d\xi} = 0\,, \tag{3}$$

$$\rho_0\frac{\dot{R}^2}{R}f[\frac{R}{\dot{R}}\frac{d}{dt}\ln\frac{\dot{R}^2}{\rho_0^{\gamma-1}} + (v-\xi)\frac{d}{d\xi}\ln\frac{f}{g^\gamma}] = (\gamma-1)Q\,, \tag{4}$$

where Q is the heat loss function.

2.1. *Heat Fronts*

For heat fronts $\alpha = \rho_1^0/\rho_2^0 > 1$, where ρ_1^0 and ρ_2^0 refer to the densities ahead and behind the heat front, respectively and assuming the Zel'dovich et al. solution ahead the spherical front, $u_1^0 = \alpha_1 u_b/\xi^2$, i.e., $\dot{R}(t) = u_b$, $v(\xi) = \alpha_1/\xi^2$ and $\alpha_1 = (\alpha - 1)/\alpha$ from equation (2) follows that $g(\xi) = 1 + \left[(1-\alpha_1)/(\xi^3-\alpha_1)\right]^{\beta/3}$ if $\beta \neq 0$and $g(\xi) = 1$ if $\beta = 0$. Additionally, $\rho_0(t) = \rho_*(t_*/t)^\beta$, where ρ_* is a constant. Note that when $\beta = 0$ (case considered in Zel'dovich et al. 1985) $\rho_1^0 = \rho_*$. The reason by which for $\beta > 0$, $g(\xi)$ is chosen of the above form is to assure that when $\xi \to \infty$ the density remains $\neq 0$, i.e., $\rho_\infty = \rho_*$, otherwise $\rho \to 0$ and $T \to \infty$ when $\xi \to \infty$ which is physically meaningless.

From the motion equation (3) one obtains the solution for $f(\xi)$ and the solution for the dimensionless pressure $(p_1^0 - p_\infty)/\rho_\infty u_b^2 = (t_*/t)^\beta f(\xi)$, where p_∞ and ρ_∞ are the gas pressure and density at $\xi \to \infty$, respectively.

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2.2. *Cooling Fronts*

The analysis made in the former subsection can be straightforwardly expanded to spherical cooling front. In consequence, condensation of the gas in the spherical cool front acts as a sink (instead of a source, as in the flame problem) for the gas at $r > R(t)$ (region 1 – the hot medium), the velocity can be written as

$$\mathbf{u}_1^0 = \nabla\varphi_1^0 = -\frac{1-\alpha}{\alpha} u_b^3 \frac{t^2}{r^2}\frac{\mathbf{r}}{r}, \quad r > R\,, \qquad (5)$$

therefore for cooling fronts $\alpha < 1$ and the flow is converging. On the other hand, from equation (5) follows that the speed of the flow ahead the cool front becomes $v(\xi) = -\alpha_1/\xi^2$ where $\alpha_1 = (1-\alpha)/\alpha$, i.e. as that corresponding to a flame front, but with a converging instead of diverging flow and stronger intensity. Henceforth, integrating the continuity equation, $g(\xi) = 1$ if $\beta = 0$ and $g(\xi) = 1 + (\alpha_1 + \xi^3)^{-}\beta/3$ if $\beta \neq 0$ and from the motion equation (3) one obtains the quadrature for $f\,(\xi)$ as well as the dimensionless presure $p_1^0/\rho_* u_b^2$.

Figure 1a are plots of the dimensionless pressure as functions of ξ for cooling fronts with $\beta = 0$ and with front compressions $\alpha = 10^{-2}$ (solid lines), 5×10^{-2} (dashed lines), 0.1 (dashed-point lines) and 0.5 (dotted lines). Contrary to the heat front, where the pressure increases when $\xi \to 1$, at the cooling front a deep well of pressure is formed close to the boundary $\xi = 1$ which produces the converging flow. The corresponding temperature also decays towards $\xi \to 1$. Due to the fact that the density is a constant when $\beta = 0$, it is clear that the temperature profile follows that of the pressure (Figure 1a). Figure 1b corresponds to the distribution of temperature when $\beta = 1$. The depth of the well of pressure as well as the temperature increases with α. In conclusion, the point source model proposed by Zel'dovich et al. (1985) for analyzing the flows ahead and behind a spherical flame front is generalized to study arbitrary thermal fronts propagating in nonhomogeneous plasmas. The pressure ahead of heat fronts in non-homogeneous plasmas shows similar behavior as in the Zel'dovich model. However, for densities decaying from the front, a maximum of the temperature ahead (but close to) the front appears. Cooling fronts instead, generate gas sinks leaving a deep pressure (and temperate) well with minimum pressure (and temperature) at the cooling front. The above qualitative results hold, regardless the self-similar power or rates values.

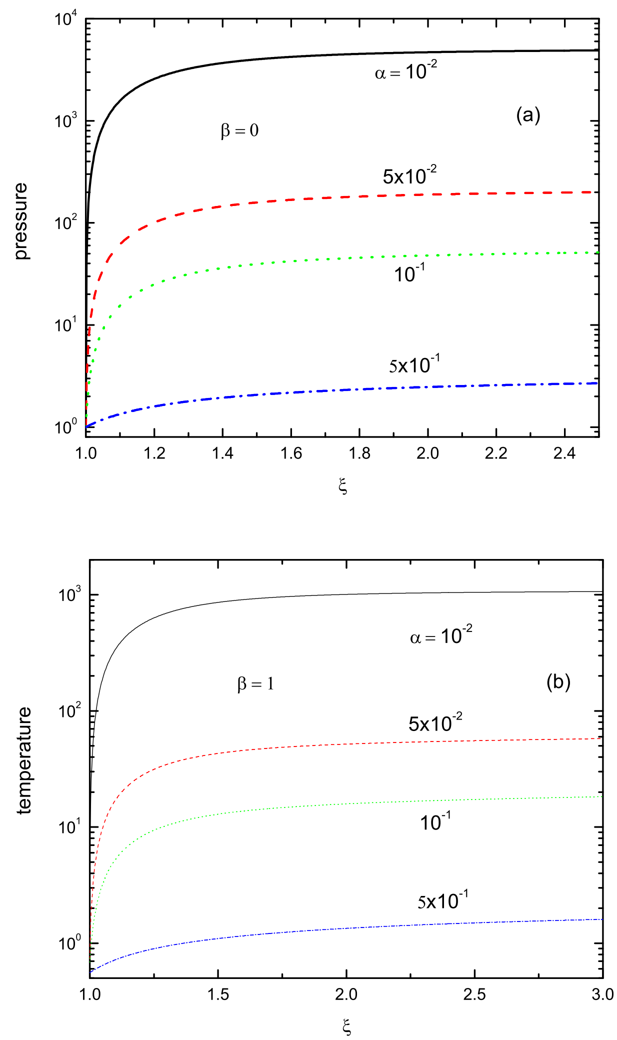

Fig. 1. The dimensionless pressure (a) and temperature (b) as functions of ξ for $\beta = 0$ and $\beta = 1$, respectivelly, for cooling fronts with different α values.

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